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Amplitude Death observed from Coupled Oscillators in Several Types of Polyhedron

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Abstract—The study of Amplitude Death (AD) and amplitude change are important for understanding the control mechanisms and efficient regulators of a system's dynamics. Previously, the mechanism of AD observed from two-dimensional coupled polygonal oscillatory networks was made clear by using mathematical analysis. In this study, we extend the polygonal oscillatory networks to three-dimensional space. We investigate the occurrence of AD by using several types of 3D oscillatory network models.

1. Introduction

Synchronization phenomena in coupled oscillators are suitable models for analyzing a number of natural occurrences [1],[2]. Therefore, many researchers have proposed different coupled oscillatory networks, and some interesting synchronization phenomena have been discovered [3]-[6].

Oscillation quenching (oscillation and amplitude death), another fundamental emergent phenomenon in coupled nonlinear systems, can be caused by several factors [7],[8]. Amplitude death (AD) occurs in strongly coupled nonlinear oscillators when their interaction causes a pair of fixed points to become stable and attracting. Setou et al. reported AD in ring coupled oscillators when the frequencies of the coupled units differ [9].

We have investigated synchronization phenomena in coupled polygonal oscillatory networks that share branches [10], [11]. In this system, van der Pol oscillators are connected to every corner of each polygonal network. The first and the second oscillators, which are connected to both polygonal networks, are called "shared oscillators," and each polygonal network has an odd number of oscillators. We then observe N-phase synchronization. Through computer simulations and theoretical analysis, we confirmed that the coupled oscillators tended to synchronize to minimize the power consumption of the whole system. The phase difference of the shared oscillators was determined by finding the minimum value of the power consumption function. Additionally, we proposed a new polygonal circuit system that includes actual inductor models (with loss) at all ground parts [12]. Synchronization phenomena in coupled polygonal oscillatory networks with strong frustration are investigated.

We confirmed that the amplitude of the oscillators decreases as the value of the coupling strength increases, and that AD occurs in the polygonal oscillatory networks. We explained the mechanism by which AD occurs using a theoretical approach.

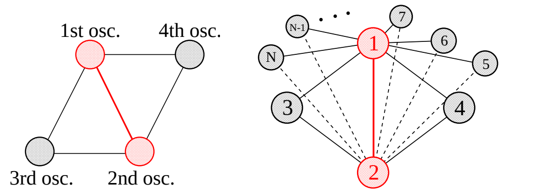
In this study, we investigate the occurrence of AD of coupled polygonal oscillatory networks in three-dimensional space. For considering the three-dimensional networks, we propose two types of network models. The first model is composed of a regular tetrahedron. As the second model, we propose a 3D model by adding depth to the N–N coupling network models that has been proposed in the two-dimensional space. By using computer simulations, we observe similar AD phenomena in the proposed three-dimensional coupled polygonal network with previous studies. We also find unique AD phenomena in 3D oscillatory network model.

2. Coupled Oscillatory Networks in Two-Dimensional Space [13]

In our previous study, we investigated AD in two coupled oscillatory networks in two-dimensional space. Two polygonal oscillatory networks are coupled by sharing a branch. And we investigate AD when the number of triangular or pentagonal networks connected to shared branch is increased. Examples of the network models used in the previous study are shown in Figs. 1 and 2. Here, the network model is composed of triangular oscillatory networks as shown in Fig. 1. While, Fig. 2 shows the circuit model which is composed of pentagonal oscillatory networks.

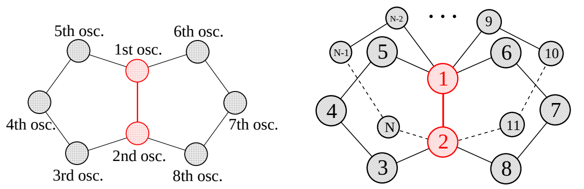
In these circuit models, we consider the coupling method in which two adjacent oscillators tend to synchronize in the anti-phase state. The number of oscillators coupled to left and the right polygonal networks is set to odd numbers to produce anti-phase synchronization between adjacent oscillators. The first and second oscillators, which are connected to both sides of the polygonal networks, are called "shared oscillators."

Figure 3 shows the circuit model of the 3–3 coupling network. The earth resistances are inserted to the 3rd and 4th oscillators to model actual inductors and realize symmetry in the circuit network model. Tiny resistors (r_m) are inserted to avoid an L-loop in the computer simulations.



(a) 3–3 coupling network. (b) N–triangular coupling network.

Figure 1: Coupled oscillatory networks composed of triangular network.



(a) 5–5 coupling network. (b) N–pentagonal coupling network.

Figure 2: Coupled oscillatory networks composed of pentagonal network.

Next, we develop an expression for the circuit equations of the N–M coupling oscillatory network. The $v_k - i_{Rk}$ characteristics of the nonlinear resistor are approximated by the following third-order polynomial equation:

$$i_{Rk} = -g_1 v_k + g_3 {v_k}^3 \quad (g_1, g_3 > 0), \qquad (1)$$
$$(k = 1, 2, 3, ..., N + M - 2).$$

Tthe normalized circuit equations governing the circuit are expressed as:

$$\begin{cases} \dfrac{dx_k}{d\tau} = \varepsilon\left(1 - \dfrac{1}{3}{x_k}^2\right)x_k - (y_{ak} + y_{bk} + y_{ck}) \\ \dfrac{dy_{ak}}{d\tau} = \dfrac{1}{3}\left\{x_k - \eta y_{ak} - \gamma(y_{ak} + y_n)\right\} \\ \dfrac{dy_{bk}}{d\tau} = \dfrac{1}{3}\left\{x_k - \eta y_{bk} - \gamma(y_{bk} + y_n)\right\} \\ \dfrac{dy_{ck}}{d\tau} = \dfrac{1}{3}\left\{x_k - \eta y_{ck} - \gamma(y_{ck} + y_n)\right\} \\ \qquad\qquad (k = 1, 2, 3, ..., N + M - 2). \end{cases} \qquad (2)$$

In this equation, γ is the coupling strength, ε denotes the nonlinearity of the oscillators, and y denotes the current of the inductor of the connected oscillator with the kth oscillator. For the computer simulations, we calculate Eq. (2) using the fourth-order Runge–Kutta method with step size $h = 0.005$. We set the parameters of this circuit model to $\varepsilon = 0.1$ and $\eta = 0.0001$. The coupling strength γ between the oscillators changes from a small to a large value.

Figure 4 shows the change in amplitude according to the network type. We confirm that global AD occurs at the same time. In the case of 3–3 coupling network, the amplitude of shared oscillators is smaller than the others. While, by increasing the triangular network as shown in Fig. 4(b), the amplitude of shared oscillators is larger than the others. We consider that the number of triangular networks affect to frustration for whole network.

Figure 5 shows the simulation results of amplitude change from the network model composed of pentagonal networks. In these models, AD first occurs in the oscillators located farthest from the shared oscillators, and then the other oscillators stop oscillating at the same time.

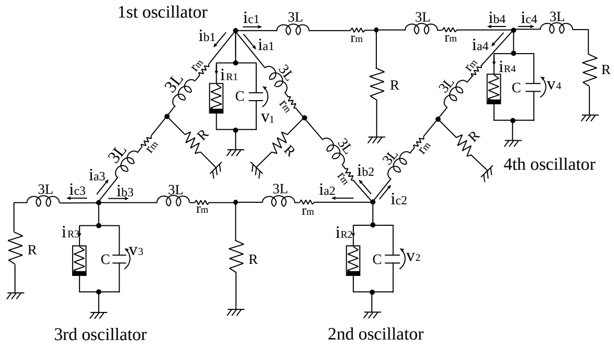
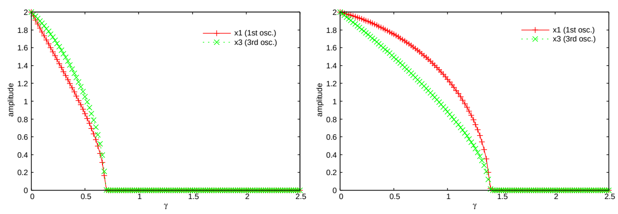
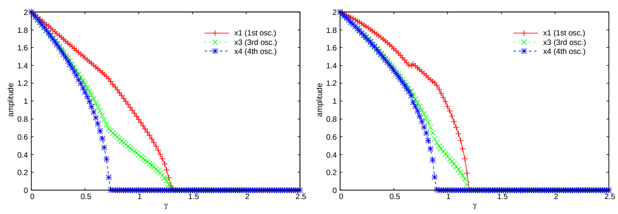


Figure 3: Coupling model (3–3 coupling network).



(a) 3–3 coupling network. (b) 7–triangular coupling network.

Figure 4: Amplitude change of coupled triangular oscillatory networks.



(a) 5–5 coupling network. (b) 17–pentagonal coupling network.

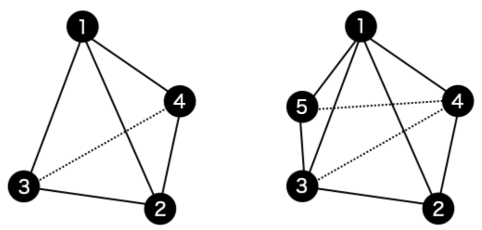
Figure 5: Amplitude change of coupled pentagonal oscillatory networks.

3. Coupled Oscillatory Networks in Three-Dimensional Space

Here, we investigate the amplitude change of the oscillators when the system model is extended to three-dimensional space.

3.1. Coupled Oscillatory Networks using Tetrahedrons

Two types of network models are proposed as shown in Fig. 6. These models are composed of a triangular network. The first model is a regular tetrahedron with four congruent equilateral triangles. The second model is a combination of two regular tetrahedrons sharing one regular triangle.



(a) Tetrahedron network. (b) Two tetrahedrons network.

Figure 6: Coupled oscillatory networks in three-dimensional space.

Figure 7 shows the simulation results of amplitude change of tetrahedron network with the coupling strength. From this result, it was found that global AD was observed in the same way as in the case of a triangular oscillatory network connected two-dimensionally. The amplitude of the all oscillators has same value. The coupling strength causing AD is the lowest of all previous models. It is therefore considered the tetrahedron network has large frustration. Figure 8 shows the simulation results of amplitude change of two tetrahedron networks with the coupling strength. In this model, there are two groups depending on the amplitude value. The first group is 1st, 2nd and 3rd oscillators. The second groups is 4th and 5th oscillators. We confirm that global AD occurs at the same time. In this network, the amplitude change is very similar with 3–3 coupling network, because the amplitude of shared oscillators is smaller than the others.

To summarize the obtained results, we proposed a regular tetrahedron network as a 3D model, however we found that the occurrence of AD phenomena is similar with the 2D models.

3.2. Coupled Oscillatory Networks adding Depth

Next, we propose a 3D model by adding depth to the N–N coupling network models that has been proposed in the two-dimensional space. Two types of 3D models are shown in Fig. 9. In these models, the square face is the shared face of the networks on both sides.

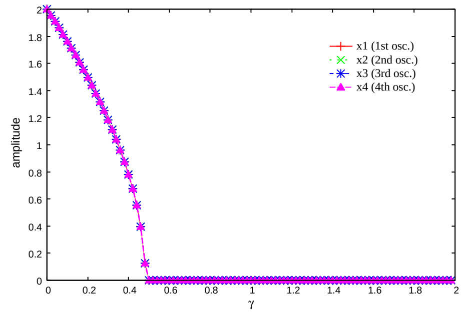


Figure 7: Amplitude change in three-dimensional coupled oscillatory networks (tetrahedron network).

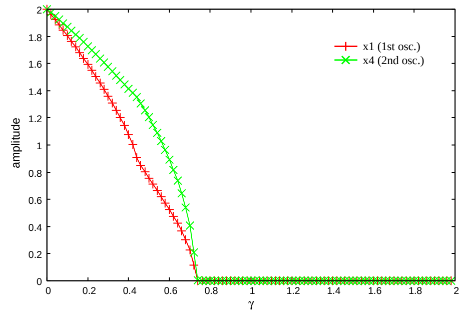
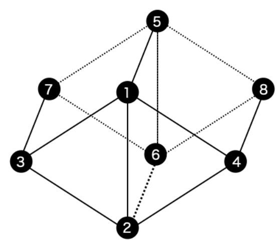
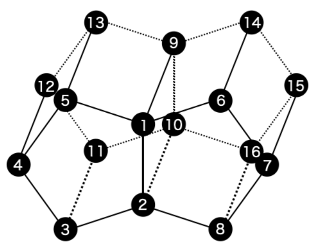


Figure 8: Amplitude change in three-dimensional coupled oscillatory networks (two tetrahedrons network).



(a) 3D triangular oscillatory network (N=8).



(b) 3D pentagonal oscillatory network (N=16).

Figure 9: Coupled oscillatory networks in three-dimensional space.

Figure 10 shows the simulation results of amplitude change of 3D triangular oscillatory network with the coupling strength. From this result, it was found that global AD was observed in the same way as in the case of 3–3 coupling network.

Figure 11 shows the simulation results of amplitude change of 3D pentagonal oscillatory network with the coupling strength. In this model, there are three groups depending on the amplitude value. One group (4th, 7th, 12th and 15th oscillators) stop their oscillation around γ=1.4. The amplitude of the other two groups decreases with the coupling strength. However, we can not observe the global AD in 3D pentagonal oscillatory network. This is a phenomenon unique to 3D models.

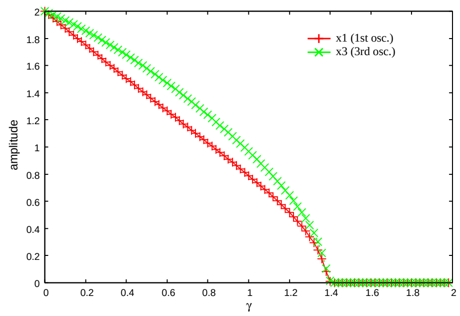


Figure 10: Amplitude change in three-dimensional coupled oscillatory networks (triangular oscillatory network).

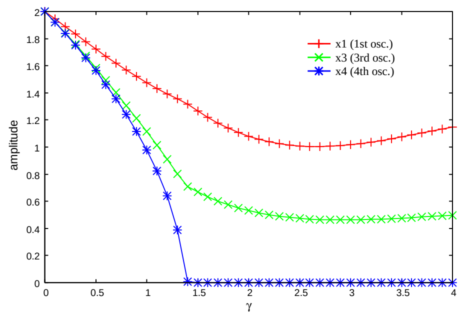


Figure 11: Amplitude change in three-dimensional coupled oscillatory networks (pentagonal oscillatory network).

4. Conclusions

In this study, we investigated the occurrence of AD observed from coupled oscillatory networks in three-dimensional space. Two types of network models are proposed. The first models is composed of a tetrahedron network and the second model is adding the depth to 2D network models. By using computer simulations, we observed similar AD phenomena in the tetrahedron network in three-dimensional space with two-dimensional space. In the case of the 3D pentagonal network model, we observe only partial AD and could not observe the global AD.

For the future works, we would like to make clear the mechanism of AD using theoretical analysis. Further investigation of AD for larger and more complex polygonal networks is one of the future issues.

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