

# Clustering Phenomena in Coupled Chaotic Circuits

## Located in 3-Dimensional Space

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### 1. Introduction

Synchronization phenomena are the most familiar phenomena that exist in nature and they have been studied in various fields. Synchronization phenomena can be observed everywhere in our life. For example, we can confirm flashing firefly lights and so on. Especially, synchronization phenomena of oscillatory networks are interesting. In addition, complex networks attract attention from various fields. The feature of networks is the degree distribution, the path length and the clustering coefficient. Therefore, we focus on the clustering phenomena in this research.

In addition, clustering phenomena are one of interesting nonlinear phenomena observed from coupled chaotic circuits. The clustering phenomena are to divide the set to be classified into subsets. Previously, many of the studies for clustering have been carried out for discrete time model [1]-[2]. However, analysis of using a continuous time model has not almost studied. Therefore, we focus on research on clustering phenomena using electronic circuits in continuous time model.

On the other hand, coupled chaotic circuits that are electronic circuits can be observed various amusing phenomena. In recent years, many methods are studied to apply to clustering and synchronization phenomena observed in coupled chaotic circuits for natural sciences. At the same time, synchronization phenomena and clustering have been studied associated with the chaotic phenomena [3]-[4].

In this study, we focus on the clustering phenomena in the network of coupled chaotic circuits.

### 2. Circuit model

Figure 1 shows the circuit model which is called Shinriki-Mori circuit. This circuit consists of a negative resistor, an inductor, two capacitors and dual-directional diodes.

The circuit equation of this circuit is given as follows:

$$\begin{cases} L \frac{di_L}{dt} = v_2 \\ C_1 \frac{dv_1}{dt} = gv_1 - i_{dn} \\ C_2 \frac{dv_2}{dt} = i_{dn} - i_L \end{cases} \quad (1)$$

The nonlinear function  $i_{dn}$  corresponds to the  $i$ - $v$  characteristics of the nonlinear resistors consisting of the diodes and are given as follows:

$$i_{dn} = \begin{cases} G_d(v_1 - v_2 - V) & (v_1 - v_2 > V) \\ 0 & (|v_1 - v_2| \leq V) \\ G_d(v_1 - v_2 + V) & (v_1 - v_2 < -V) \end{cases} \quad (2)$$

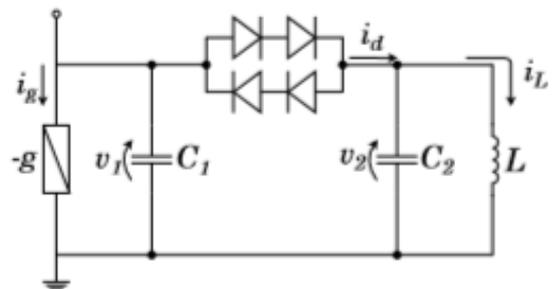


Fig. 1. Circuit model

By changing the variables and parameters such that

$$i_L = \sqrt{\frac{C_2}{L}} Vx, v_1 = Vy, v_2 = Vz, \alpha = \frac{C_2}{C_1}, \beta = G_d \sqrt{\frac{L}{C_2}}, \gamma = g \sqrt{\frac{L}{C_2}}, t = \sqrt{LC_2} \tau$$

The normalized equation of chaotic circuit is given as follows:

$$\begin{cases} \frac{dx}{d\tau} = z \\ \frac{dy}{d\tau} = \alpha(\gamma y - \beta f) \\ \frac{dz}{d\tau} = \beta f - x. \end{cases} \quad (3)$$

The nonlinear function  $f$  corresponds to the characteristics of the nonlinear resistor consisting of the diodes and described as follows:

$$f = \begin{cases} y - z - 1 & (y - z > 1) \\ 0 & (|y - z| \leq 1) \\ y - z + 1 & (y - z < -1). \end{cases} \quad (4)$$

For the computer simulation, we set the parameters as  $\alpha = 0.50$ ,  $\beta = 20.00$  and  $\gamma = 0.50$ .

### 3. Results and discussion

We investigate the case of 3-dimensional networks. Thirty chaotic circuits are located in 3-dimensional, including the positional information. The location of 30 chaotic circuits is shown in Fig. 2 and Table I.

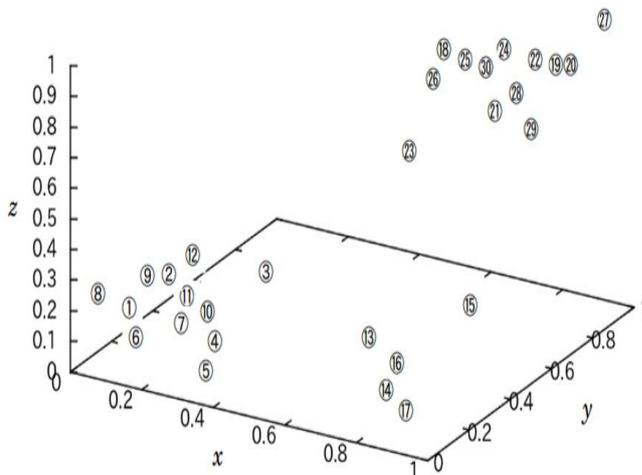


Fig. 2. Location of 30 chaotic circuits in 3-dimensional space.

Table. I.

The location of chaotic circuits in the 3-dimensional space.

1	0.15	0.05	0.15	16	0.80	0.20	0.15
2	0.20	0.25	0.30	17	0.85	0.15	0.05
3	0.35	0.35	0.25	18	0.70	0.60	0.95
4	0.25	0.25	0.05	19	0.90	0.80	0.85
5	0.30	0.15	0.05	20	0.80	0.95	0.75
6	0.05	0.20	0.10	21	0.75	0.85	0.70
7	0.15	0.30	0.15	22	0.85	0.80	0.85
8	0.05	0.05	0.25	23	0.60	0.60	0.60
9	0.20	0.05	0.35	24	0.80	0.65	0.90
10	0.25	0.10	0.20	25	0.65	0.80	0.80
11	0.35	0.05	0.25	26	0.65	0.65	0.85
12	0.25	0.05	0.35	27	0.95	0.95	0.95
13	0.75	0.15	0.25	28	0.90	0.65	0.75
14	0.80	0.25	0.10	29	0.70	0.85	0.65
15	0.95	0.30	0.35	30	0.75	0.80	0.85

All circuits connected each other by resistors.

We consider the coupled chaotic circuits:

$$\begin{cases} \frac{dx_i}{d\tau} = z_i \\ \frac{dy_i}{d\tau} = \alpha(\gamma y_i - \beta f - \sum_{i,j=1}^N r_{i,j}(y_i - y_j)) \\ \frac{dz_i}{d\tau} = \beta f - x_i. \end{cases} \quad (5)$$

The nonlinear function  $f$  corresponds to the  $i$ - $v$  characteristics of the nonlinear resistors consisting of the diodes and are given as follows

$$f = \begin{cases} y_i - z_i - 1 & (y_i - z_i > 1) \\ 0 & (|y_i - z_i| \leq 1) \\ y_i - z_i + 1 & (y_i - z_i < -1). \end{cases} \quad (6)$$

where,  $i$  in the equation represents the circuit itself, and  $j$  is the coupling with other circuits. The parameter  $r$  represents the coupling strength between the circuits. In this simulation, we set the coupling parameter value  $r_{i,j}$  to correspond the distance between the circuits by the following equation:

$$r_{i,j} = \frac{q}{(d_{i,j})^2} \quad (7)$$

$d_{i,j}$  represents the Euclidean distance between the  $i$ -th and the  $j$ -th circuits. Further, the parameter  $q$  is the weight parameter that determines the coupling strengths. In this case, we set parameter  $q = 0.004724$ .

Figure 3 shows the computer simulation results obtained from the 30 chaotic circuits located as shown in Fig. 2. From these results, we confirm that the 1st and the 2nd circuits are synchronized at in-phase state. However, the 1st chaotic circuit and the 13th chaotic circuit are not synchronized. Also the 1st chaotic circuit and the 18th chaotic circuit are not synchronized. Similarly, between the 13th and the 14th chaotic circuit are synchronized, between the 18th and the 19th chaotic circuit are synchronized. However, between the 13th and the 18th chaotic circuit are not synchronized. From these results, the circuits can form 3 clusters defined by chaotic synchronization as shown in Fig. 4.

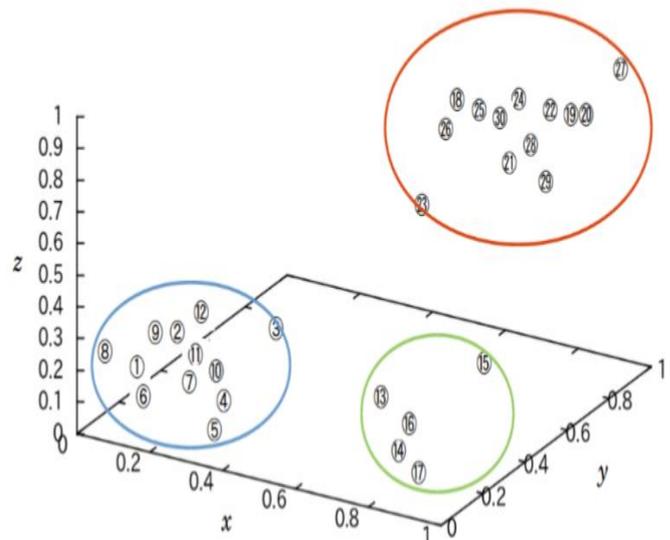
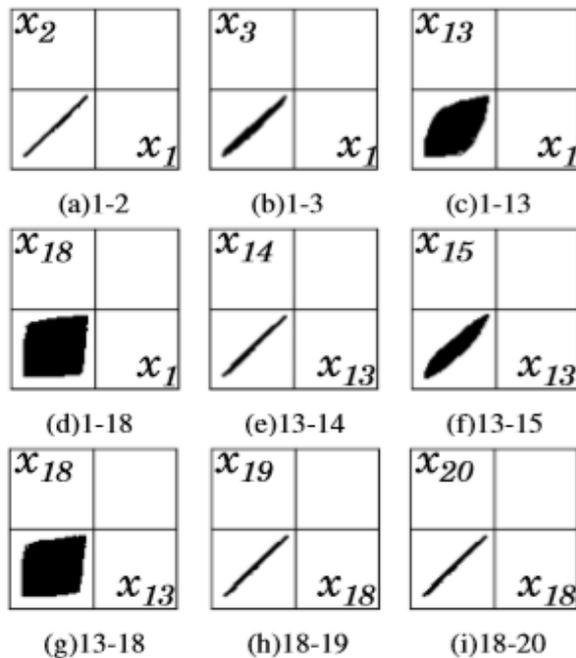


Fig. 3. Location of 30 chaotic circuits in 3-dimensional space. Fig. 4. The clustering result of 30 chaotic circuits.

#### 4. Conclusion

In this study, we investigated synchronization phenomena when the chaotic circuits are located in 3-dimensional space. Synchronization phenomena were seen between circuits at near distance, and synchronization phenomena could not be seen between circuits at far distance. With these results, it was confirmed that the chaotic circuits were different from synchronization phenomena by distance information and the clustering phenomena were observed.

In the future works, we would like to increase the number of chaotic circuits and the cluster. Moreover, we consider that we would like to increase the dimensional space. We also would like to investigate changes in the clusters by changing the value of  $q$ .

#### References

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