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Realization of Associative Memory by Oscillators Changing the Coupling Strength

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Abstract

In this paper, we adapt the theory of the Associatron to the coupling strength of Kuramoto model and realize associative memory in Kuramoto model. We consider that this realization enables associative memory in the coupling system of the oscillators as well. Therefore, we solve the circuit equations of the coupling system of the van der Pol oscillators and realize associative memory by using oscillation and synchronization. In this study, we propose that nine van der Pol oscillators are connected by resistors and simulate 3×3 image processing.

1. Introduction

There are various synchronization phenomena in nature, such as the emission of fireflies and the movement of pendulums. Similarly, synchronization phenomena have been observed in the coupling system of oscillators. Interestingly, it has been proven that pattern recognition and associative memory can be realized by using this oscillator synchronization phenomena[1-2]. It is also known that pattern recognition and associative memory can be realized by using this technology. Pattern recognition and associative memory are excellent human abilities, but there is still a big difference in their abilities compared to computers. For a long time, many researchers have been actively developing and studying CNN and other technologies to solve this problem. In recent years, the synchronization of oscillator coupling systems begin to attract attention, and research on pattern learning, recall, and recognition using various oscillators begin to be conducted [3-4]. In this study, we first use Kuramoto model to realize associative memory[5]. Next, we verify whether the same results can be obtained with a coupling system of van der Pol oscillators.As a result, associative memory was achieved in both cases. However, it was found that the coupling strength had to be reduced when using van der Pol oscillators.

2. Associative memory using Kuramoto model

We conduct an experiment with nine oscillators W1$\sim$W9 mutually coupled, which is shown in Fig.1. We observe the synchronization of the coupling system of these oscillators.

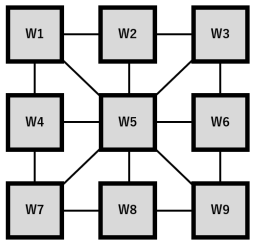

Figure 1: Coupling system of oscillators

We use Kuramoto model, which is a well-known mathematical model of synchronization. Since there are nine oscillators to be used this time, N=9. We devise the coupling strength K in this equation based on the Associatron[6].

$$\frac{\partial \theta_i}{\partial t} = \omega + \frac{K}{9}\sum_{j=1}^{9} \sin(\theta_j - \theta_i) \qquad i = 1, 2, ..., 9 \tag{1}$$

For Eq.(1), we redefine

$$K = \begin{cases} E_0 \times s \\ E \times s \end{cases}$$

where s is the coupling strength and E is the coupling strength matrix determined based on each pattern; E_0 is the storage matrix for the input pattern and E is the storage matrix for the stored pattern. Since we have nine oscillators, we use a 3×3 pattern. First, each square of the pattern corresponds to the oscillator W1$\sim$W9 in Fig.1 from the upper left, and f^0 and f^p are created with the black area of the image as 1 and the white area as -1. In this case, p is the number of patterns to be stored, and f^p is created for the number of patterns.

$$\begin{cases} f^0 = f^0_{W1}, f^0_{W2}, f^0_{W3}, ..., f^0_{W9} \\ f^p = f^p_{W1}, f^p_{W2}, f^p_{W3}, ..., f^p_{W9} \end{cases}$$

This f^0 and f^p are used to define E_0 and E.

$$E_0 = f^0 f^{0T}$$
$$= \begin{pmatrix} f^0_{W1} f^0_{W1} & f^0_{W1} f^0_{W2} & \cdots & f^0_{W1} f^0_{W9} \\ f^0_{W2} f^0_{W1} & f^0_{W2} f^0_{W2} & \cdots & f^0_{W2} f^0_{W9} \\ \vdots & \vdots & \ddots & \vdots \\ f^0_{W9} f^0_{W1} & f^0_{W9} f^0_{W2} & \cdots & f^0_{W9} f^0_{W9} \end{pmatrix} \quad (2)$$

$$E = \sum_{k=1}^{p} f^k f^{kT}$$
$$= \sum_{k=1}^{p} \begin{pmatrix} f^0_{Wk} f^k_{W1} & f^k_{W1} f^k_{W2} & \cdots & f^k_{W1} f^k_{W9} \\ f^k_{W2} f^k_{W1} & f^k_{W2} f^k_{W2} & \cdots & f^k_{W2} f^k_{W9} \\ \vdots & \vdots & \ddots & \vdots \\ f^k_{W9} f^k_{W1} & f^k_{W9} f^k_{W2} & \cdots & f^k_{W9} f^k_{W9} \end{pmatrix} \quad (3)$$

Now we are ready for associative memory. First, we use the memory matrix E_0 of the input pattern to synchronize it. After that, we change to the storage matrix E of the storage pattern and synchronize it again. The memory recall start from the point of the change. From the calculated phase, we derive the phase difference from the reference oscillator of W1. When the phase difference is 0, 2π..., it is said to be synchronized with W1. On the other hand, when the phase difference is π, 3π, ..., it can be said to be asynchronous. The phase difference is converted into a pattern by determining whether it is synchronized with W1 or not.

3. Simulation of associative memory using Kuramoto model

The two patterns to be stored in the system are shown in Fig.2, and the input pattern is shown in Fig.3.

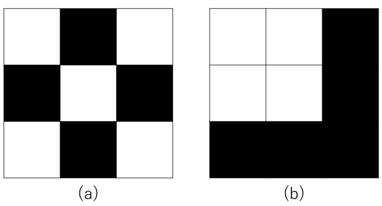

Figure 2: Stored patterns.

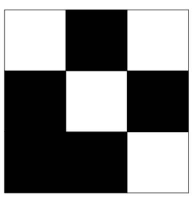

Figure 3: Input pattern.

Based on these condition, E_0 and E are calculated. The initial value of θ is given randomly between 0 and 0.1[rad], and $\omega = \pi$[rad/s], $s = 0.4$. The calculation is performed until $t = 0 \sim 60$[s], and at $t = 25$[s], the storage matrix is changed from E_0 to E for the memory recall. The oscillation waveform obtained from the calculated phase is shown in Fig.4, and the graph showing the phase difference of each oscillator with oscillator W1 is shown in Fig.5.

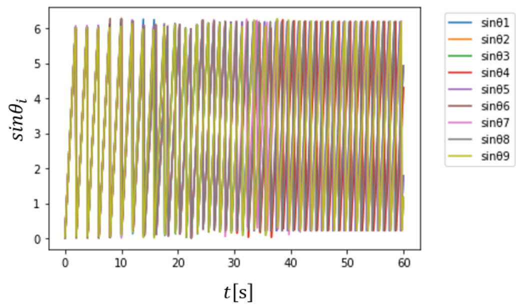

Figure 4: The oscillation waveforms (Kuramoto Model).

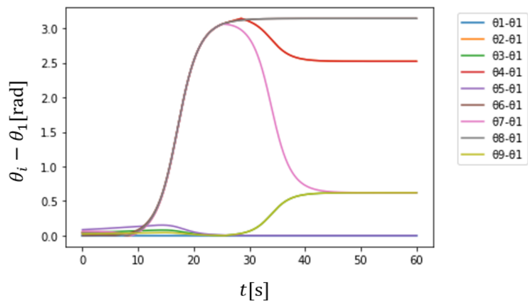

Figure 5: The phase difference (Kuramoto Model).

Figure 6 shows the pattern we created based on the phase difference at $t = 60$[s]. It can be seen that associative memory is achieved.

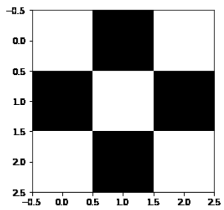

Figure 6: The recall pattern (Kuramoto Model).

4. Associative memory by a coupling system of van der Pol oscillators

In the previous section, we were able to achieve associative memory by devising the coupling strength of Kuramoto model. Next, we perform the same verification on the coupling system of van der Pol oscillators as shown in Fig.7. Nine oscillators are connected by resistors, so the circuit equations are Eq.(4). i_G is the equation of the nonlinear element, which is Eq.(5).

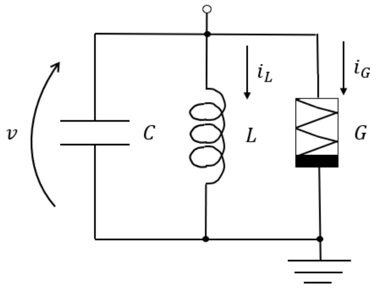

Figure 7: Circuit model of Van der Pol Oscillator

$$\begin{cases} C\frac{dv_n}{dt} = i_L - i_G - \sum_{k=1}^{9} \frac{1}{R}(v_n - v_k) \\ L\frac{di_n}{dt} = v_n \qquad\qquad n = 1, 2, ..., 9 \end{cases} \tag{4}$$

$$i_G = -g_1 v + g_3 v^3 \tag{5}$$

The parameters for normalization of Eq.(4) are follows:

$$v = \sqrt{\frac{g_1}{g_3}}x, \qquad i = \sqrt{\frac{g_1 C}{g_3 L}}y, \qquad t = \sqrt{LC}\tau,$$

$$\epsilon = g_1\sqrt{\frac{L}{C}}, \qquad \gamma = \frac{1}{R}\sqrt{\frac{L}{C}}.$$

The normalized circuit equations are described as Eq.(6):

$$\begin{cases} \frac{dx_n}{d\tau} = \epsilon x_n(1 - x_n^2) - y_n + \sum_{k=1}^{9} K(v_n - v_k) \\ \frac{dy_n}{d\tau} = x_n \qquad\qquad n = 1, 2, ..., 9. \end{cases} \tag{6}$$

In this case, K is

$$K = \begin{cases} E_0 \times s \\ E \times s \end{cases}$$

as in the previous section. The calculated x may have different amplitudes depending on each oscillator. Therefore, instead of considering the phase difference as in the associative memory of Kuramoto model, we consider the difference between the oscillation waveform x_1 of oscillator W1. When it is synchronous, the difference be zero, and when it is asynchronous, the amplitude have doubled from the original oscillation waveform. In this way, determine whether it is synchronized with W1, and convert it into a pattern.

5. Simulation of associative memory using van der Pol oscillator

The image to be stored and input are exactly the same as in the experiment in section 3, so $E \cdot E_0$, and s are the same values. The initial values of x and y are given randomly between 0 and 0.1, $\varepsilon = 0.1$, and $s = 0.01$. At $\tau = 100$, the storage matrix is changed from E_0 to E for the memory recall. From the above, calculations are performed for $\tau = 0 \sim 200$, and the graph of x_n is shown in Fig.8. And Fig.9 shows the result of calculating the difference between the oscillation waveform of oscillator W1 and x_1.

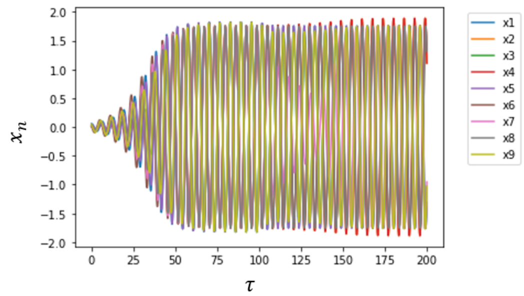

Figure 8: The oscillation waveforms (van der Pol oscillator).

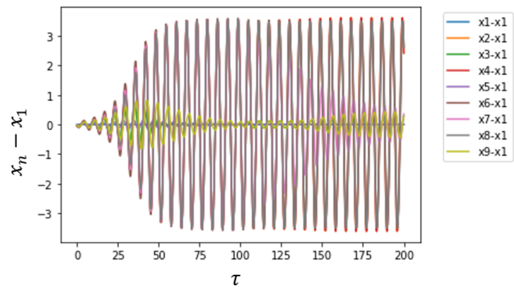

Figure 9: The synchronization state (van der Pol oscillator).

The pattern created based on the synchronization state at $\tau = 200$ is shown in Fig.10. It can be seen that associative memory can be realized even with a coupling system of van der Pol oscillators.

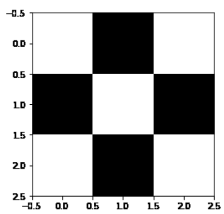

Figure 10: The recall pattern (van der Pol oscillator).

In this section, we experimented with $s = 0.01$, because recall was not successful with $s = 0.4$, the same condition as in section 3. Under this condition, we confirmed that the oscillator W1 cyclically repeats synchronously and asynchronously from the moment it is changed to the storage matrix E. An example of this is shown in Fig.11. After switching the storage matrix to E at $\tau = 30$, the oscillation waveforms become asynchronous around $\tau = 80, 180$ and synchronous around $\tau = 130, 230$.

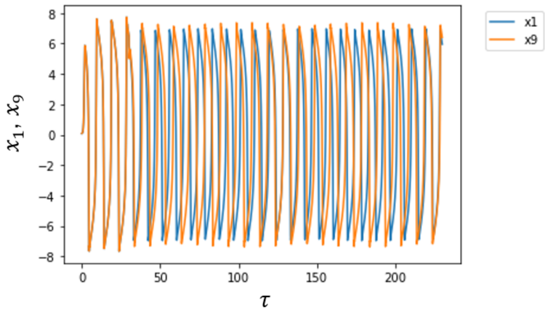

Figure 11: The oscillation waveforms(Van der Pol oscillator, W1,W9)

In addition to W9, this phenomenon was also observed in four other oscillators W2, W3, W4, and W7. Therefore, we created a pattern based on the synchronization states of $\tau = 80, 130, 180, 230, \ldots$ which are periodically synchronized and asynchronized. As a result, the two patterns shown in Fig.2(a) and 2(b) were repeatedly recalled, confirming that associative memory cannot be realized at $s = 0.4$. These results indicate that associative memory can be achieved with a van der Pol oscillator, but the coupling strength needs to be reduced to achieve this.

6. Conclusion

In this study, we have realized associative memory for a 3×3 pattern by changing the coupling strength. Using Kuramoto model to realize associative memory, we consider that associative memory can be realized in any coupling system of oscillators where the coupling strength can be changed. Therefore, as an example, we realized associative memory with a van der Pol oscillator. However, it was also confirmed that in the coupled system of the van der Pol oscillator, when the coupling strength was too large, it was not recalled well.

However, this study was not able to investigate the extent to which the coupling strength affects the associative memory. Therefore, the task is to verify the effects of changes in the coupling strength and storage matrix on associative memory. It is also necessary to verify the results with analog circuits.

References

[1] Paolo Maffezzoni, Bichoy Bahr, Zheng Zhang, and Luca Daniel, " Oscillator Array Models for Associative Memory and Pattern Recognition ", IEEE Transactions on Circuits and Systems I: Regular Papers (Volume: 62, Issue: 6, June 2015) pp.1591-1598.

[2] Frank. C. Hoppensteadt and Eugene. M. Izhikevich, " Oscillatory neurocomputers with dynamic connectivity ", Physical Review Letters (Volume: 82, Issue: 14, April 1999) pp. 2983-2986.

[3] Ting Zhang, Mohammad R. Haider, Iwan D. Alexander, Yehia Massoud, "A Coupled Schmitt Trigger Oscillator Neural Network for Pattern Recognition Applications", 2018 IEEE 61st International Midwest Symposium on Circuits and Systems (August 2018) pp. 238-241.

[4] Tetsuya Yamamoto, Takashi Amemiya, Tomohiko Yamaguchi, "Learning and recalling of phase pattern in coupled BVP oscillators ", 2005 IEEE International Symposium on Circuits and Systems (Volume: 3, July 2005) pp. 2839 – 2842.

[5] Yoshiki Kuramoto, "Self-entrainment of a population of coupled non-linear oscillators ", International Symposium on Mathematical Problems in Theoretical Physics (January 1975) pp. 420-422.

[6] Kaoru Nakano, " Associatron- A Model of Associative Memory ", IEEE Transactions on Systems, Man, and Cybernetics (Volume: SMC-2, Issue: 3, July 1972) pp. 380-388.