Investigation of Synchronization Phenomena in Coupled Two-degrees-of-Freedom Chaotic Circuits

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Abstract—**This paper considers synchronization phenomena in coupled two-degrees-of-freedom chaotic circuits by a resister. It is considered that studying various cases of synchronization phenomena when using chaotic circuits showing asynchronous simultaneous oscillation will be useful in clarifying non-linear phenomena that exist around us. By means of the circuit experiments and computer simulations, chaotic attractors and Lissajous figures are shown. From the results, synchronization phenomenon was confirmed between the circuits farthest from the connection part.**

Keywords; Synchronization; Asynchronous simultaneous oscillation; Two-degrees-of-freedom chaotic circuit

I. INTRODUCTION

There are many synchronization phenomena in nature. Representatives include frog chorus and firefly issuance. As for engineering applications, communication encryption and decryption, and synchronization phenomena in neurons of the human brain have been confirmed, so application to brain computers is expected. Therefore, research on synchronization phenomena in the field of chaos has attracted much attention. Observation and analysis of these phenomena using electric circuits is one of the useful means because it is able to correspond to the actual physical system. Studies have been conducted on the variation of synchronization phenomena and the complexity of generated chaos for each parameter when two three-dimensional chaotic circuits are connected [1].

Asynchronous simultaneous oscillation is one of the typical non-linear phenomena and is considered to exist in various high-dimensional systems in nature. Therefore, several synchronous phenomena have been investigated when connecting an asynchronous simultaneous oscillator with resistors [2] or inductors [3]. High-dimensional chaotic circuits are actively studied, and a two-degrees-of-freedom chaotic circuit has been proposed [4]. The circuit shows asynchronous simultaneous oscillation when connecting two sets of chaotic circuits with different natural frequencies in series. A set of circuits in a two-degrees-of-freedom chaotic circuit is called a subcircuit. In this study, we investigate the synchronization phenomenon using two chaotic circuits with asynchronous simultaneous oscillation connected by resistors. In the circuit experiment and computer simulation, chaotic attractors and Lissajous figures are shown for three natural frequency cases.

II. SYSTEM MODEL

The circuit model is shown in Fig. 1. Two two-degrees-of-freedom chaotic circuits are coupled by a resistor R. The chaotic circuit consists of negative resistor and two Inaba's circuits. In order to change the frequencies of chaotic circuits, we change the value of L and C.

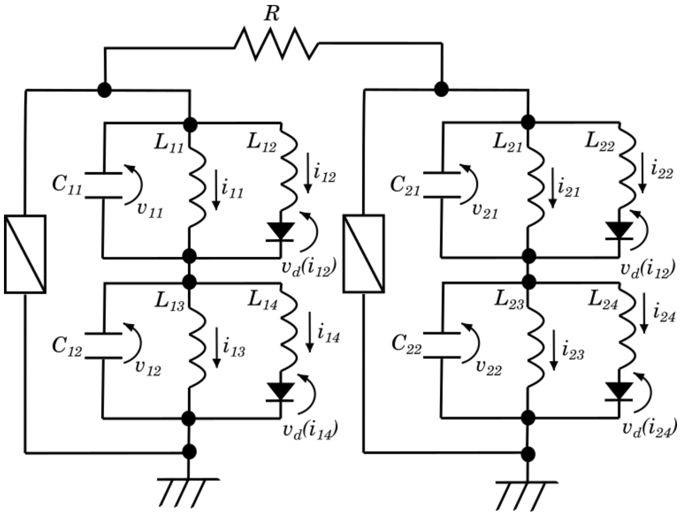

Figure 1: Circuit model.

The parameters are described as follows:

$$\alpha = g\sqrt{\frac{L_{i1}}{C_{i1}}}\ ,\ \beta_{11} = \frac{L_{11}}{L_{12}}\ ,\ \beta_{12} = \frac{L_{11}}{L_{13}}\ ,\ \beta_{13} = \frac{L_{11}}{L_{14}}\ ,$$

$$\beta_{21} = \frac{L_{11}}{L_{21}}\ ,\ \beta_{22} = \frac{L_{11}}{L_{22}}\ ,\ \beta_{23} = \frac{L_{11}}{L_{23}}\ ,\ \beta_{24} = \frac{L_{11}}{L_{24}}$$

$$\gamma_1 = \frac{C_{11}}{C_{12}}\ ,\ \gamma_{21} = \frac{C_{11}}{C_{21}}\ ,\ \gamma_{22} = \frac{C_{11}}{C_{22}},\ \delta = \frac{1}{R}\sqrt{\frac{L_{i1}}{C_{i1}}}\ ,\ \varepsilon = \frac{1}{r_d}\sqrt{\frac{L_{i1}}{C_{i1}}}\ .$$

The normalized circuit equations are described as follows:

$$\begin{cases} \dot{x}_{11} = -(y_{11} + z_{11}) + \alpha(x_{11} + x_{12}) - \delta(x_{11} + x_{12} - x_{21} - x_{22}) \\ \dot{y}_{11} = x_{11} \\ \dot{z}_{11} = \beta_{11}(x_{11} - f(z_{11})) \\ \dot{x}_{12} = -\gamma_1(y_{12} + z_{12}) + \alpha\gamma_1(x_{11} + x_{12}) - \gamma_1\delta(x_{11} + x_{12} - x_{21} - x_{22}) \\ y_{12} = \beta_{12}x_{12} \\ \dot{z}_{12} = \beta_{13}(x_{11} - f(z_{12})) \\ \dot{x}_{21} = -\gamma_{21}(y_{22} + z_{21}) + \alpha\gamma_{21}(x_{21} + x_{22}) + \gamma_{21}\delta(x_{11} + x_{12} - x_{21} - x_{22}) \\ \dot{y}_{21} = \beta_{21}x_{21} \\ \dot{z}_{21} = \beta_{22}(x_{21} - f(z_{21})) \\ \dot{x}_{22} = -\gamma_{22}(y_{22} + z_{22}) + \alpha\gamma_{22}(x_{21} + x_{22}) + \gamma_{22}\delta(x_{11} + x_{12} - x_{21} - x_{22}) \\ \dot{y}_{22} = \beta_{23}x_{22} \\ \dot{z}_{22} = \beta_{24}(x_{21} - f(z_{22})) \end{cases}$$

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III. RESULTS

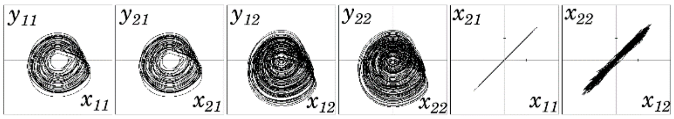
In this study, various natural frequency cases are considered. The frequencies of x_{11}, x_{21}, x_{12}, and x_{22} are corresponding to ω_1, ω_2, ω_3, and ω_4, respectively.

A. Computer simulation results

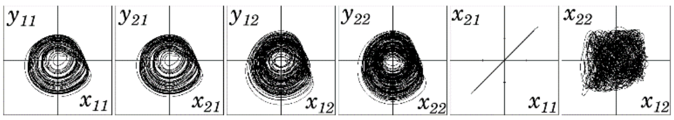
Figure 2 shows the computer simulation results.

In the case Fig. 2(a), in-phase synchronization phenomena are confirmed between the upper subcircuits and between the lower subcircuits. The circuit parameters are chosen as $\alpha = 0.2$, $\beta_{11} = 6.0$, $\beta_{12} = 3.0$, $\beta_{13} = 18.0$, $\beta_{21} = 1.0$, $\beta_{22} = 6.0$, $\beta_{23} = 3.0$, $\beta_{24} = 18.0$, $\gamma_1 = 3.0$, $\gamma_{21} = 1.0$, $\gamma_{22} = 3.0$, δ=0.1, and $\varepsilon = 0.01$.

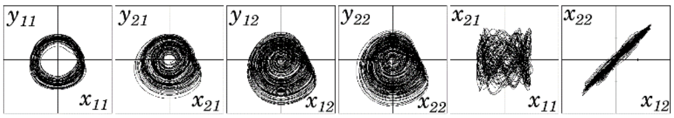
In the case Fig. 2(b), in-phase synchronization phenomena are confirmed only between the upper subcircuits. The circuit parameters are changed from case Fig. 2(a) to $\beta_{12} = 2.0$, $\beta_{13} = 12.0$, $\gamma_1 = 2.0$.



(a) $\omega_1 = \omega_2$, $\omega_3 = \omega_4$.



(b) $\omega_1 = \omega_2$, $\omega_3 \neq \omega_4$.



(c) $\omega_1 \neq \omega_2$, $\omega_3 = \omega_4$.

Figure 2: Attractors and synchronization phenomena in the computer simulation.

In the case Fig. 2(c), although the subcircuits closest to the connection is asynchronous, it is confirmed that the lower subcircuits far from the connection are in-phase synchronized. The circuit parameters are changed from case Fig. 2(a) to $\beta_{21} = 2.0$, $\beta_{22} = 12.0$, $\gamma_{21} = 2.0$

B. Circuit experiment results

Figure 3 shows the circuit experiment results. As shown in Fig. 3, in-phase synchronization is able to be observed between the two underlining subcircuits, when natural frequencies are set to $\omega_1 \neq \omega_2$, $\omega_3 = \omega_4$.

Element values are $C_{11} = 100.0$[nF], $C_{12} = 33.0$[nF], $L_{11} = 180.0$[mH], $L_{12} = 30.0$[mH], $L_{13} = 60.0$[mH], $L_{14} = 10.0$[mH], $C_{21} = 68.0$[nF], $C_{22} = 33.0$[nF], $L_{21} = 120.0$[mH], $L_{22} = 20.0$[mH], $L_{23} = 60.0$[mH] $L_{24} = 10.0$[mH], $R = 12$[kΩ].

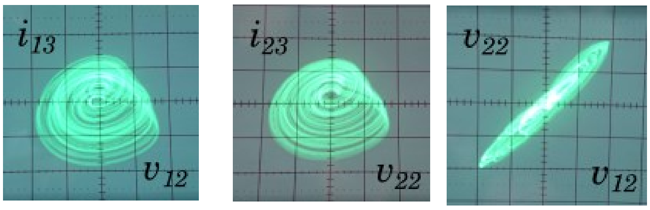


Figure 3: Attractors and synchronization phenomena in the circuit experiment.

IV. CONCLUSION

In this study, synchronization phenomena using two two-degrees-of-freedom chaotic circuits are investigated. We confirmed that directly connecting two previously separated circuits via a resistor induced synchronization between the two underlining subcircuits farthest from the newly created connection, while the two closest subcircuits stayed asynchronous. As our future works, we will investigate synchronization phenomena in the circuit experiment and we will observe synchronization phenomena when connecting circuits with an inductor.

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