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Study on Precision Recall by Using van der Pol Oscillators with Third-Power and Fifth-Power for Associative Memory

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Abstract

It is a great goal for many researchers to give computers associative memory functions. This study compares associative memory functions between the coupling circuit model with van der Pol oscillators with third-power and the model with fifth-power. As a result, synchronization state of the model with fifth-power was much stabler. Besides, the model with fifth-power outperforms the model with third-power in terms of recall response time and recall rate.

1. INTRODUCTION

Synchronization phenomenon is that two or more objects do the same motions at the same time. It is observed in nature, such as luminescence of firefly and cry of frogs. Besides, synchronization phenomenon is applied in development studies by various researchers. In the field of medical science, heartbeat is one of the synchronization phenomena, myocardial cells synchronized each other. Belousov-Zhabotinsky (BZ) reaction is popular for chemical researchers as chemical oscillation. Similarly, synchronization phenomenon is observed on the coupling circuit via resistors [1].

Associative memory is to recall more closely related things from a part of memory. It is particular to human ability, and then superior to computer. On the other hand, some people cannot recognize the names and emotions when they see human face without age relationship. This illness is called prosopagnosia. To assist the patients, many researchers have studied associative memory and pattern recognition by using coupling circuits [2], [3].

In previous study, associative memory was realized by using the coupling circuit model of van der Pol oscillators with third-power [4]. Besides, we confirmed that associative memory was also realized by using van der Pol oscillator with fifth-power [5], and then investigated the properties of van der Pol oscillators with fifth-power [6].

In this study, we discussed average of recall response time and recall rate by analyzing phase differences time evolution on both models.

2. COUPLING CIRCUIT MODEL

Figure 1 shows the coupling circuit model of van der Pol oscillators via resistors. The current-voltage characteristics of nonlinear resistor in van der Pol oscillator is also shown in Fig. 2.

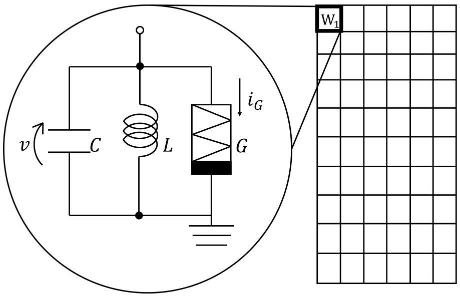
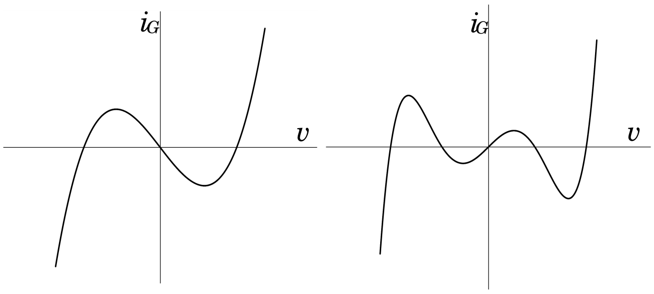


Figure 1: Coupling circuit model.



(a) Third-power.

(b) Fifth-power.

Figure 2: Current-voltage characteristics.

The circuit equations of the model are Eq. (1) and the equations of current-voltage characteristics of nonlinear resistors is Eq. (2).

$$\begin{cases} C\dfrac{dv_n}{dt} = -i_{G_n} - i_{L_n} + \displaystyle\sum_{k=1}^{60}\frac{1}{R}(v_n - v_k) \\ L\dfrac{di_{L_n}}{dt} = v_n \quad (n = 1, 2, \cdots, 60). \end{cases} \tag{1}$$

$$i_G = \begin{cases} -g_1 v + g_3 v^3 & \text{(Third-power)} \\ g_1 v - g_3 v^3 + g_5 v^5 & \text{(Fifth-power).} \end{cases} \tag{2}$$

The parameters for normalization of Eq. (1) with third-power are follows:

$$v = \sqrt{\frac{g_1}{g_3}}x,\ i = \sqrt{\frac{g_1 C}{g_3 L}}y,\ t = \sqrt{LC}\tau$$

$$\varepsilon = g_1\sqrt{\frac{L}{C}},\ K = \frac{1}{R}\sqrt{\frac{L}{C}}.$$

The normalized circuit equations with third-power are described as Eq. (3).

$$\begin{cases} \dfrac{dx_n}{d\tau} = \varepsilon x_n(1 - x_n^2) - y_n + \displaystyle\sum_{k=1}^{60} K(x_n - x_k) \\ \dfrac{dy_n}{d\tau} = x_n \quad (n = 1, 2, \cdots, 60). \end{cases} \tag{3}$$

On the other hand, Eq. (1) with fifth-power are normalized by using these parameters:

$$v = \sqrt[4]{\frac{g_1}{g_5}}x,\ i = \sqrt[4]{\frac{g_1 C^4}{g_5 L^4}}y,\ t = \sqrt{LC}\tau$$

$$\varepsilon = g_1\sqrt{\frac{L}{C}},\ K = \frac{1}{R}\sqrt{\frac{L}{C}},\ \delta = \frac{g_3}{\sqrt[4]{{g_1}^2{g_5}^2}}.$$

The normalized circuit equations with fifth-power are described as Eq. (4).

$$\begin{cases} \dfrac{dx_n}{d\tau} = -\varepsilon x_n(1 - \delta x_n^2 + x_n^4) - y_n \\ \qquad\qquad + \displaystyle\sum_{k=1}^{60} K(x_n - x_k) \\ \dfrac{dy_n}{d\tau} = x_n \\ \qquad\qquad (n = 1, 2, \cdots, 60). \end{cases} \tag{4}$$

Here, K is redefined as Eq. (5) based on associatron [4, 5, 6].

$$K = \begin{cases} E_0 \times s \\ E\ \times s. \end{cases} \tag{5}$$

Parameter s is the coupling strength. E_0 and E are the storage matrixes for input pattern and stored patterns respectively. Each pixel corresponds to van der Pol oscillators in Fig. 1. The oscillator W1 in the top left is reference. f^0 and f^p are created by setting white pixel to +1 and black pixel to -1. p is the number of stored patterns and f^p is created for each stored pattern.

$$\begin{cases} f^0 = (f^0_{W1} f^0_{W2} \cdots f^0_{W60}) \\ f^p = (f^p_{W1} f^p_{W2} \cdots f^p_{W60}). \end{cases} \tag{6}$$

Storage matrixes E_0 and E are defined as Eq. (7) and Eq. (8) using f^0 and f^p.

$$E_0 = f^0 \times f^{0T}. \tag{7}$$

$$E = \frac{1}{p}\sum_{k=1}^{p} f^p \times f^{pT}. \tag{8}$$

Based on the above, recall process is described below. First, parameter of normalized circuit equations is set, and then calculate the solution by using Runge-Kutta method. Hence, input pattern is recognized on the coupling circuit model by using Eq. (7). Second, each oscillator on the model begin to synchronize or asynchronized with the reference oscillator W1 by using Eq. (8). Finally, pixels output white and black by using Eq. (6). White is output when each oscillator is synchronized with the reference oscillator W1, and black when each oscillator is asynchronized.

Phase difference time evolution is calculated by using Poincaré map. Figure 3 shows the solution space for normalized circuit equations. Besides, z_n and θ_n are defined as Eq. (9).

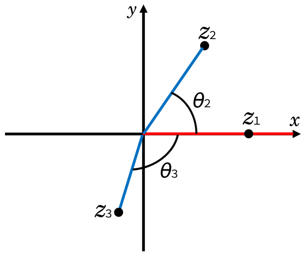


Figure 3: Solution space.

$$\begin{aligned} z_n &= (x_n, y_n) \\ \theta_n &= \arctan\left(\frac{y_n - y_1}{x_n - x_1}\right) \quad (n = 1, 2, \cdots, 60). \end{aligned} \tag{9}$$

Poincaré section is the positive half of the x-axis in Fig. 3, so $x_1 > 0$ and $y_1 = 0$. Absolute value of phase difference $|\theta_n|$ is more less than or equal to 0° and greater than or equal to 90°, nth-oscillator outputs white. $|\theta_n|$ is more less than 90° and greater than or equal to 180° , nth-oscillator outputs black.

In this study, three stored patterns are used in Fig. 4.

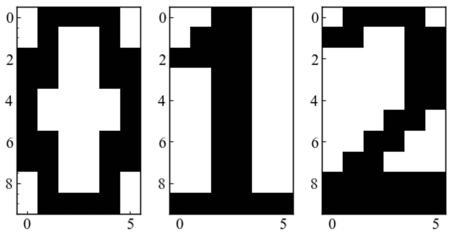
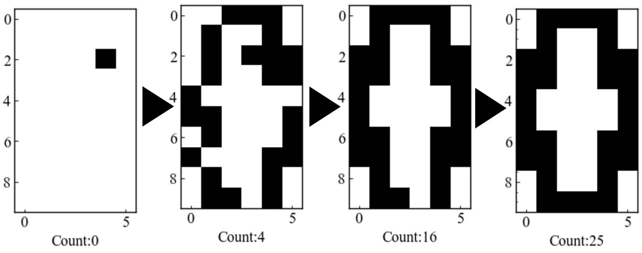


Figure 4: Stored patterns.

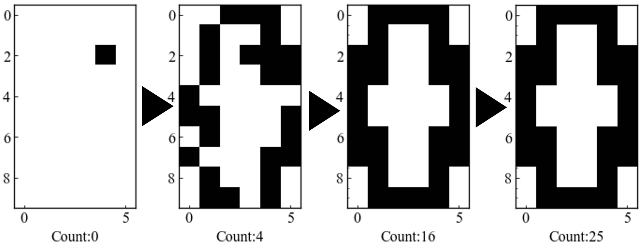
3. RESULTS

In these simulations, parameters of the model are set to $\varepsilon = 0.1$, $s = 0.01$, $\tau = 625$ and $\delta = 1$.

First, we investigated recall response time by using 10 kinds of input patterns whose hamming distance for stored pattern is 10. Recall response time is defined as first count that the model recall stored pattern. Figure 5 shows recall process of both models when input broken 0 whose hamming distance is 10. In addition, synchronization state of both models are shown in Fig. 6. Count is the number of times that z_1 pass through poincaré section.

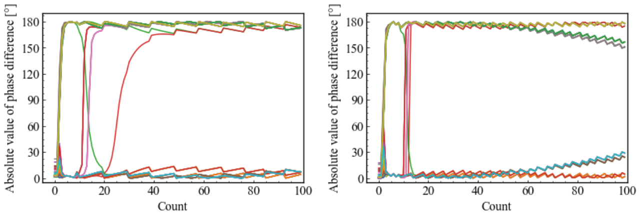


(a) Third-power.



(b) Fifth-power.

Figure 5: Recall process.



(a) Third-power. (b) Fifth-power.

Figure 6: Synchronization state.

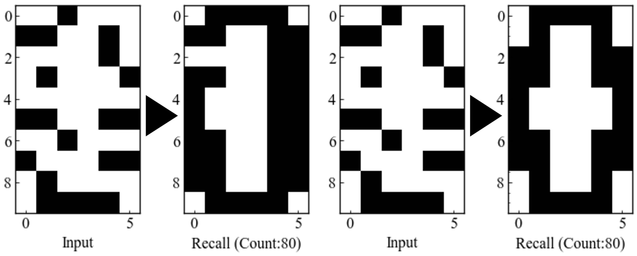
As a result, both models recalled stored pattern. Besides, at count 16, the model with fifth-power had already recalled stored pattern but the model with third-power had not yet. So, we calculated average of recall response time by using 10 input patterns whose hamming distance is 10, 15 and 20. Table1 shows average of recall response time.

Table 1: Average of recall response time.

Hamming distance	10		15		20	
Characteristics	Third	Fifth	Third	Fifth	Third	Fifth
Ave. recall response time	26.4	15.5	33.3	16.1	25.3	14.5

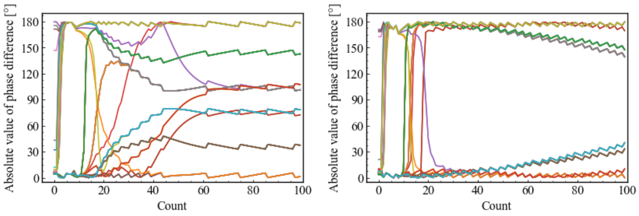
In Table 1, average of recall response time of the model with fifth-power is faster than the model with third-power without input patterns.

Next, we investigated recall rate by using 10 kinds of input patterns whose hamming distance for stored pattern is 20. Figure 7 shows recall results of both models when input broken 0 whose hamming distance is 20, and the synchronization state is shown in Fig. 8.



(a) Third-power. (b) Fifth-power.

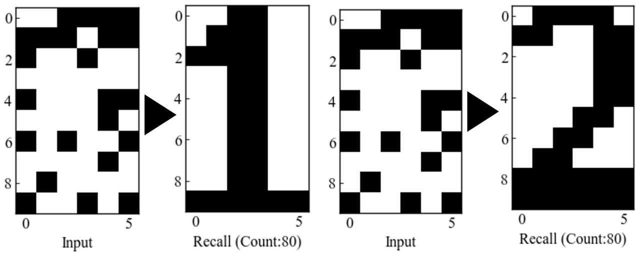
Figure 7: Recall results.



(a) Third-power. (b) Fifth-power.

Figure 8: Synchronization state.

As you can see, the model with fifth-power recalled stored pattern although the model with third-power recalled broken pattern. Besides, phase differences of the model with third-power split into away from 0° or 180°. On the other hand, phase differences of the model with fifth-power split into 0° or 180° clearly. It has same tendency when input broken 1 whose hamming distance is 20. However, both models recalled stored pattern every times when input broken 2 whose hamming distance is 20. Therefore, we reinvestigated that recall rate when input broken 2 whose hamming distance is 25. Recall results of both models when input broken 2 whose hamming distance is 25 are shown in Fig. 9.



(a) Third-power. (b) Fifth-power.

Figure 9: Recall results.

As a result, the model with third-power recall 1 and the model with fifth-power recall 2. In this simulation, each hamming distance of input pattern for 1 and 2 is 30 and 25 respectively. Hence, the model with fifth-power recalled correct pattern but the model with third-power recalled failed pattern. Table 2 shows recall rate. Recall rate is defined as an index of recall times from count 0 to 80 divided by the number of trials.

Table 2: Recall rate.

Hamming distance	20		25	
Characteristics	Third	Fifth	Third	Fifth
Recall rate 0	0.7	1	none	none
Recall rate 1	0.4	1	none	none
Recall rate 2	1	1	0.2	0.9

In all cases, recall rate of the model with fifth-power is higher than the model with third-power. In other words, the model with fifth-power recall correct pattern even when input more broken pattern.

Here, we discussed the cause of the difference of the synchronization state between the model with third- power and the model with fifth-power. The difference of the synchronization state is caused by numerical results of Eqs. (3) and (5) arising from the current-voltage characteristics of nonlinear resistor Eq. (2). We confirmed that the model with third-power and the model with fifth-power oscillated while converging on a stable limit cycle, when examined the synchronization state. In addition, we used same values of parameters for all experiments except for δ. The difference between the model with third-power and fifth-power is only presence or absence of δ. Therefore, we consider that the cause of the difference of the synchronization state between the model with third- power and the model with fifth-power is numerical results of Eqs. (3) and (5) resulting δ.

4. CONCLUSION

We investigated recall response time and recall rate by using the coupling circuit model with third-power and the model with fifth-power. First, the model with fifth-power recalled stored patterns faster than the model with third-power. Second, synchronization state of the model with third-power has a tendency to split into 0°or 180°when input broken pattern. Otherwise, the one of the model with fifth-power is stable even when input more broken pattern. Consequently, recall rate of the model with fifth-power is higher than the model with third-power. In the future, we would like to examine the solution flow in detail, and realize associative memory on the model with fifth-power by using unstable limit cycle.

References

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