2022 International Symposium on Nonlinear Theory and Its Applications, NOLTA2022, Virtual, December 12-15, 2022

Clustering Using Chaos Synchronization with Learning Algorithm

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Abstract—
In our previous studies, we have proposed the method of clustering using synchronization of coupled chaotic circuits networks. It was confirmed that applied learning algorithm like Hebbian learning is effective for advanced clustering when the data patterns of clusters are overlapped.

In this study, we investigate the effect of coupling strength of whole network for clustering performance. Also, the effect of the learning parameters is investigated. From computer simulations results, we confirm that clustering with learning algorithm has no effect for the clustering error.

1. Introduction

Some observed phenomena from coupled oscillatory networks can be a good models to express the physical phenomena in our living life. Therefore many researchers investigate models using coupled nonlinear circuits and observe several interesting phenomena.

In our previous studies [1]-[4], we have investigated clustering in two-dimensional networks of complex chaotic circuits, where the coupling strength reflects the network distance information. We showed that circuits that are arranged close to each other can achieve phase synchronization, whereas coupled circuits located far away from each other cannot be synchronized. We also observed that these networks of coupled chaotic circuits can be split into different synchronized groups, thus revealing a clustering phenomenon. To achieve more complex and advanced clustering, we proposed a clustering method based on a new chaotic coupled circuit network applying learning rule. For a more complex clustering example, we consider a circuit layout with overlapping clusters. By using computer simulations, we confirmed that chaotic circuit network with the learning rule is more effective than the standard chaotic circuit network.

In this study, we investigate the effect of the scaling parameter of the coupling strength for the whole network. We confirm that in the case of the method without learning rule, the number of miss edges between the two clusters are increased with the scaling parameter of the coupling strength. While, in the case of the method with learning rule, there are no affection of miss edges.

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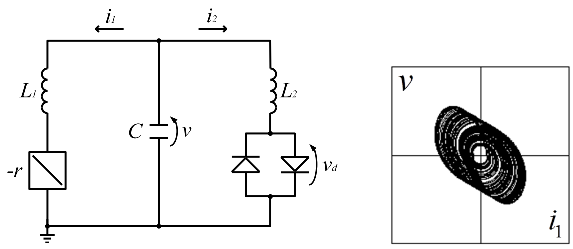
2. Chaotic Circuits Model

The chaotic circuit model and obtained chaos attractor are shown in Fig. 1. The chaotic circuit has been investigated in the literature [5]-[7]. This circuit consists of three memory elements, one linear negative resistance element, and one nonlinear resistance element consisting of two diodes. The negative resistance is realized using the linear region of a negative impedance converter made from an operational amplifier.



(a) Chaotic circuit. (b) Chaotic attractor.

Figure 1: Chaotic circuit model.

The approximate $I - V$ characteristics of the nonlinear resistance element are indicated by the following equation, where the parameter r_d is the slope of the nonlinear resistance.

$$v_d(i_2) = \frac{r_d}{2}\left(\left|i_2 + \frac{V}{r_d}\right| - \left|i_2 - \frac{V}{r_d}\right|\right). \quad (1)$$

By changing the variables, such that

$$i_1 = \sqrt{\frac{C}{L_1}}Vx;\ i_2 = \frac{\sqrt{L_1 C}}{L_2}Vy;\ v = Vz;$$

$$r\sqrt{\frac{C}{L_1}} = \alpha;\ \frac{L_1}{L_2} = \beta;\ r_d\frac{\sqrt{L_1 C}}{L_2} = \delta; t = \sqrt{L_1 C}\tau \qquad (2)$$

The normalized circuit equations can be described by

$$\begin{cases} \dfrac{dx}{d\tau} = \alpha x + z \\ \dfrac{dy}{d\tau} = z - f(y) \\ \dfrac{dz}{d\tau} = -x - \beta y \end{cases} \qquad (3)$$

where $f(y)$ is described as follows:

$$f(y) = \frac{\delta}{2}\left(\left|y + \frac{1}{\delta}\right| - \left|y - \frac{1}{\delta}\right|\right). \qquad (4)$$

The following equations represent the circuit equations when all the chaotic circuits are coupled globally with each other (all-to-all coupling).

$$\begin{cases} \dfrac{dx_i}{d\tau} = \alpha x_i + z_i \\ \dfrac{dy_i}{d\tau} = z_i + f(y) \\ \dfrac{dz_i}{d\tau} = -x_i - \beta y_i - \displaystyle\sum_{j=1}^{N} \gamma_{ij}(z_i - z_j) \\ \qquad (i, j = 1, 2, \cdots, N) \end{cases} \qquad (5)$$

In the computer simulations, we set the parameters to be $\alpha = 0.460$, $\beta = 3.0$ and $\delta = 470$. The characteristic function $f(y)$ can be described as a three-segment piecewise-linear function. In this study, the value of γ_{ij} reflects the distance between the circuits in an inverse manner, as described using the following equation:

$$\gamma_{ij} = \frac{g}{(d_{ij})^2}. \qquad (6)$$

Here, d_{ij} denotes the Euclidean distance between the $i-th$ circuit and the $j-th$ circuit, while g is a scaling parameter that determines the coupling strengths.

3. Clustering Arrangement and Learning

The circuit arrangement with overlapped clusters is shown in Fig. 2. This time, we consider a more difficult case: a circuit layout with overlapping clusters. The number of clusters is set to three, and the chaotic circuits are randomly placed according to a normal distribution from the center of each cluster.

The purpose of clustering for this circuit layout is not to divide it into three clusters, but to extract as many nodes as possible from the central part of the cluster.

In this study, we use clustering method that applies Hebbian rule [8] as well as the determination of clusters by

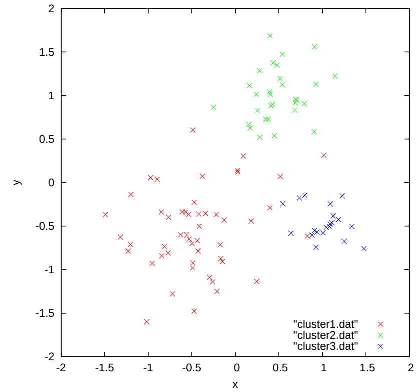


Figure 2: Circuit arrangement with overlapped clusters (N=100, the number of clusters: 3, red points: 30 (cluster-1), green points: 30 (cluster-2), blue points: 40 (cluster-3)).

synchronization of chaotic circuit networks. The Hebbian rule states that synapses, which connect neurons become more efficient when neurons fire repeatedly, and less efficient when they do not fire for long periods of time. We apply this Hebbian rule to chaotic circuit synchronization. In other words, the coupling between the synchronized chaotic circuits is made stronger, and the coupling between the un-synchronized chaotic circuits is made weaker. The Hebbian rule is applied to the chaotic circuits network as following steps.

[step-1] At the initial state, all nodes are fully connected with coupling strengths depending on distance.

[step-2] After a transient phase, we apply two rules for a sequence of generations. Each generation has length τ_h = 10,000.

- **(check synchronization:)** In order to check whether two nodes are alike, we calculate the synchronization ratio for every pair of oscillators. If the synchronization ratio is larger than 80, 60, and 50%, the corresponding coupling strength becomes stronger with $\Delta\gamma$ = 0.00001. And, if the synchronization ratio is smaller than 20, 40, and 49the corresponding coupling strength becomes weaker with $\Delta\gamma = -0.00001$.

  In order to analyze the synchronization ratio, we define a synchronization state as

$$|x_k - x_n| < 0.3 \quad (k \in S_n)$$

[step-3] Step-2 is repeated until 10 iterations are reached (H = 10).

[step-4] At the final state (G = 10), we check the synchronization ratio for every pair of oscillators.

4. Simulation Results

Figures 3 and 4 show the example of clustering results of chaotic circuit network with/without learning rule. The lines in the figure represent edges between circuits with high synchronization rates. In the method with learning rule, there are many lines within clusters. In contrast, the method without learning rule has many lines between clusters, therefore it is difficult to determine the clusters. These results show that changing the weights of the couplings, it is effective for clustering.

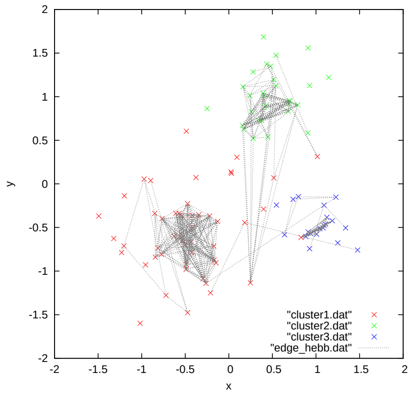


Figure 3: Example of clustering result with learning rule ($g = 0.00008$).

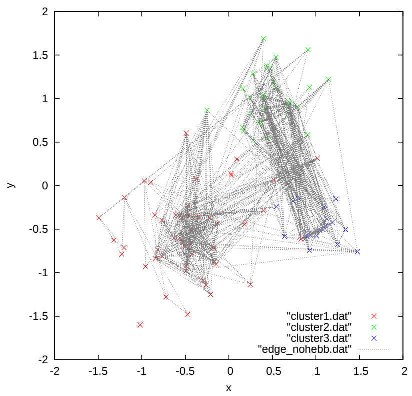
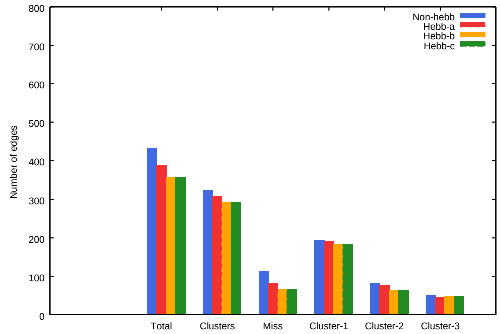


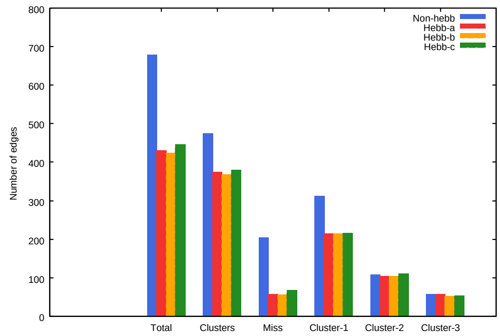
Figure 4: Example of clustering result without learning rule ($g = 0.00008$).

Figure 5 shows the number of edges with high synchronization rates in each category. It can be seen that the method without learning rule has more synchronized edges in all categories. However, the number of miss categories (edges between clusters) is also high, and the accuracy of the clusters is not good. In contrast, the method with learning rule has fewer edges in the miss category, which can be said to be good for clustering accuracy.

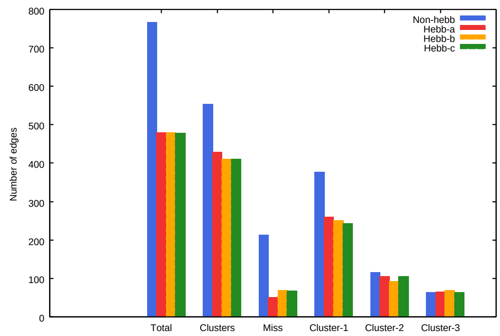
Furthermore, in the case of the method without learning rule, the number of miss edges between the two clusters are increased with the scaling parameter of the coupling strength. While, in the case of the method with learning rule, there are no affection of miss edges. Totally, the number of high-synchronized edges are increased with the scaling parameter g.



(a) $g = 0.00006$.



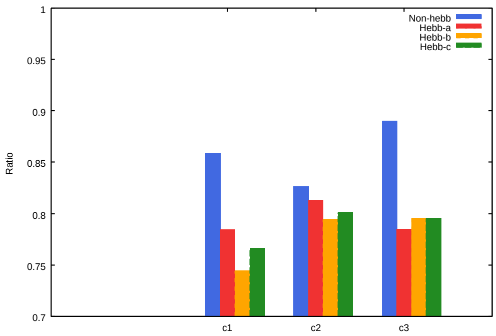
(b) $g = 0.00008$.



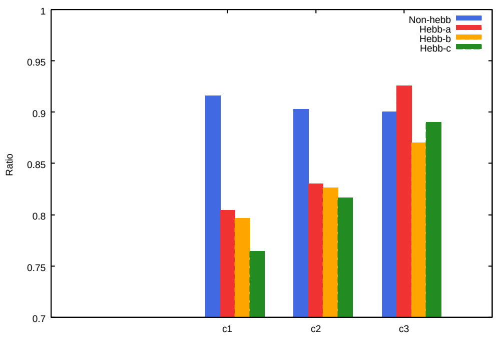
(c) $g = 0.00010$.

Figure 5: Clustering results of number of edges.

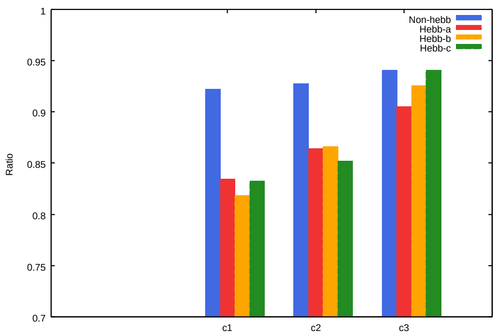
Finally, we investigate the detection ratio of clusters by counting the nodes which has edge within same cluster. The simulation results are shown in Fig. 6. We confirm that the detection ratio of cluster is increased when the scaling parameter of the coupling strength has large value. However, it was difficult to observe the any differences depending on the learning parameters. Therefore, we would like to investigate the effect of the learning parameters in detail as the future work.



(a) g = 0.00006.



(b) g = 0.00008.



(c) g = 0.00010.

Figure 6: Clustering results of detection ratio of clusters.

5. Conclusion

In this study, we used the clustering method based on a chaotic coupled circuit network applying Hebbian rule in order to achieve more complex and advanced clustering. We also investigated the effect of coupling strength of whole network for clustering performance. By using computer simulations, we confirmed that chaotic circuit network with the learning rule is more effective than the standard algorithm.

In future work, we would like to study clustering phenomena in large-scale networks. Furthermore, we hope to apply the clustering methods developed here to data mining, image processing and other applications in real life situations.

References

[1] Y. Takamaru, H. Kataoka, Y. Uwate & Y. Nishio, "Clustering Phenomena in Complex Networks of Chaotic Circuits," *Proc. ISCAS'12*, Vol. 22, pp. 320-334, 2012.

[2] Y. Takamaru, Y. Uwate, T. Ott & Y. Nishio, "Dependence of Clustering Patterns on Density of Chaotic Circuits in Networks," *RISP Journal of Signal Processing*, vol. 17, no. 4, pp. 103-106, Jul. 2013.

[3] K. Ago, Y. Uwate & Y. Nishio, "Investigation of Clustering in Complex Networks Using Coupled Chaotic Circuits," *Proc. of IFAT'15*, pp. 101-102, 2015.

[4] K. Oi, Y. Uwate & Y. Nishio, "Synchronization and Clustering in Coupled Parametrically Excited Oscillators with Small Mismatch," *Proc. of ISCAS'15*, pp. 910-913, 2015.

[5] Y. Nishio, N. Inaba, S. Mori & T. Saito, "Rigorous Analyses of Windows in a Symmetric Circuit," *IEEE Transactions on Circuits and Systems*, vol. 37, no. 4, pp. 473-487, Apr. 1990.

[6] C. Bonatto & J. A. C. Gallas, "Periodicity Hub and Nested Spirals in the Phase Diagram of a Simple Resistive Circuit," *Phys. Rev. Lett.*, 101, 054101, Aug. 2008.

[7] A. Koseska, E. Volkov, & J. Kurths, "Real-World Existence and Origins of the Spiral Organization of Shrimp-Shaped Domains," *Phys. Rev. Lett.*, 105, 074102, Aug. 2010.

[8] D. O. Hebb, "Organization of Behavior," 1949.