Frustrated Complex Networks of Nonlinear Circuits with Stochastically Coupling

Yoko Uwate and Yoshifumi Nishio
Dept. of E. E. Eng., Tokushima University
2-1 Minami Josanjima
Tokushima, Japan
e-mail: {uwate, nishio}@ee.tokushima-u.ac.jp

Thomas Ott
Institute of Applied Simulation
Zurich University of Applied Sciences
Zurich, Switzerland
e-mail: ottt@zhaw.ch

Abstract—**In this study, we focus on an effect of frustration to polygonal oscillatory network with stochastically coupling. We propose a coupled nonliear circuit network with stochastically coupling. Frustration as environmental factor is occurred by network topology which is composed from polygonal structure. We investigate synchronization of the proposed network using different frustration levels by changing the coupling strength. By using computer simulations, the effect of frustration to polygonal oscillatory networks with stochastically coupling is shown.**

Keywords; frustrated synchronization, coupled oscillators

I. Introduction

Coupled oscillatory circuits are excellent models for describing high-dimensional nonlinear phenomena occurring in our living world. In particular, synchronization is one of the most important functions that can be explained and explored with the help of an oscillator. This is because, when oscillators are coupled, a strong correlation rhythm between oscillators called a synchronized state appears. Therefore, many different types of coupled oscillatory networks were proposed and many interesting synchronization phenomena have been discovered.

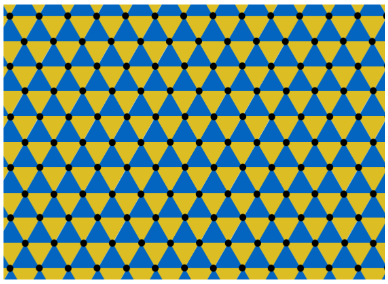
In our research group, we have been studied the synchronization phenomena observed from the nonlinear oscillatory networks [1]. We consider that the synchronization in complex oscillator networks are useful a deeper understanding of control methods in smart grid, communication systems and so on [2]. However, many of the networks that have been studied so far are static models, and it is necessary to propose a model that changes the network topology like a network observed in the real world including environmental factors.

In this study, we focus on an effect of frustration to polygonal oscillatory network with stochastically coupling. We propose a coupled nonliear circuit network with stochastically coupling. Frustration as environmental factor is occurred by network topology which is composed from polygonal structure. In this system, van der Pol oscillators are connected to every node of a polygonal network. In this oscillatory system, two adjacent oscillators tend to synchronize with anit-phase. Hence, if the number of nodes is odd, frustration occurs in the polygonal network. Each node has a coupling probability which is key factor to connect or not (on/off coupling). At every certain time in a simulation, the network topology is changed by the coupling probability. We investigate synchronization of the proposed network using different frustration levels by changing the coupling strength. By using computer simulations, the effect of frustration to polygonal oscillatory networks with stochastically coupling is shown.

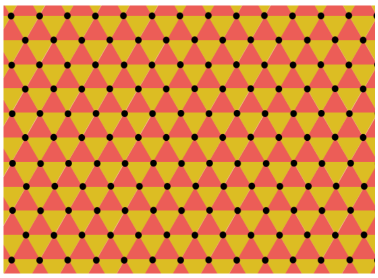
II. Network Model using van der Pol Oscillato

The conceptual network models with different frustration levels used in this study are shown in Fig. 1. In this figure, triangular oscillators are coupled with edges on 2-dimensional space and black circle denotes a van der Pol oscillators.

Each edge has a coupling probability (p) which is key factor to connect or not (on/off coupling). At every certain time (τ=500) in a simulation, the network topology is changed by the coupling probability.



(a) Frustrate network with strong coupling.



(b) Frustrate network with strong coupling.

Fig. 1. Network models using triangular oscillators.

Figure 2 shows a van der Pol oscillator. This oscillator is composed by an inductor, a negative resistance and a condenser. Two triangular oscillatory networks are coupled by a resistor via inductors as shown in Fig. 3.

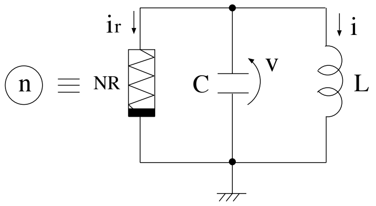


Fig. 2. van der Pol oscillator.

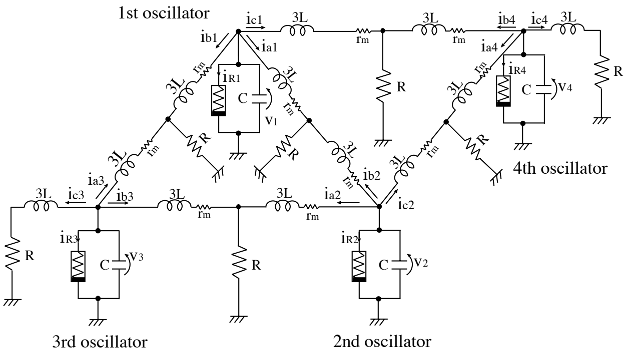


Fig. 3. Coupling scheme of two triangular oscillatory networks.

III. Simulation Results

For the computer simulations, 20 van der Pol oscillators are coupled in triangular oscillatory space like Fig. 1 as the first step. By changing the coupling strength (g) and the coupling probability (p) in the network, the average amplitude of coupled oscillators are investigated.

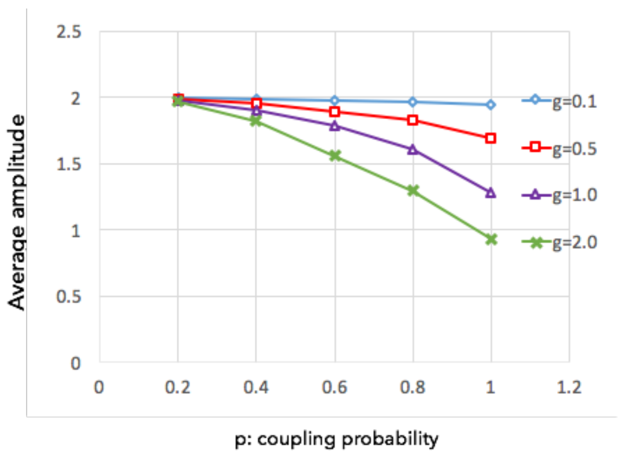
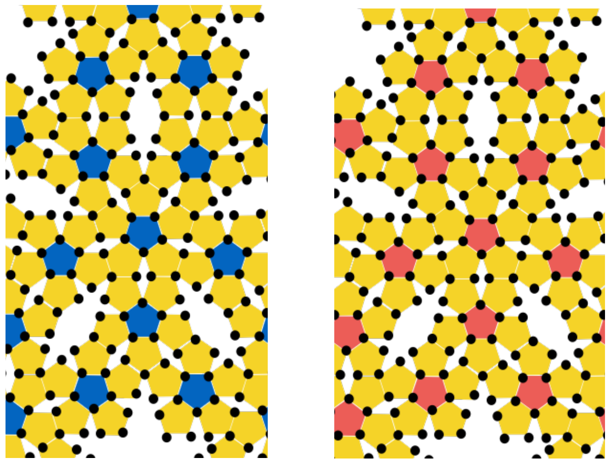


Fig. 4. Simulation results of average amplitude depending on the coupling strength (N=20).

Figure 4 shows the simulation results of the average amplitude with coupling probability (p). In the case of static network (p=1.0), the average amplitude of 4 networks has big difference from 0.93 to 1.94. While if the coupling probability is set to 0.2 (dynamical networks), the average amplitude of all 4 networks has similar value around 1.99. From this result, we confirm that the frustrated network (g=2.0) is affected a lot by stochastically coupling. By decreasing the coupling probability, the network topology is changed dynamically, then the frustration effect is defused.



(a) (b)

Fig. 5. Conceptual network models using pentagonal oscillators. (a) Frustrate network with strong coupling. (b) Non-frustrated network with weak coupling.

IV. Conclusions

In this study, we proposed coupled frustrated polygonal oscillators with a stochastically coupling. This coupling is switched on/off state depending on the coupling probability. As the first step, 20 van der Pol oscillators are coupled in triangular oscillatory network was investigated. We calculate the amplitude of coupled oscillators in order to capture the effect of frustrations.

From computer simulations, we confirmed that by decreasing the coupling probability in the proposed networks, the average amplitude of all networks reached to 2.0 which means that network has no frustration.

References

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