Synchronization Phenomena of Coupled Oscillators with Node and Edge Weights in Two-Dimensional Complex Networks

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Abstract—**Complex networks have attracted a great deal of attention, and research is being conducted in various fields. However, most of these studies have analyzed networks by coupling strength is kept constant between nodes for all connections. In this study, we focus on the Euclidean distance of edges and the degree of nodes to give weight to the network, and we set the coupling strength. We performed networks analysis in terms of synchronization phenomena for complex networks constructed with van der Pol oscillators by computer simulations.**

Keywords; Oscillator; Synchronization; Complex networks

I. INTRODUCTION

Complex networks have received much attention in various fields such as sociology, biology and engineering [1]-[3]. Furthermore, in the field of engineering, complex networks using circuits have been studied, and interesting phenomena such as synchronization between circuits have been observed. It was confirmed that the topological structure of the network influences the synchronization [4],[5]. However, most of these studies have examined synchronization phenomena in networks where the coupling strength is kept constant for all the connections [6].

In this study, we construct a network model with scale-free property in two-dimensional space using 100 van der Pol oscillators. We analyze the network from the synchronization phenomenon between circuits when the network is weighted by considering the degree of nodes and the Euclidean distance of the edges.

II. SYSTEM MODEL

Figure 1 shows a van der Pol oscillator. This oscillator is a simple circuit, consisting of only a capacitor, an inductor and a nonlinear element. Figure 2 shows the Barabási Albert model (BA model) used in this study. For this model, five networks with different topologies are constructed under the same conditions. We set the average degree of node to be close to 4.0. The circuit in Fig. 1 is considered as a single node. By connecting these circuits with resistors, edges are created to form networks.

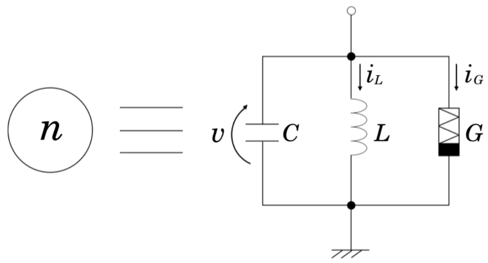


Figure 1. van der Pol oscillator.

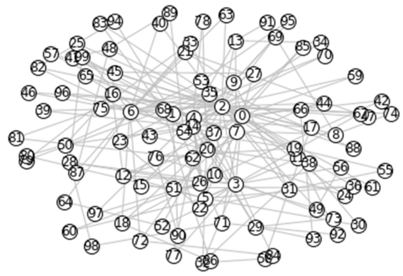


Figure 2. Barabási Albert model (BA model).

The characteristic equation of the nonlinear element and circuit equation are obtained from the circuit in Fig. 1. By using the normalization parameter and the variables, the normalized circuit equation described as follows:

$$\begin{cases} \dfrac{dx_n}{d\tau} = \alpha \left\{ \varepsilon x_n (1 - {x_n}^2) - y_n - \displaystyle\sum_{n,k=1}^{100} E_{nk} \gamma_{nk} (x_n - x_k) \right\} \\ \dfrac{dy_n}{d\tau} = x_n \end{cases} \quad (1)$$

$$(n, k = 1, 2, \cdots, 100).$$

E_{nk} represents the adjacency matrix of the network. This is a matrix that indicates whether node n and node k are connected or not. $E_{nk} = 1$ if node n and node k are connected, and $E_{nk} = 0$ if they are not connected. The coupling strength γ_{nk} is determined by using the parameter q as follows:

$$\gamma_{nk} = \frac{1}{R_{nk}}\sqrt{\frac{L}{C}} = \frac{q \cdot w_n w_k}{(d_{nk})^2}. \quad (2)$$

Here, d_{nk} is the Euclidean distance of the edge between node n and node k, and w is the degree of each node. In this study, we compare the following three cases:

1) when the only nodes are weighted ($d_{nk} = 1$ in Eq. (2))
2) when the only edges are weighted ($w_n w_k = 1$ in Eq. (2))
3) when both node and edge are weighted.

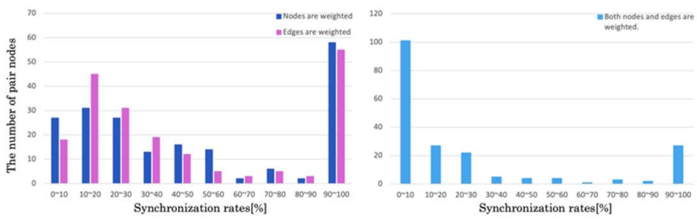
III. RESULTS

The parameter of the van der Pol oscillator is set to $\varepsilon = 0.1$. Moreover, α represents the small error of the capacitor, in the range of [0.975:1.025] in increments of 0.0005. The parameter q, which determines the coupling strength, is set so that the coupling strength takes close to $\gamma = 0.5$ for each network.

The average synchronization rates for each network (BA① ~BA⑤) for coupling strength determination method are shown in Table 1. The distribution of synchronization rates for Network 1 is also shown in Fig. 3.

TABLE I. AVERAGE SYNCHRONIZATION RATE FOR EACH COUPLING STRENGTH DETERMINATION METHOD.

coupling strength	BA①	BA②	BA③	BA④	BA⑤	avg
1)	49.95	49.82	50.73	53.73	48.61	50.57
2)	47.26	45.87	47.26	50.38	47.80	47.72
3)	26.15	31.31	21.49	30.17	25.96	27.02



(a) Only nodes and only edges. (b) Both node and edge.

Figure 3. Synchronization rate distribution by coupling strength determination method in BA① network.

First, Table 1 shows that each network has almost the same average synchronization rate for each method of determining the coupling strength. The average synchronization rate is about 50% when only nodes and only edges are weighted. On the other hand, it drops to less than about 30% when both are weighted. Next, Figures 3(a) and 3(b) show that for the synchronization rate distribution of BA ① among the five networks, the distribution is biased toward larger and smaller values for the coupling strength weighted for both node and edge than for the coupling strength weighted for only nodes and only edges.

IV. CONCLUSION

In this study, we constructed multiple networks of BA model using 100 van der Pol oscillators and investigated the synchronization phenomenon between circuits. Here, the degree of nodes and the Euclidean distance of edges were used as the coupling strength to weight the networks. The results show that when both node and edge are weighted for all networks, the synchronization rate decreases compared to when only each of them is weighted. We also find that when both are weighted, the synchronization rate distribution is divided into two patterns: one is close to perfect synchronization and the other is almost out of synchronization.

In the future, we would like to further investigate these two patterns and create a clustering index to see if we can form clusters with higher precision.

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