Analysis of Synchronization Phenomena in Complex Networks Consisting of van der Pol Oscillators

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Abstract—**Complex networks have been the subject of great interest. Some characteristics of complex networks are closely related to the real world network and have been studied in terms of network topology and interactions between nodes. In this study, we build three networks which are composed of van der Pol oscillator and investigate synchronization phenomena in each network. As a result, it was confirmed that the synchronization states are different depending on the network structure.**

1. Introduction

Complex networks have attracted a lot of attention from various fields such as sociology, biology and engineering [1]. Further in the field of engineering, research on complex networks using circuits has been studied, and interesting phenomena such as synchronization between circuits have been observed. For example, studies have focused on the structure and degree distribution of networks [2],[3]. However, there are still many unclear points, and studies in networks close to the real world will become even more important in the future.

In this study, we build three network models by using oscillators and investigate synchronization phenomena between nodes in networks.

2. System Model

Figure 1 shows van der Pol oscillator. This oscillator consists of a capacitor, an inductor and a nonlinear element. Further we use three network models in this study. Figure 2(a) shows regular lattice network, Fig. 2(b) shows Erdos-Renyi model (ER random) and Fig. 2(c) shows Barabasi-Albert model (BA scale-free). Table 1 shows each network properties.

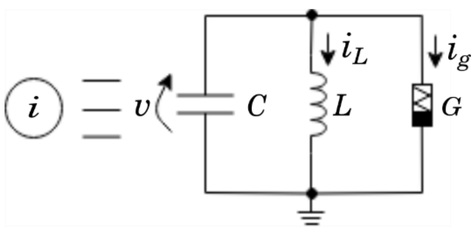
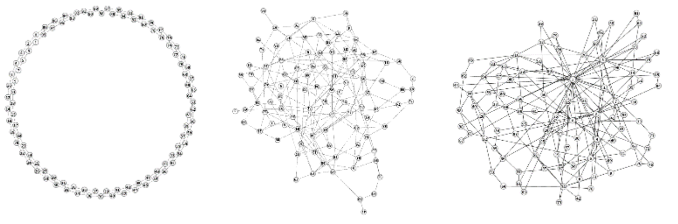


Figure 1: van der Pol oscillator.



(a) (b) (c)

Figure 2: Network model.

(a) Regular. (b) ER random. (c) BA scale-free.

Table 1: Node number N, Average node degree d_{avg}, Maximum node degree d_{max}, Minimum node degree d_{min} and Average distance D_{avg} of three network models.

network	Regular	ER random	BA scale-free
N	100	100	100
d_{avg}	4.0	3.74	3.92
d_{max}	4.0	9.0	32.0
d_{min}	4.0	1.0	2.0
D_{avg}	12.87	3.68	2.98

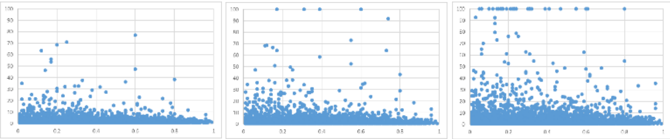
In this study, we set the average node degree to be close to 4.0 each network model. Further as features of the network, the maximum node degree is largest for BA scale-free, the average distance is largest for Regular and smallest for BA scale-free.

The normalized circuit equations are described as follows:

$$\begin{cases} \frac{dx_i}{d\tau} = \alpha\{\varepsilon x_i(1 - x_i{}^2) - y_i - \sum_{i,j=1}^{100} \gamma_{ij}(x_i - x_j)\} \\ \frac{dy_i}{d\tau} = x_i \qquad (i, j = 1, 2, \cdots, 100). \end{cases} \quad (1)$$

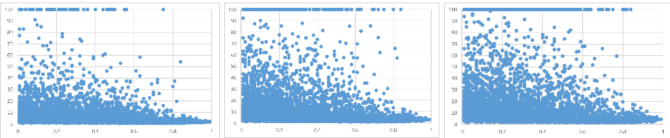
3. Results

We set the parameters of van der Pol oscillator as $\varepsilon = 0.1$ and $\alpha = 1.0$. Here, the mismatch is added to α. The mismatch is generated by random and the range of the mismatch is set to [-0.5:0.5]. In this study, we vary the coupling strength γ in each network model and show results on the relation between mismatch differences and synchronization rates in the three network models.



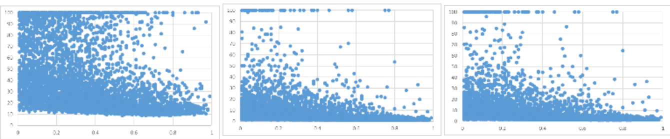
(i) $\gamma = 1.0$ (ii) $\gamma = 2.0$ (iii) $\gamma = 3.0$

(a) Regular.



(i) $\gamma = 1.0$ (ii) $\gamma = 2.0$ (iii) $\gamma = 3.0$

(b) ER random.



(i) $\gamma = 0.5$ (ii) $\gamma = 1.0$ (iii) $\gamma = 1.2$

(c) BA scale-free.

Figure 3: The relation between mismatch differences, synchronization rates and coupling strength γ.
(The vertical axis is synchronization rates and the horizontal axis is mismatch differences.)

First, when the coupling strength is varied, the synchronization rates do not change significantly in Fig. 3(a). Therefore it is not confirmed that synchronization rates and mismatch differences are heavily related in regular network. Second, when the coupling strength is higher, the number of pair of nodes with higher synchronization rates increases as the mismatch difference is smaller in Fig. 3(b). Therefore it is confirmed that synchronization rates and mismatch differences are related in ER random. Third, when the coupling strength is higher, the number of pair of nodes with higher synchronization rates decreases, and the synchronization rates vary even when the coupling strength is small compared to other networks in Fig. 3(c). Therefore it is confirmed that synchronization rates and mismatch differences are related in BA scale-free. These results confirm that the optimal coupling strength in synchronization between circuits depends on the characteristics of the network structure.

4. Conclusion

In this study, we built three network models by using oscillators and investigated synchronization phenomena between nodes in networks. It is not confirmed that synchronization rates and mismatch differences are heavily related in regular network. Further it is confirmed that synchronization rates and mismatch differences are related in ER random and BA scale-free.

5. Future work

In the future, we will investigate how the synchronization rate changes when the coupling strength of ER random is further increased. Further it was confirmed the number of pair of nodes decreased when the coupling strength was increased in BA scale-free. Therefore it is assumed that the optimal coupling strength exists around 0.5 and will investigate in detail.

6. Reference

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