NCSP'22
RISP International Workshop on Nonlinear circuits, Communications and Signal Processing 2022
February 28 - March 1

Synchronization and Clustering of Chaotic Circuit Networks with Hebbian Rule

Yoko Uwate†, Thomas Ott‡ and Yoshifumi Nishio†

†Dept. of Electrical and Electronic Engineering, Tokushima University
2-1 Minami-Josanjima, Tokushima, Japan
Email: {uwate, nishio}@ee.tokushima-u.ac.jp

‡Zurich University of Applied Sciences
Einsiedlerstrasse 31a, 8820 Waedenswil, Switzerland
Email: thomas.ott@zhaw.ch

Abstract

In this study, we investigate clustering phenomena in a network composed of coupled chaotic circuits. In this network model, the coupling strength is reflected by the distance information when the chaotic circuits are placed in a two-dimensional space. For more advanced clustering, we propose a new method that applies Hebbian rule to synchronization. By applying Hebbian rule, it is confirmed that clusters that cannot be extracted by standard chaotic circuit networks can be extracted.

1. Introduction

Today, every time we open a newspaper, the page contains Artificial Intelligence:AI characters, and new technologies that apply deep learning are being developed everywhere in the industry. One of the basic laws of learning in the brain is the Hebbian rule. This rule has proposed by Hebb in 1949 [1]. This is a fundamental rule of learning and long-term memory based on the hypothesis that synapses become more efficient at transmitting electrical stimuli each time a neuron fires, and conversely, less efficient if they do not fire for a long time. The mechanism as to the neurobiology of the Hebbian rule has also been clarified. Coupling strengths are enhanced by the electrical signals and strengthened the part of more communications. The neurons have output part called axon. Some axons are covered with oligodendrocyte on the Myelin sheaths. Oligodendrocyte is white material which is rich in lipid. Covered axons can communicate quicker than non-covered ones. This phenomenon is called Myelination [2]. For example, we practice to achievement something when we usually try to do it. This action from challenge to achievement is myelination. Therefore, there is a big relation between learning and myelination. The study about myelination has possibility to make more performance for learning by adapting application.

In recent years, it has become necessary to handle increasingly large amounts of information in our daily lives. To enable structuring and analysis of such data, it is useful to partition each data set into clusters. The aim of a clustering algorithm is to find data clusters that consist of similar elements. Clustering algorithms have widespread applications in a variety of fields, including data mining, image processing and biological data analysis [3]-[5]. Various different clustering algorithms are thus available, along with many different applications. Previously, many clustering studies have been performed using discrete time models, such as coupled map lattices (CMLs) and self-organizing maps (SOMs) [6]-[8]. However, few studies of clustering have been performed using a continuous time model. Therefore, this work focuses on research into clustering phenomena using real electronic circuits in a continuous time model.

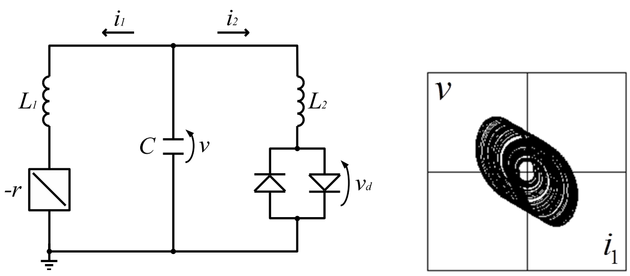
Coupled chaotic circuits can be realized using electronic circuits and various interesting phenomena can be observed in these circuits. In recent years, many studies have reported on application of the clustering and synchronization phenomena that can be observed in coupled chaotic circuits to natural sciences. The reason for this interest is that the characteristics of the chaos phenomena observed in coupled chaotic circuits also exist in real life, in phenomena such as human behavior, emotions and heartbeats. At the same time, synchronization and clustering phenomena have been studied associated with the chaos phenomena. Coupled chaotic circuits thus have the potential to be applied to a variety of different fields. We believe that we can apply the synchronization phenomena of coupled chaotic circuits to social networks in real life if we can clear up the chaos phenomena. Therefore, our study considers a new approach to investigation of the synchronization and clustering phenomena that occur in coupled chaotic circuits.

In our previous studies [12]-[15], we have investigated clustering in two-dimensional networks of complex chaotic circuits, where the coupling strength reflects the network distance information. We showed that circuits that are arranged close to each other can achieve phase synchronization, whereas coupled circuits located far away from each other cannot be synchronized. We also observed that these networks of coupled chaotic circuits can be split into different synchronized groups, thus revealing a clustering phenomenon.

However, the layout of the clustering we were targeting was easy, and we needed to consider a more practical and difficult problem. To achieve more complex and advanced clustering, we propose a clustering method based on a new chaotic coupled circuit network applying Hebbian rule. For a more complex clustering example, we consider a circuit layout with overlapping clusters. By using computer simultions, we confirm that chaotic circuit network with the Hebbia rule is more effective than the starndard chaotic circit network.

2. Chaotic Circuits Model

In this section, we explain the chaotic circuit model. Figure 1 shows the chaotic circuit model that has been investigated in the literature [9]-[11]. This circuit consists of three memory elements, one linear negative resistance element, and one nonlinear resistance element consisting of two diodes. The negative resistance is realized using the linear region of a negative impedance converter made from an operational amplifier.



(a) Chaotic circuit. (b) Chaotic attractor.

Figure 1: Chaotic circuit model.

The approximate $I-V$ characteristics of the nonlinear resistance element are indicated by the following equation, where the parameter r_d is the slope of the nonlinear resistance.

$$v_d(i_2) = \frac{r_d}{2}\left(\left|i_2 + \frac{V}{r_d}\right| - \left|i_2 - \frac{V}{r_d}\right|\right). \tag{1}$$

By changing the variables, such that

$$i_1 = \sqrt{\frac{C}{L_1}}Vx;\ i_2 = \frac{\sqrt{L_1 C}}{L_2}Vy;\ v = Vz;$$
$$r\sqrt{\frac{C}{L_1}} = \alpha;\ \frac{L_1}{L_2} = \beta;\ r_d\frac{\sqrt{L_1 C}}{L_2} = \delta; t = \sqrt{L_1 C}\tau \tag{2}$$

The normalized circuit equations can be described by

$$\begin{cases} \dfrac{dx}{d\tau} = \alpha x + z \\ \dfrac{dy}{d\tau} = z - f(y) \\ \dfrac{dz}{d\tau} = -x - \beta y \end{cases} \tag{3}$$

where $f(y)$ is described as follows:

$$f(y) = \frac{\delta}{2}\left(\left|y + \frac{1}{\delta}\right| - \left|y - \frac{1}{\delta}\right|\right). \tag{4}$$

The following equations represent the circuit equations when all the chaotic circuits are coupled globally with each other (all-to-all coupling).

$$\begin{cases} \dfrac{dx_i}{d\tau} = \alpha x_i + z_i \\ \dfrac{dy_i}{d\tau} = z_i + f(y) \\ \dfrac{dz_i}{d\tau} = -x_i - \beta y_i - \displaystyle\sum_{j=1}^{N} \gamma_{ij}(z_i - z_j) \\ \qquad (i, j = 1, 2, \cdots, N) \end{cases} \tag{5}$$

In the computer simulations, we set the parameters to be $\alpha = 0.460$, $\beta = 3.0$ and $\delta = 470$. The characteristic function $f(y)$ can be described as a three-segment piecewise-linear function. In this study, the value of γ_{ij} reflects the distance between the circuits in an inverse manner, as described using the following equation:

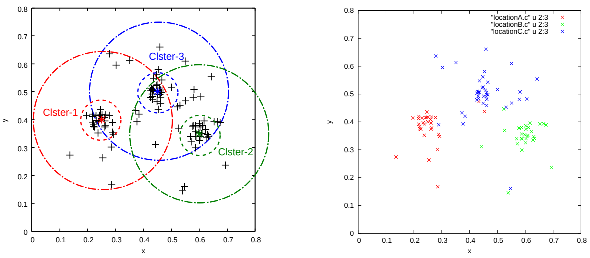
$$\gamma_{ij} = \frac{g}{(d_{ij})^2}. \tag{6}$$

Here, d_{ij} denotes the Euclidean distance between the $i-th$ circuit and the $j-th$ circuit, while g is a scaling parameter that determines the coupling strengths.

3. Overlapped Clustering Arrangement and Hebbian Learning

The previous circuit layout was a case where clustering was easy. This time, we consider a more difficult case: a circuit layout with overlapping clusters. The number of clusters is set to three, and the chaotic circuits are randomly placed according to a normal distribution from the center of each cluster. The circuit arrangement with overlapped clusters is shown in Fig. 2.

The purpose of clustering for this circuit layout is not to divide it into three clusters, but to extract as many nodes as possible from the central part of the cluster.



(a) Overlapped clusters. (b) Circuit arrangement.

Figure 2: Circuit arrangement with overlapped clusters (N=100, the number of clusters: 3, red points: 30 (cluster-1), green points: 30 (cluster-2), blue points: 40 (cluster-3)).

In this study, we propose a new clustering method that applies Hebbian rule as well as the determination of clusters by synchronization of chaotic circuit networks. The Hebbian rule states that synapses, which connect neurons become more efficient when neurons fire repeatedly, and less efficient when they do not fire for long periods of time. We apply this Hebbian rule to chaotic circuit synchronization. In other words, the coupling between the synchronized chaotic circuits is made stronger, and the coupling between the unsynchronized chaotic circuits is made weaker. The Hebbian rule is applied to the chaotic circuits network as following steps.

[step-1] At the initial state, all nodes are fully connected with coupling strengths depending on distance.

[step-2] After a transient phase, we apply two rules for a sequence of generations. Each generation has length $\tau_h = 10,000$. The conceptual diagram of the computer simulation is shown in Fig. 3.

- **(check synchronization:)** In order to check whether two nodes are alike, we calculate the synchronization ratio for every pair of oscillators. If the synchronization ratio is larger than 50%, the corresponding coupling strength becomes stronger with $\Delta\gamma = 0.01$.

  In order to analyze the synchronization ratio, we define a synchronization state as

$$|x_k - x_n| < 0.3 \quad (k \in S_n)$$

[step-3] Step-2 is repeated until 10 iterations are reached ($H = 10$).

[step-4] At the final state ($G = 10$), we check the synchronization ratio for every pair of oscillators.

4. Simulation Results

Figures 4 and 5 show the clustering results of chaotic circuit network with/without Hebbian rule. In the case of clustering with Hebbian rule, it was successful in extracting more nodes from the center of the cluster.

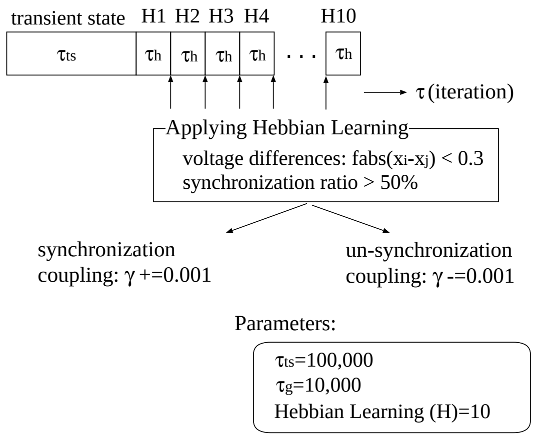


Figure 3: Process of Hebbian Learning.

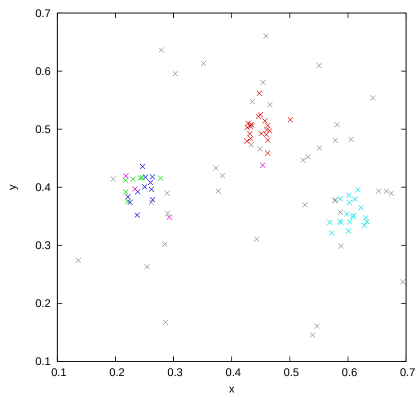


Figure 4: Clustering result with Hebbian rule.

The extraction rates of the clusters are summarized in the Tabs. I and II. By using the Hebbian rule, over half of the nodes in the two clusters have been successfully detected. While in the case of the clustering without Hebbian rule, the number of detected nodes in the every cluster was less than half. Therefore, it can be said that the clustering method for chaotic circuit networks using the Hebbian rule is effective.

Table 1: Clustering result with Hebbian rule.

	n_c	n_{en}	%
cluster-1	30	22	**73.3**
cluster-2	30	19	**63.3**
cluster-3	40	19	47.5

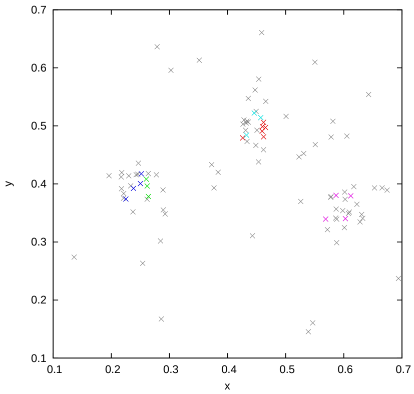


Figure 5: Clustering result without Hebbian rule.

Table 2: Clustering result without Hebbian rule.

	n_c	n_{en}	%
cluster-1	30	7	23.3
cluster-2	30	4	13.3
cluster-3	40	9	22.5

5. Conclusion

In this study, we propose a clustering method based on a new chaotic coupled circuit network applying Hebbian rule in order to achieve more complex and advanced clustering, For a more complex clustering example, we considered a circuit layout with overlapping clusters. By using computer simultions, we confirmed that chaotic circuit network with the Hebbian rule is more effective than the starndard chaotic circit network. By using the synchronization state between nodes, we were able to confirm that it can be applied to more advanced clustering.

In future work, we would like to study clustering phenomena in large-scale networks. Furthermore, we hope to apply the clustering methods developed here to data mining, image processing and other applications in real life situations.

References

[1] D. O. Hebb, "Organization of Behavior," 1949.

[2] K. A. Nave & H. B. Werner, "Myelination of the nervous system: mechanisms and functions," *Annu Rev Cell Dev Biol*, DOI: 10.1146/annurev-cellbio-100913-013101, 2014.

[3] Daniel T. Larose & Chantal D. Larose "Discovering Knowledge in Data: An Introduction to Data Mining," *John Wiley and Sons*, 2014.

[4] N. Dhanachandra, K. Manglem & Y. J. Chanu, "Image Segmentation Using K -means Clustering Algorithm and Subtractive Clustering Algorithm," *Procedia Computer Science*, Vol. 54, pp. 764-771, 2015.

[5] G. Manogaran, V. Vijayakumar, R. Varatharajan, P. M. Kumar & R. Sundarasekar, "Machine Learning Based Big Data Processing Framework for Cancer Diagnosis Using Hidden Markov Model and GM Clustering," *Wireless Personal Communications*, Vol. 102, Issue 3, pp. 20992116, 2018.

[6] K. Kaneko, "Clustering, Coding, Switching, Hierarchical Ordering, and Control in a Network of Chaotic Elements," *Physical D*, vol. 41, pp. 137-172, 1990.

[7] T. Ott, M. Christen & R. Stoop, "An Unbiased Clustering Algorithm Based on Self-organization Processes in Spiking Neural Networks," *Proc. of NDES'06*, pp. 143-146, 2006.

[8] L. Angelini, F. D. Carlo, C. Marangi, M. Pellicoro & S. Stramaglia, "Clustering Data by Inhomogeneous Chaotic Map Lattice," *Phys. Rev. Lett.*, 85, pp. 554-557, 2000.

[9] Y. Nishio, N. Inaba, S. Mori & T. Saito, "Rigorous Analyses of Windows in a Symmetric Circuit," *IEEE Transactions on Circuits and Systems*, vol. 37, no. 4, pp. 473-487, Apr. 1990.

[10] C. Bonatto & J. A. C. Gallas, "Periodicity Hub and Nested Spirals in the Phase Diagram of a Simple Resistive Circuit," *Phys. Rev. Lett.*, 101, 054101, Aug. 2008.

[11] A. Koseska, E. Volkov, & J. Kurths, "Real-World Existence and Origins of the Spiral Organization of Shrimp-Shaped Domains," *Phys. Rev. Lett.*, 105, 074102, Aug. 2010.

[12] Y. Takamaru, H. Kataoka, Y. Uwate & Y. Nishio, "Clustering Phenomena in Complex Networks of Chaotic Circuits," *Proc. ISCAS'12*, Vol. 22, pp. 320-334, 2012.

[13] Y. Takamaru, Y. Uwate, T. Ott & Y. Nishio, "Dependence of Clustering Patterns on Density of Chaotic Circuits in Networks," *RISP Journal of Signal Processing*, vol. 17, no. 4, pp. 103-106, Jul. 2013.

[14] K. Ago, Y. Uwate & Y. Nishio, "Investigation of Clustering in Complex Networks Using Coupled Chaotic Circuits," *Proc. of IFAT'15*, pp. 101-102, 2015.

[15] K. Oi, Y. Uwate & Y. Nishio, "Synchronization and Clustering in Coupled Parametrically Excited Oscillators with Small Mismatch," *Proc. of ISCAS'15*, pp. 910-913, 2015.