Effect of Stochastically Coupling on Frustrated Triangular Oscillatory Network

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2021 IEEE International Symposium on Circuits and Systems (ISCAS) | 978-1-7281-9201-7/20/$31.00 ©2021 IEEE | DOI: 10.1109/ISCAS51556.2021.9401462

***Abstract*— In this study, we focus on an effect of frustration to triangular oscillatory network with stochastically coupling. We propose a coupled nonliear circuit network with stochastically coupling. Frustration as environmental factor is occurred by network topology which is composed from polygonal structure. We investigate synchronization of the proposed network using different frustration levels by changing the coupling strength. By using computer simulations, the effect of frustration to triangular oscillatory networks with stochastically coupling is shown.**

I. Introduction

Coupled oscillatory circuits are excellent models for describing high-dimensional nonlinear phenomena occurring in our living world. In particular, synchronization is one of the most important functions that can be explained and explored with the help of an oscillator. This is because, when oscillators are coupled, a strong correlation rhythm between oscillators called a synchronized state appears. Therefore, many different types of coupled oscillatory networks were proposed and many interesting synchronization phenomena have been discovered.

In our research group, we have been studying the synchronization phenomena observed from the nonlinear oscillatory networks. We have investigated synchronization phenomena in coupled polygonal oscillatory networks. Through computer simulations and theoretical analysis, we confirmed that the coupled oscillators tended to synchronize to minimize the power consumption of the whole system. The phase difference of the shared oscillators was determined by finding the minimum value of the power consumption function. We consider that the synchronization in complex oscillator networks are useful for a deeper understanding of control methods in smart grid, communication systems and so on.

However, many of the networks that have been studied so far are static models, and it is necessary to propose a model that changes the network topology like a network observed in the real world including environmental factors. In this study, we focus on an effect of frustration to polygonal oscillatory network with stochastically coupling. We propose a coupled nonliear circuit network with stochastically coupling. Frustration as environmental factor is occurred by network topology which is composed from polygonal structure. In this system, van der Pol oscillators are connected to every node of a polygonal network. I n this oscillatory system, two adjacent oscillators tend to synchronize with anit-phase. Hence, if the number of nodes is odd, frustration occurs in the polygonal network. Each node has a coupling probability which is key factor to connect or not (on/off coupling). At every certain time in a simulation, the network topology is changed by the coupling probability. We investigate synchronization of the proposed network using different frustration levels by changing the coupling strength. By using computer simulations, the effect of frustration to polygonal oscillatory networks with stochastically coupling is shown.

II. Network Model using van der Pol Oscillators

The conceptual network models with different frustration levels used in this study is shown in Fig. 1. In this figure, triangular oscillators are coupled with edges on 2-dimensional space and a circle denotes a van der Pol oscillator. Each edge has a coupling probability (p) which is key factor to connect or not (on/off coupling). At every certain time (τ=500) in a simulation, the network topology is changed by the coupling probability.

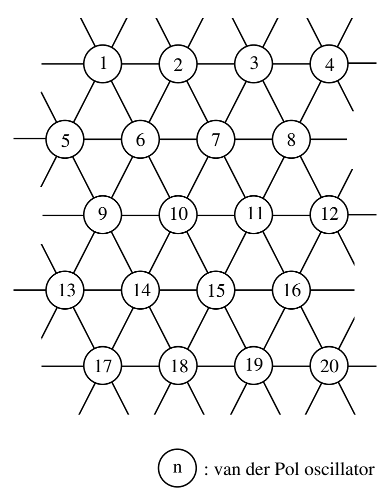

Fig. 1. Network model with triangular oscillators.

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Figure 2 shows a van der Pol oscillator. This oscillator is composed by an inductor, a negative resistance and a condenser. The oscillator component is very simple, however the oscillator could generate oscillation time wave. When the parameter of the nonlinearity is set to small value (e.g. ε=0.1), time wave form behaves similar to $\sin$ wave.

The circuit realization (target on 1st, 5th and 6th oscillators) of triangular oscillatory networks is shown in Fig. 3. In this circuit model, we use unique coupling method for two adjacent osicllators. Two adjacent oscillators are coupled by a resistor via a inductor which originally belongs to each van der Pol oscillators. The inductor of one van der Oscillator is devided to six to connect the next oscillators. If the oscillators are located at boundary position, the inductors connect to the earth resistance via resistor R.

By using this coupling scheme, two oscillators tend to synchronize at anti-phase state. However, in the triangular oscillatory network, two oscillators can not synchronized with anti-phase state because of the network structure. Then, the coupled oscillators synchronize with phase difference to minimize the enegy consumption. This is really original part compared with the other networks focusing on synchronization.

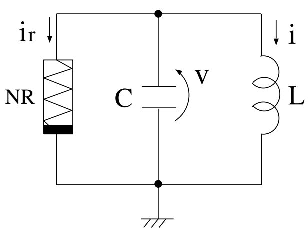

Fig. 2. van der Pol oscillator.

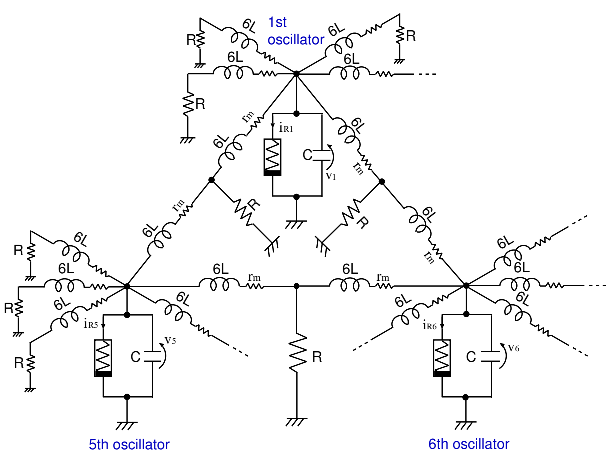

Fig. 3. Circuit realization for 1st, 5th and 6th oscillators in Fig. 1.

We develop the expression of the circuit equations of this model. The $v_k - i_{Rk}$ characteristics of the nonlinear resistor are assumed to be the following third order polynomial equation;

$$i_{Rk} = -g_1 v_k + g_3 {v_k}^3 \quad (g_1, g_3 > 0),\ (k = 1, 2, 3, 4). \tag{1}$$

Then the circuit dynamics is described by the following ordinary differential equation:

$$\begin{cases} C\dfrac{dv_k}{dt} = -i_{Rk} - i_{ak} - i_{bk} - i_{ck} \\ 6L\dfrac{di_{ak}}{dt} = v_k - r_m i_{ak} - R(i_{ak} + i_n) \\ 6L\dfrac{di_{bk}}{dt} = v_k - r_m i_{bk} - R(i_{bk} + i_n) \\ 6L\dfrac{di_{ck}}{dt} = v_k - r_m i_{ck} - R(i_{ck} + i_n) \\ 6L\dfrac{di_{dk}}{dt} = v_k - r_m i_{dk} - R(i_{dk} + i_n) \\ 6L\dfrac{di_{ek}}{dt} = v_k - r_m i_{ek} - R(i_{ek} + i_n) \\ 6L\dfrac{di_{fk}}{dt} = v_k - r_m i_{fk} - R(i_{fk} + i_n) \end{cases} \tag{2}$$

Where, i_n denotes the current from the neighbor oscillator over the corresponding coupling resistor. By using the variables and the parameters,

$$t = \sqrt{LC}\tau, \quad v_k = \sqrt{\frac{g_1}{3g_3}}x_k,$$

$$i_{ak} = \sqrt{\frac{g_1}{3g_3}}\sqrt{\frac{C}{L}}y_{ak}, \quad i_{bk} = \sqrt{\frac{g_1}{3g_3}}\sqrt{\frac{C}{L}}y_{bk},$$

$$i_{ck} = \sqrt{\frac{g_1}{3g_3}}\sqrt{\frac{C}{L}}y_{ck}, \quad i_{dk} = \sqrt{\frac{g_1}{3g_3}}\sqrt{\frac{C}{L}}y_{dk},$$

$$i_{ek} = \sqrt{\frac{g_1}{3g_3}}\sqrt{\frac{C}{L}}y_{ek}, \quad i_{fk} = \sqrt{\frac{g_1}{3g_3}}\sqrt{\frac{C}{L}}y_{fk},$$

$$i_n = \sqrt{\frac{g_1}{3g_3}}\sqrt{\frac{C}{L}}y_n,$$

$$\varepsilon = g_1\sqrt{\frac{L}{C}}, \quad \gamma = R\sqrt{\frac{C}{L}}, \quad \eta = r_m\sqrt{\frac{C}{L}},$$

The normalized circuit equations governing the circuit are expressed as

$$\begin{cases} \dfrac{dx_k}{d\tau} = \varepsilon\left(1 - \dfrac{1}{3}{x_k}^2\right)x_k - (y_{ak} + y_{bk} + y_{ck}) \\ \dfrac{dy_{ak}}{d\tau} = \dfrac{1}{6}\left\{x_k - \eta y_{ak} - \gamma(y_{ak} + y_n)\right\} \\ \dfrac{dy_{bk}}{d\tau} = \dfrac{1}{6}\left\{x_k - \eta y_{bk} - \gamma(y_{bk} + y_n)\right\} \\ \dfrac{dy_{ck}}{d\tau} = \dfrac{1}{6}\left\{x_k - \eta y_{ck} - \gamma(y_{ck} + y_n)\right\} \\ \dfrac{dy_{dk}}{d\tau} = \dfrac{1}{6}\left\{x_k - \eta y_{dk} - \gamma(y_{dk} + y_n)\right\} \\ \dfrac{dy_{ek}}{d\tau} = \dfrac{1}{6}\left\{x_k - \eta y_{ek} - \gamma(y_{ek} + y_n)\right\} \\ \dfrac{dy_{fk}}{d\tau} = \dfrac{1}{6}\left\{x_k - \eta y_{fk} - \gamma(y_{fk} + y_n)\right\} \end{cases} \tag{3}$$

In these equations, γ is the coupling strength, ε denotes the nonlinearity of the oscillators. For the computer simulations, we calculate Eq. (3) using the fourth-order Runge-Kutta

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method with the step size $h = 0.005$. The parameters of this circuit model are fixed as $\varepsilon = 0.1$ and $\eta = 0.0001$.

III. Simulation Results

For the computer simulations, 20 van der Pol oscillators are coupled in triangular oscillatory space like Fig. 1. By changing the coupling strength (γ) and the coupling probability (p) in the network, the average amplitude of coupled oscillators are investigated.

A. Amplitude Change

Figure 4 shows the simulation results of the average amplitude with coupling probability (p). In the case of static network (p=1.0), the average amplitude of 4 networks has big difference from 0.93 to 1.94. While if the coupling probability is set to 0.2 (dynamical networks), the average amplitude of all 4 networks has similar value around 1.99. From this result, we confirm that the frustrated network (γ=2.0) is affected a lot by stochastically coupling. By decreasing the coupling probability, the network topology is changed dynamically, then the frustration effect is defused.

The every amplitude are calclated as shown in Fig. 5. When the coupling strength is weak (γ=0.1), all amplitude of coupled oscillaotr does not change by changing the coupling probability (Fig. 5(a)). By increasing the coupling strength, the amplitude of some oscillators decrease with the coupling probability. The amplitude of the oscillators which are located in the middle of the network more decrease than the others (Fig. 5(b)-(d)).

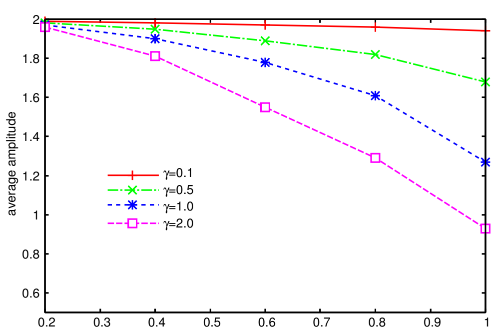
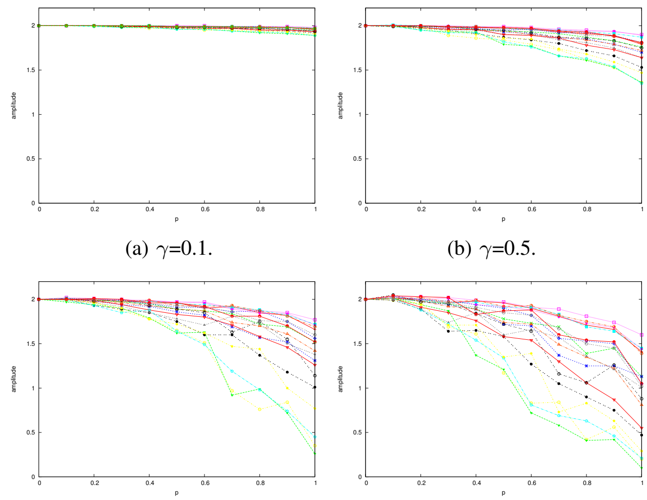


Fig. 4. Average amplitude.

B. Phase Difference

Next, we investigate the phase difference between 1st oscillator and the others when the coupling probability is changed. For the simulation, initial condition is set to that two adjacent oscillattors synchronize with anti-phase state. The simulation results of the phase differences are shown in Figs. 6 and 7. In the case of Fig. 6, the coupling strength is fixed with γ=0.5. The phase difference converges several points when the coupling probability is set to p=1.0. By decreasing the value of the coupling probability, the phase difference starts to vibrate with small amplitude. And, some oscillators do not synchronize when the coupling probability p is smaller than 0.4.



(a) γ=0.1. (b) γ=0.5.

(c) γ=1.0. (d) γ=2.0.

Fig. 5. Amplitude change.

In the case of Fig. 7, the coupling strength is fixed with γ=1.0. The coupled oscillatory network has frustration. Therefore, the phase difference can not converge one point as shown in Fig. 7(a),(b). By decreasing the value of the coupling probability (p=0.6), the phase difference converges with small vibrations at some points (Fig. 7(c),(d)). For the frustrated network, the stocastically coupling is effective to obtained the similar phase difference with the non-frustrated network.

IV. Conclusion

In this study, we proposed coupled frustrated polygonal oscillators with a stochastically coupling. This coupling is switched on/off state depending on the coupling probability. As the first step, 20 van der Pol oscillators are coupled in triangular oscillatory network was investigated. We calculate the amplitude of coupled oscillators in order to capture the effect of frustrations. From computer simulations, we confirmed that by decreasing the coupling probability in the proposed networks, the average amplitude of all networks reached to 2.0 which means that network has no frustration.

Next, we also investigated the phase diffrence by changing the coupling strength. When the network with weak coupling, the oscillators does not synchronize with small value of the coupling probability. While, the case of the network with strong coupling, the original phase difference can be obtained by using the appropriate coupling probability.

References

[1] J. D. Watts and S. H. Strogatz, "Collective Dynamics of Small-World Networks," *Nature*, 393, 440, 1998.
[2] S. H. Strogatz, "Exploring Complex Networks," *Nature*, 410, 268, 2001.
[3] E. Bullmore and O. Sporns, "The Economy of Brain Network Organization," *Nat. Rev. Neurosci.*, 10, 186, 2009.
[4] T. Nishikawa, A. E. Motter, Y. C. Lai and F. C. Hoppensteadt, "Heterogeneity in Oscillator Networks: Are smaller worlds easier to synchronize?," *Phys. Rev. Lett.*, 91, 014101, 2003.

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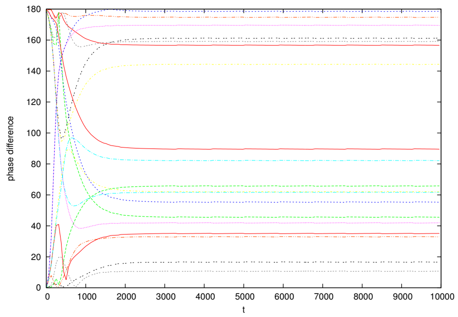
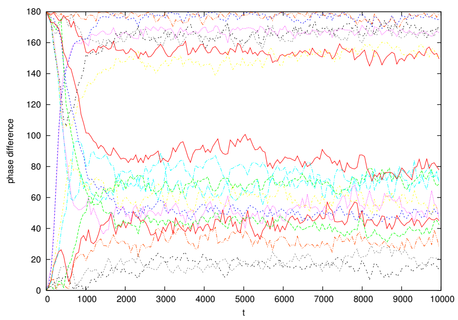
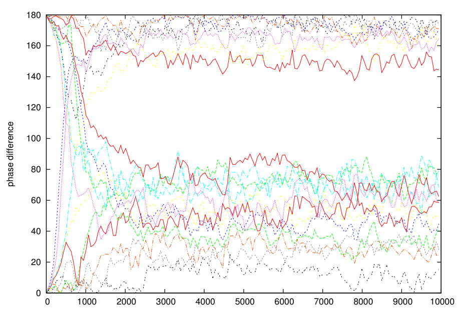


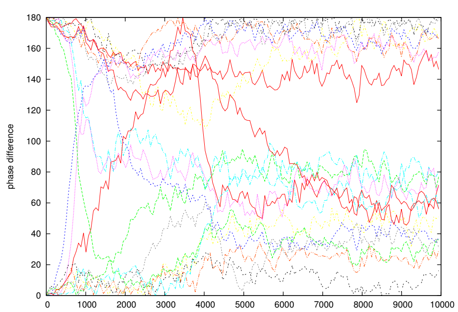
(a) p=1.0.



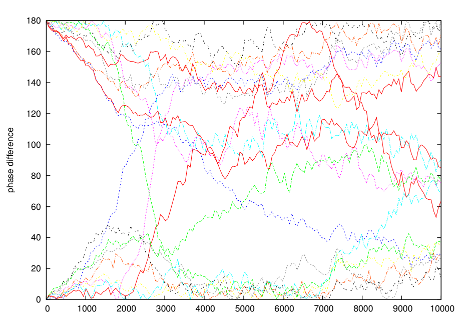
(b) p=0.8.



(c) p=0.6.

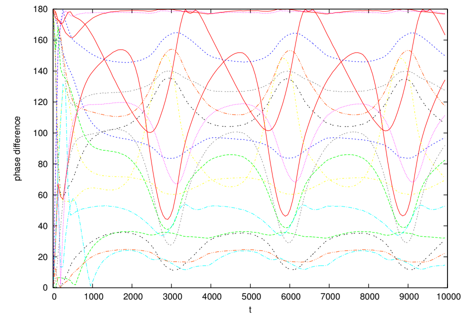


(d) p=0.4.

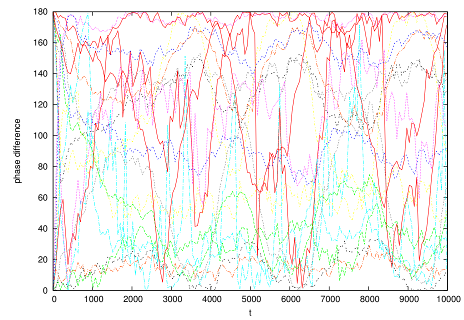


(e) p=0.2.

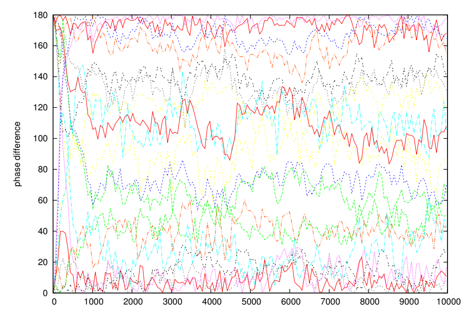
Fig. 6. Phase difference (γ=0.5).



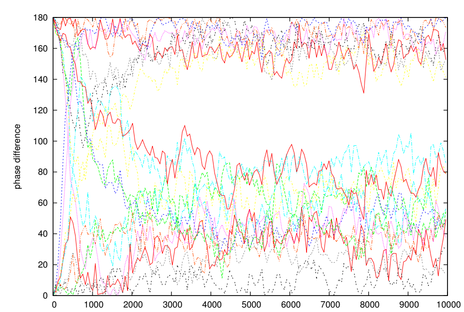
(a) p=1.0.



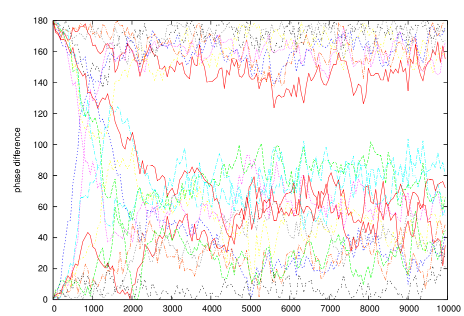
(b) p=0.8.



(c) p=0.6.



(d) p=0.4.



(e) p=0.2.

Fig. 7. Phase difference (γ=1.0).

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[5] Y. Moreno and A. F. Pacheco, “Synchronization of Kuramoto Oscillators in Scale-Free Networks,” *Europhys. Lett.*, 68, 603, 2004.
[6] M. Chavez, D. -U. Hwang, A. Amann, H. G. E. Hentschel and S. Boccaletti, “Synchronization is Enhanced in Weighted Complex Networks,” *Phys. Rev. Lett.*, 94, 218701, 2005.
[7] A. Arenas, A. Díaz-Guilera and C. J. Perez-Vicente, “Synchronization Reveals Topological Scales in Complex Networks” *Phys. Rev. Lett.*, 96, 114102, 2006.
[8] C. Zhou, A. E. Motter and J. Kurths, “Universality in the Synchronization of Weighted Random Networks,” *Phys. Rev. Lett.*, 96, 034101, 2006.
[9] J. Gómez-Gardeñes, Y. Moreno and A. Arenas, “Paths to Synchronization in Complex Networks” *Phys. Rev. Lett.*, 034101, , 2007.
[10] I. Lodato, S. Boccaletti and V. Latora, “Synchronization Properties of Network Motifs” *Europhys. Lett.*, 78, 28001, 2007.
[11] A. Arenas, A. Díaz-Guilera, J. Kurths, Y. Moreno and c. Zhou, “Synchronization in Complex Networks” *Phys. Rep.*, 469, 93, 2008.
[12] J. Gómez-Gardeñes, S. Gomez, A. Arenas and Y. Moreno, “Explosive Synchronization Transitions in Scale-Free Networks” *Phys. Rev. Lett.*, 106, 128701, 2011.
[13] L. V. Gambuzza, A. Cardillo, A. Fiasconaro, L. Fortuna, J. Gomez-Gardenes and M. Frasca, “Analysis of Remote Synchronization in Complex Networks,” *Chaos*, 23, 043103, 2013.
[14] S. Boccaletti, J. Kurths, G. Osipov, D. Valladares and C. Zhou, “The Synchronization of Chaotic Systems” *Physics Reports*, 366, pp. 1-101, 2002.
[15] A. Arenas, A. Diaz-Guilera, J. Kurths, Y. Moreno and C. Zhou, “Phase Synchronization of Chaotic Oscillators” *Physics Reports*, 469, pp. 93-153, 2008.
[16] M.G. Rosenblum, A.S. Pikovsky and J. Kurths, “Phase Synchronization of Chaotic Oscillators” *Physical Review Letters*, vol.76, no.11, pp.1804-1807, Mar. 1996.
[17] W. Wang, I.Z. Kiss and J.L. Hudson, “Experiments on Arrays of Globally Coupled Chaotic Electrochemical Oscillators: Synchronization and Clustering” *Chaos*, vol.10, no.1, pp.248-256, Mar. 2000.
[18] M. Yamauchi, Y. Nishio and A. Ushida, ”Phase-waves in a Ladder of Oscillators” *IEICE Trans. Fundamentals*, vol.E86-A, no.4, pp.891-899, Apr. 2003.
[19] H.B. Fotsina and J. Daafouza, “Adaptive Synchronization of Uncertain Chaotic Colpitts Oscillators based on Parameter Identification” *Physics Letters A*, vol.339, pp.304-315, May 2005.
[20] M. Shinriki, M. Yamamoto and S. Mori, “Multimode Oscillations in a Modified van der Pol Oscillator Containing a Positive Nonlinear Conductance,” *Proc. of IEEE*, vol. 69, pp. 394-395, 1981.

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