Clustering in Globally Coupled Chaotic Circuits with Changing Weights

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Abstract— **In this study, we focus on clustering phenomena in a network composed of coupled chaotic circuits. In this circuit network, the coupling strength is reflected by the distance information when the chaotic circuits are placed in a two-dimensional space. We propose a method that applies Hebbian rule to synchronization. Namely, the coupling weights are changed depending on synchronization states. By using the computer simulations, it is confirmed that the proposed methods is more effective than the standard method for clustering.**

Keywords; synchronization; coupled circuits; clustering

I. Introduction

In recent years, we need to deal with increasing amounts of information in our daily lives. In order to structure and analyze such data, it is effective to divide each data into clusters. The goal of a clustering algorithm is to find data clusters that consist of similar elements. Previously, many clustering studies have been performed using discrete time models, such as coupled map lattices (CMLs) and self-organizing maps (SOMs). However, few studies of clustering have been performed using a continuous time model. Therefore, this study focuses on the study of clustering phenomena in continuous-time models using actual electronic circuits.

Coupled chaotic circuits can be realized using electronic circuits and various interesting phenomena can be observed in these circuits. In recent years, many studies have reported on application of the clustering and synchronization phenomena that can be observed in coupled chaotic circuits to natural sciences. The reason for this interest is that the characteristics of the chaos phenomena observed in coupled chaotic circuits also exist in real life, in phenomena such as human behavior, emotions and heartbeats. Coupled chaotic circuits thus have the potential to be applied to a variety of different fields. In our previous studies, we proposed several types of clustering methods [1], [2] and recently we proposed a new clustering method based on new chaotic coupled circuit network applying Hebbian rule [3]. By using computer simulations, we confirmed that chaotic circuit network with changing weights is more effective than the conventional chaotic circuit network. However, the circuit layout was so complex that it was difficult to evaluate the accuracy of the clustering.

In this study, we focus on the accuracy of clustering by using coupled chaotic circuits with Hebbian rule.

II. Circuit Model and Circuit Arrangements

In this section, we explain the chaotic circuit model. Figure 1 shows the chaotic circuit model that has been investigated in the literature. This circuit consists of three memory elements, one linear negative resistance element, and one nonlinear resistance element consisting of two diodes. The negative resistance is realized using the linear region of a negative impedance converter made from an operational amplifier.

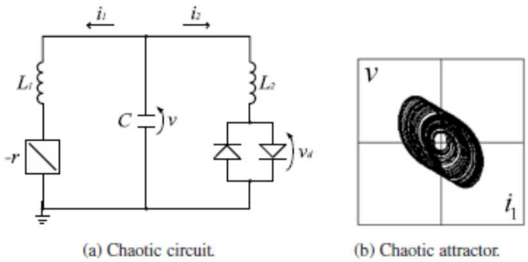


Fig. 1 Chaotic circuit.

The following equations represent the circuit equations when all the chaotic circuits are coupled globally with each other (globally coupling).

$$\begin{cases} \dfrac{dx_i}{d\tau} = \alpha x_i + z_i \\ \dfrac{dy_i}{d\tau} = z_i + f(y) \\ \dfrac{dz_i}{d\tau} = -x_i - \beta y_i - \displaystyle\sum_{j=1}^{N} \gamma_{ij}(z_i - z_j) \\ \qquad (i, j = 1, 2, \cdots, N) \end{cases}$$

In the computer simulations, we set the parameters of circuit system to be $\alpha = 0.460, \beta = 3.0$ and $\delta = 470$. The characteristic function $f(y)$ can be described as a three-segment piecewise-linear function. In this study, the value of γ_{ij} reflects the distance between the circuits in an inverse manner, as the following equation:

$$\gamma_{ij} = \frac{g}{(d_{ij})^2}.$$

Here, d_{ij} denotes the Euclidean distance between i-th circuit and j-th circuit, where g is a scaling parameter that determines the coupling strengths.

Figure 2 shows the circuit arrangement using 100 chaotic circuits. There are three clusters.

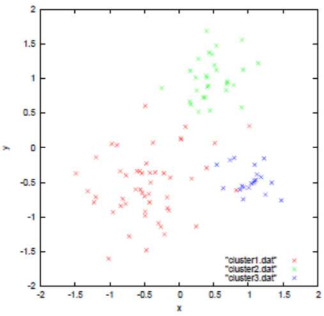


Fig. 2 Circuit arrangement. (N=100, the number of clusters: 3, red points: 50 (cluster-1), green points: 30 (cluster-2), blue points: 20 (cluster-3)).

III. PROPOSED ALGORITHM

In this study, we propose a new clustering method that applies Hebbian rule as well as the determination of clusters by synchronization of chaotic circuit networks. The Hebbian rule states that synapses, which connect neurons become more efficient when neurons fire repeatedly, and less efficient when they do not fire for long periods of time. We apply this Hebbian rule to chaotic circuit synchronization. In other words, the coupling between the synchronized chaotic circuits is made stronger, and the coupling between the unsynchronized chaotic circuits is made weaker. The Hebbian rule is applied to the chaotic circuits network as following steps.

[step-1] At the initial state, all edges are fully connected with the coupling strength depending on distance.

[step-2] After a transient phase, we apply two rules for a sequence of generations.

(check synchronization:) In order to check whether two nodes are alike, we calculate the synchronization ratio for every pair of oscillators. If the synchronization ratio is larger than 60%, the corresponding coupling strength becomes stronger with $\Delta\gamma = 0.01$. While the synchronization ratio is smaller, the coupling strength becomes weaker with $\Delta\gamma = 0.01$.

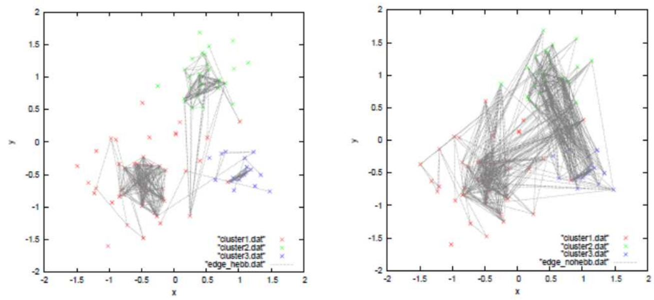
[step-3] step-2 is repeated until 10 iterations are reached.

[step-4] At the final state, we check the synchronization ratio for ever pair of chaotic circuits.

IV. CLUSTERING RESULTS

Figure 3 shows one example of clustering results obtained by the proposed and conventional models. The lines in the figure represent edges between circuits with high synchronization rates. In the proposed model, there are many lines within clusters. In contrast, the conventional model has many lines between clusters, therefore it is difficult to determine the clusters. These results show that changing the weights of the couplings, it is effective for clustering.

Figure 4 shows the number of edges with high synchronization rates in each category. It can be seen that the conventional model has more synchronized edges in all categories. However, the number of miss categories (edges between clusters) is also high, and the accuracy of the clusters is not good. In contrast, the proposed model has fewer edges in the miss category, which can be said to be good for clustering accuracy.



(a) Proposed model. (b) Conventional model.

Fig. 3 Examples of clustering results.

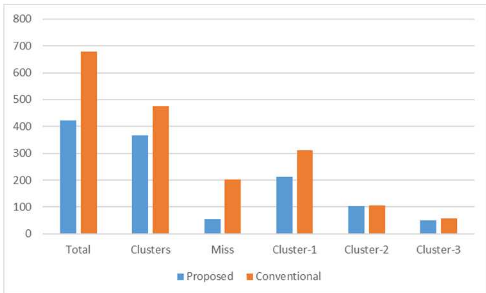


Fig. 4 Number of edges.

V. CONCLUSION

In this study, we focus on a clustering method based on a new chaotic coupled circuit network applying Hebbian rule in order to achieve more complex and advanced clustering. By using computer simulations, we confirmed that chaotic circuit network with the Hebbian rule is more effective than the conventional chaotic circuit network for clustering.

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