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Investigation of Nonlinear Characteristics of Oscillators Using Gyrator

Yoko Uwate and Yoshifumi Nishio

Dept. of Electrical and Electronic Engineering, Tokushima University
2-1 Minami-Josanjima, Tokushima, Japan
Email: {uwate, nishio}@ee.tokushima-u.ac.jp

Abstract

In this study, we investigate the change in nonlinear features when the coil of van der Pol oscillator is replaced by gyrator and capacitor. The oscillator with a gyrator has two more parameters than the standard oscillator. We investigate the nonlinearity features when those two parameters are changed. And we discuss whether the oscillator using a gyrator exhibits the same phenomena as the standard oscillator, and whether new phenomena can be observed.

1. Introduction

The coupled oscillator system is an excellent model for representing high-dimensional nonlinear phenomena. The synchronization phenomena observed in the coupled oscillator system depend on the topology of the network and the oscillators at the nodes, and a variety of oscillator networks have been proposed and interesting synchronization phenomena have been observed.

In our research group, we have been focusing on the synchronization of coupled oscillaotrs and chaotic circuits as several types of network topologies. In Ref. [1], we have reported the synchronization phenomena of van der Pol oscillators observed from polygonal oscillatory networks. The obtained synchronization were explained by using the theoretical analysis of power consumptions. There are several engineering applications of the coupled polygonal oscillatory networks, for example, synchronization problem of complex networks [2] including the Internet, the World Wide Web, sensor networks, and power grid networks, and generation and control of gait patterns of multi-legged robots [3], [4].

In recent years, advances in semiconductor technology have led to the incorporation of various circuits as integrated circuits. However, the van der Pol oscillation circuit contains a coil. It is extremely difficult to create an inductor using a coil in an integrated circuit, so there is a method of expressing the inductor characteristics using a gyrator and a capacitor. In this study, we use a circuit in which the inductor of a conventional van der Pol oscillation circuit is represented by a gyrator and a capacitor, and investigate the nonlinear characteristics when the parameters are chenged.

2. van der Pol oscillator using Gyrator

Figure 1 shows a standard van der Pol oscillator which is composed of one inductor, one capacitor and one negative resister. The circuit equation is expredded by the following equation.

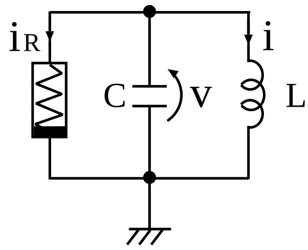


Figure 1: A standard van der Pol oscillator.

$$\begin{cases} L\dfrac{di}{dt} = v \\ C\dfrac{dv}{dt} = -i - i_R \end{cases} \tag{1}$$

where

$$i_R = -g_1 v + g_3 v^3. \tag{2}$$

By using the variables and the parameters,

$$t = \sqrt{LC}\tau, \quad v = \sqrt{\frac{g_1}{3g_3}}x,$$

$$i = \sqrt{\frac{g_1}{3g_3}}\sqrt{\frac{C}{L}}y, \quad \varepsilon = g_1\sqrt{\frac{L}{C}},$$

the normalized circuit equations are obteined as follows.

$$\begin{cases} \dfrac{dx}{d\tau} = \varepsilon x(1 - \dfrac{1}{3}x^2) - y \\ \dfrac{dy}{d\tau} = x \end{cases} \tag{3}$$

For the computer simulations, we calculate Eq. (3) using the fourth-order Runge-Kutta method with the step size $h = 0.01$.

In Eq. (3), the parameter ε denotes the nonlinearlity of oscillator. When the value of the parameter ε is close to zero, the nonlinearity of the capacitor voltage waveform is weak, and the waveform closely resembles a sine wave. In contrast, as the value of ε increases, the nonlinearity becomes stronger and the waveform becomes similar to a square wave.

Figure 2 shows the how to change the time wave form by changing the nonlinearity parameter ε.

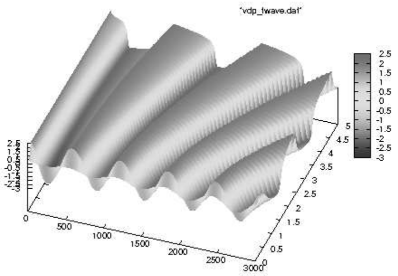


Figure 2: Nonlinear Characteristcis of time wave form obtained from a starndard van der Pol oscillator.

Next, we consider the oscillator which is composed of one negative resistor, two capacitors and one gyrator as shown in Fig. 3. A gyrator is a circuit network that inverts impedance. By using a gyrator, it is possible to convert capacitance into inductance.

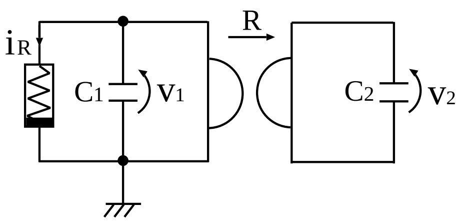


Figure 3: van der Pol using Gyrator.

The circuit equation is expredded by the following equation.

$$\begin{cases} C_1 \dfrac{dv_1}{dt} = -\dfrac{1}{R} v_2 - i_R \\ C_2 \dfrac{dv_2}{dt} = \dfrac{1}{R} v_1 \end{cases} \tag{4}$$

where

$$i_R = -g_1 v_1 + g_3 v_1^3. \tag{5}$$

By using the variables and the parameters,

$$t = \sqrt{C_1 C_2}\tau, \quad v_1 = \sqrt{\frac{g_1}{3g_3}}x,$$

$$v_2 = \sqrt{\frac{g_1}{3g_3}}y, \quad \varepsilon = g_1\sqrt{\frac{C_2}{C_1}},$$

$$\alpha = \sqrt{\frac{C_2}{C_1}}, \quad \beta = \frac{1}{R}.$$

the normalized circuit equations are obteined as follows.

$$\begin{cases} \dfrac{dx}{d\tau} = \varepsilon x(1 - \dfrac{1}{3}x^2) - \alpha\beta y \\ \dfrac{dy}{d\tau} = \dfrac{\beta}{\alpha} x \end{cases} \tag{6}$$

The oscillator with a gyrator has two more parameters (α and β) than the standard van der Poloscillator. If the values of the two parameters are both zero ($\alpha = \beta = 1.0$), then the differential equation is exactly the same as that of the standard oscillator. The time series waveforms of the changing the parameter ε are shown in Fig. 4. This result is the same as for the standard van der Pol oscillator (Fig. 2).

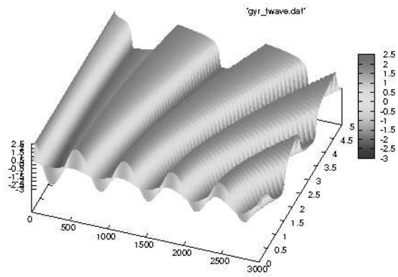


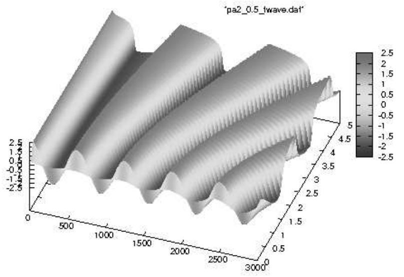
Figure 4: Nonlinear Characteristcis of time wave form obtained from oscillator using Gyrator.

3. Nonlinear Characteristics of Oscillator using Gyrator

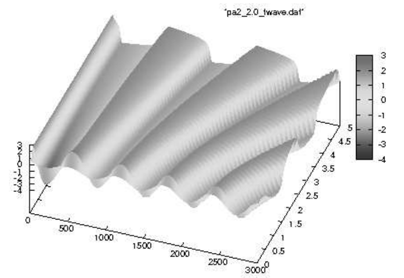
In this section, we investigate the change in the nonlinearity of the voltage waveform of the capacitor with the values of parameters α and β. Figure 5 shows the simulation results when the parameter α is set to 0.5, 2.0, and 5.0. This result shows that different alphas have no effect on the change in nonlinearity. It was found that as α increases, the amplitude of the time series increases.

Figure 8 shows the simulation results when the parameter β is set to 0.5, 2.0, and 5.0. In this case, we can see that the change in nonlinearity varies greatly with the value of β. First, it can be said that the oscillation frequency of the oscillator depends on the value of β. When the value of β is 0.5, the oscillation frequency is small, and as the beta increases, the oscillation frequency increases.

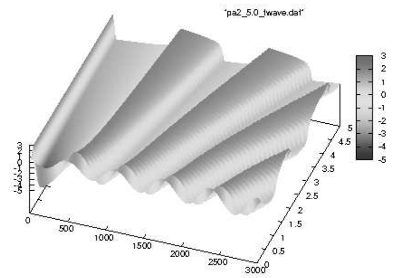
Furthermore, if we look at the change in nonlinearity, we can see that when β is 0.5, the nonlinearity becomes stronger as ϵ increases. However, when β is 2.0 and 5.0, the nonlinearity is not so strong even when ϵ is increased.



(a) α=0.5.

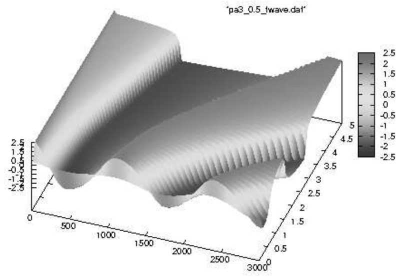


(b) α=2.0.

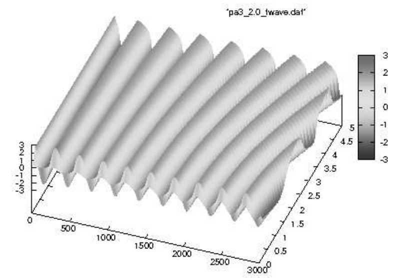


(c) α=5.0.

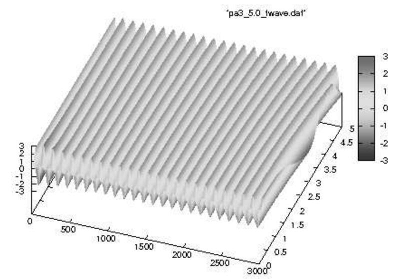
Figure 5: Time wave forms depending on α.



(a) β=0.5.



(b) β=2.0.



(c) β=5.0.

Figure 6: Time wave forms depending on β.

4. Oscillations using Circuit Simulator

Finally, the oscillation phenomenon of the gyrator-type oscillator is investigated using a circuit simulator (LTspice). The circuit model of the gyrator-type oscillator is shown in Fig. 7. The gyrator is modeled by the voltage controlled current source.

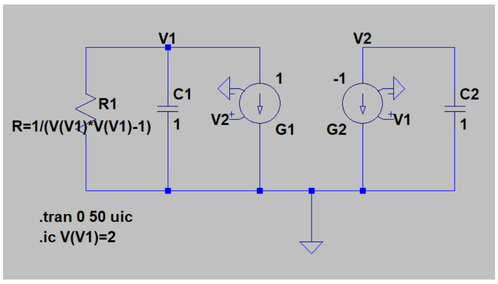
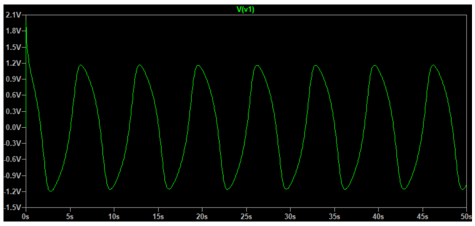


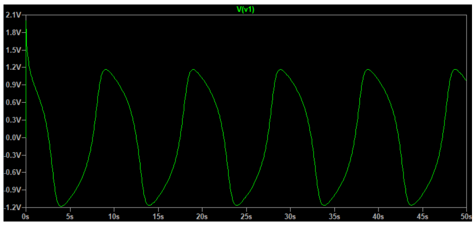
Figure 7: Gyrator type of oscillator for LTspice.

The simulation results of LTspice are shown in Fig. 8. By changing the value of the conductance (G_2) of the voltage-controlled current source, it corresponds to the change in β. From these results, it can be said that the oscillation frequency increases as the value of conductance increases, and the nonlinearity does not become stronger, which is similar to the simulation results described above. However, when $G_2 = 0.5$, the nonlinearity is stronger than in the simulation results above. The investigation of this difference will be the subject of future research.

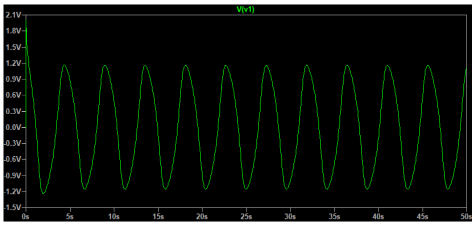
In addition, we investigated oscillation in a gyrator circuit using an operational amplifier, but we could not successfully observe the oscillation phenomenon.



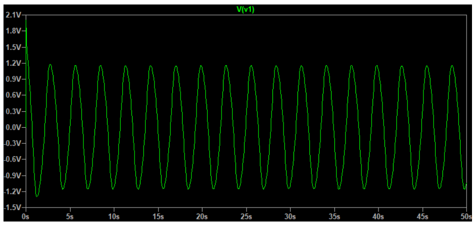
(a) G_2=1.0.



(b) G_2=0.5.



(c) G_2=2.0.



(d) G_2=5.0.

Figure 8: Simulation retuls of LTspice.

5. Conclusions

In this study, we have investigated the change in nonlinear features when the coil of van der Pol oscillator is replaced by gyrator and capacitor. By changing the parameters of the gyrator, we were able to observe changes in the nonlinearity features that could not be observed with the standard van der Pol oscillator. Oscillators using gyrators do not use coils, which makes them suitable for integrated circuits.

The investigation of synchronization phenomena in a network of coupled gyrator oscillators is our future work.

References

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