2023 RISP International Workshop on Nonlinear Circuits, Communications and Signal Processing (RISP NCSP 2023), Feb. 28 - Mar. 3, 2023

Synchronization of Bi-Directionally Coupled Chaotic Circuits with Non-Uniform Coupling Strength

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Abstract

In this study, we investigate the synchronization of bi-directionally coupled chaotic circuits with non-uniform coupling strength in proposed model. First, computer simulation of two chaotic circuits is performed. Then, the number of circuits is increased to see how it affects the synchronization phenomena. As a result, synchronization phenomena are also observed when the coupling strength is non-uniform. The number of circuits that are synchronized is related to the coupling strength that can keep the synchronization phenomena.

1. Introduction

Synchronization phenomena are commonly observed in nature, such as the resonance of a metronome, the luminescence of a firefly, or the beating of a heart. In recent years, research has been conducted in various fields, such as synchronization of human brain networks [1] and communication networks [2]. Coupled oscillators are excellent for modeling nonlinear phenomena in nature, and synchronization is one such nonlinear phenomenon. In particular, we can confirm an interesting phenomenon called chaos synchronization, in which chaotic circuits that emit complex behaviors align their behaviors by coupling. Therefore, chaos synchronization in coupled chaotic circuits is often studied [3]-[5]. Among them, many studies have focused on the topology and coupling strength of bi-directionally connected networks. Those studies have shown that the coupling strength in both directions is equal. However, social networks such as human relationships do not always have equal bi-directional coupling strength.

Therefore, in this study, we focused on the bi-directional strength between chaotic circuits. We investigated the synchronization phenomenon when the bi-directional coupling strength is non-uniform, where the coupling strength in one direction is fixed and the coupling strength in the other direction is varied. In addition, a comparison is made with the case where the bi-directional coupling strength is varied simultaneously. Moreover, we also investigate the synchronization phenomenon when the number of chaotic circuits is changed.

2. System model

Figure 1 shows the chaotic circuit used in this study. The chaotic circuit consists of a negative resistor, two capacitors, an inductor, and a bidirectional diode. The chaotic circuit is called the Mori-Shinriki circuit [6], [7].

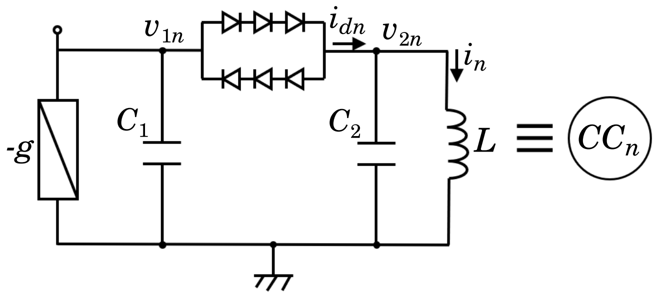

Figure 1: Circuit model.

The circuit equations of the chaotic circuit are shown as follows:

$$\begin{cases} L\dfrac{di_n}{dt} = v_{2n} \\ C_1\dfrac{dv_{1n}}{dt} = gv_{1n} - i_{dn} - \dfrac{1}{R}\displaystyle\sum_{k\in C_n}(v_{1n} - v_{1k}) \\ C_2\dfrac{dv_{2n}}{dt} = -i_n + i_{dn}, \end{cases} \quad (1)$$
$$(n = 1, 2, 3, 4),$$

The $i - v$ characteristic of nonlinear resistor consisting of the diodes is described as follows:

$$i_{dn} = \begin{cases} G_d(v_{1n} - v_{2n} - a) & (v_{1n} - v_{2n} > a) \\ 0 & (|v_{1n} - v_{2n}| \leq a) \\ G_d(v_{1n} - v_{2n} + a) & (v_{1n} - v_{2n} < -a). \end{cases} \quad (2)$$

By using the parameters and variables as follows:

$$
\begin{aligned}
&i_n = \sqrt{\frac{C_2}{L}} a x_n,\ v_{1n} = a y_n,\ v_{2n} = a z_n \\
&t = \sqrt{L C_2}\tau,\ \text{“}\cdot\text{”} = \frac{d}{d\tau},\ \alpha = \frac{C_2}{C_1} \\
&\beta = \sqrt{\frac{L}{C_2}} G_d,\ \gamma = \sqrt{\frac{L}{C_2}} g,\ \delta = \frac{1}{R}\sqrt{\frac{L}{C_2}}.
\end{aligned}
$$

The normalized circuit equations of the circuit are described as follows:

$$
\begin{cases}
\dot{x} = z_n \\
\dot{y} = \alpha\gamma y_n - \alpha\beta f(y_n - z_n) - \alpha\delta f(y_n - y_k) \\
\dot{z} = \beta f(y_n - z_n) - x_n
\end{cases}
\tag{3}
$$

Where the nonlinear function corresponding to the $i-v$ characteristic in Eq. (2) is described as follows:

$$
f(y_n - z_n) = \begin{cases}
y_n - z_n - 1 & (y_n - z_n > 1) \\
0 & (|y_n - z_n| \leq 1) \\
y_n - z_n + 1 & (y_n - z_n < -1).
\end{cases}
\tag{4}
$$

Figure 2: Model A.

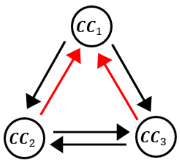

Figure 3: Model B.

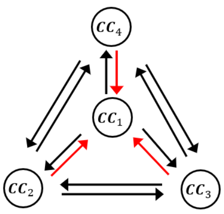

Figure 4: Model C.

Figures 2-4 show the system model used in this study. The coupling strength from other circuits to CC1 is varied, while the coupling strength between other circuits is fixed at $\delta = 0.22$ for simulation.

In order to measure the synchronization phenomena quantitatively, we defined synchronization as in the following equation.

$$
|y_i - y_j| \leq 0.01 \quad (i \neq j) \tag{5}
$$

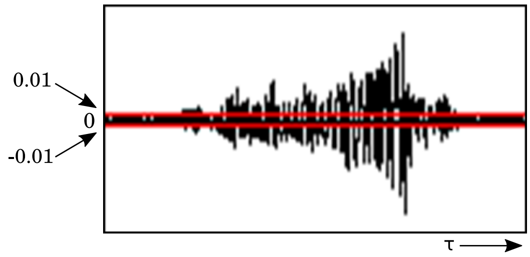

Figure 5: Voltage difference waveform.

The synchronization rate is expressed using the voltage difference waveform between circuits. When the voltage difference is within the reference value, it is considered synchronous, and when it is not within the reference value, it is considered asynchronous. The synchronization rate is then indicated by the percentage of how far within the reference value within a certain period of time.

3. Simulation results

In this study, the synchronization phenomena are investigated by the computer simulations. The parameters as $\alpha = 0.5$, $\beta = 20$, $\gamma = 0.5$ on all circuits are fixed. The synchronization rate is averaged over five changes of the initial values for each circuit. Figure 6 shows the changes in the synchronization phenomena in Model A. The coupling strength from CC1 to CC2 is constant, and when the coupling strength from CC2 to CC1 is varied, it is considered Non-Uniform, and when the coupling strength between CC1 and CC2 is both varied, it is considered Uniform. Figure 7 shows the voltage difference between CC1 and CC2 of model A at each coupling strength. The coupling strength from CC2 to CC1 is varied. Figure 6 shows that the synchronization rate remains close to 100[%] until the coupling strength of one side becomes about half of the coupling strength of the other side. In addition, the change in the synchronization phenomena for the Non-Uniform coupling strength is different from that for the Uniform coupling strength. The voltage difference waveforms in Fig. 7 show a clear distinction between synchronous and asynchronous.

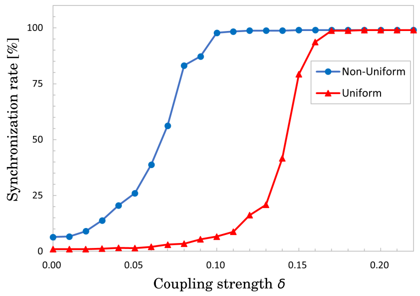

Figure 6: Relationship between synchronization rate and coupling strength (Model A).

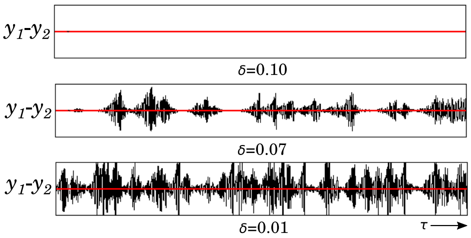

Figure 7: Voltage difference waveforms (Model A).

Figure 8 shows the change in synchronization phenomena in model B. The coupling strength from CC2 and CC3 to CC1 is varied. Figure 9 shows the voltage difference waveforms between CC1 and CC3 of model B at each coupling strength. From the Figs. 8 and 9, with uniform coupling strength, the synchronization phenomena collapse from $\delta = 0.11$. On the other hand, for non-uniform coupling strength, it is confirmed that the synchronization phenomena collapse from $\delta = 0.05$. When $\delta = 0.07$, the voltage difference in Model A appears to vary, but in Model B, no variation in the voltage difference is seen, considering that the number of circuits affects the synchronization phenomena.

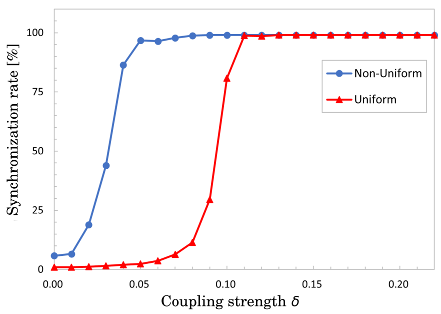

Figure 8: Relationship between synchronization rate and coupling strength (Model B).

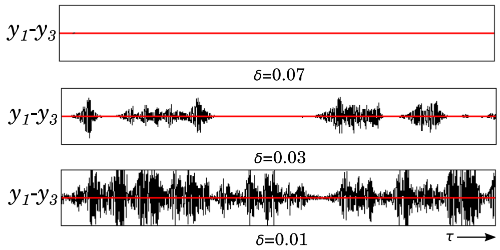

Figure 9: Voltage difference waveforms (Model B).

Figure 10 shows the change in synchronization phenomena in model C. The coupling strength from CC2, CC3, and CC4 to CC1 is varied. Figure 11 shows the voltage difference between CC1 and CC4 of Model C at each coupling strength. Figure 12 compares the three proposed models with non-uniform coupling strengths. from Fig. 10, Model C also shows similar results to Model B. Moreover, compared to Model B, the coupling strength at which the synchronization phenomena collapse is smaller in Fig. 12.

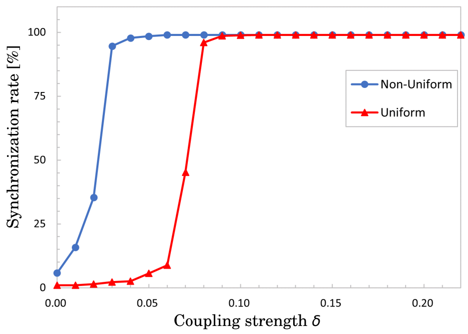

Figure 10: Relationship between synchronization rate and coupling strength (Model C).

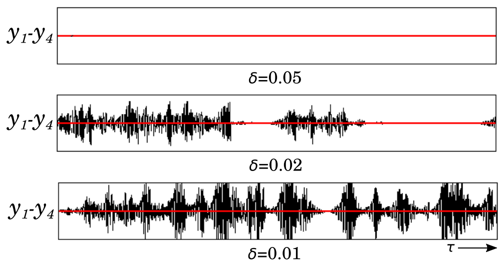

Figure 11: Voltage difference waveforms (Model C).

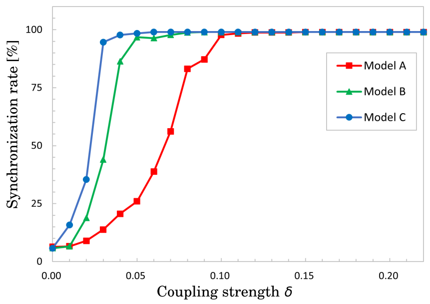

Figure 12: Comparison of synchronization rate for the three models.

4. Conclusions

In this study, we investigate the synchronization of bi-directionally coupled chaotic circuits with non-uniform coupling strength. It is confirmed that the synchronization phenomena are kept even with non-uniform coupling strength. Moreover, the non-uniform coupling strength showed the synchronization phenomena even at weak coupling strength compared to when the coupling strength is weakened uniformly. It is also confirmed that by increasing the number of circuits connected to CC1, the coupling strength at which the synchronization phenomena collapse becomes smaller.

In the future, we would like to investigate the synchronization phenomena in system model with more meaningful directions by increasing the number of connection points with non-uniform coupling strength. We would also like to check the synchronization phenomena when the connection state between synchronized groups is changed.

References

[1] M. G. Kitzbichler, M. L. Smith, S. R. Christensen and E. Bullmore, " Broadband Criticality of Human Brain Network Synchronization ", *PLoS Computational Biology (Volume: 5)*, pp. 1-13, 2009.

[2] A. Omri, M. Shaqfeh, A. Ali and H. Alnuweiri, " Synchronization Procedure in 5G NR Systems ", *IEEE Access (Volume: 7)*, pp. 41286-41295, 2020.

[3] S. Masamura, T. Iwamoto, Y. Sugitani, K. Konishi and N. Hara, " Experimental investigation of amplitude death in delay-coupled double-scroll circuits with randomly time-varying network topology ", *Nonlinear Dyn (2020) 99*, pp. 3155-3168), 2019.

[4] K. Nakabai, T. Nara, Y. Uwate and Y. Nishio, " Synchronization in Ladder-Coupled Chaotic Circuits Including Ring Structures ", *Proceedings of IEEE International Symposium on Circuits and Systems (ISCAS'19)*, DOI:10.1109/ISCAS.2019.8702350 (5 pages), 2019.

[5] K. Ago, Y. Uwate and Y. Nishio, " Influence of Local Bridge on a Complex Network of Coupled Chaotic Circuits ", *Proceedings of International Symposium on Nonlinear Theory and its Applications (NOLTA'14)*, pp. 731-734, 2014.

[6] M. Shinriki, M. Yamamoto and S. Mori, " Multimode Oscillations in a Modified van der Pol Oscillator Containing a Positive Nonlinear Conductance ", *Proc. IEEE, vol. 69*, pp. 394-395, 1981.

[7] N. Inaba, T. Saito and S. Mori, " Chaotic Phenomena in a Circuit with a Negative Resistance and an Ideal Switch of Diodes ", *IEICE Trans., vol. E-70*, pp. 744-754, 1987.