

Switching Phenomena of Synchronization Phenomena in Ladder-Coupled Chaotic Network Including Ring Structure

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1. Introduction

Synchronization in the network is one of the most interesting fields from the scientific points. Investigation of the synchronization is an important research for clarifying the nonlinear phenomenon in the natural world, which has been observed in various fields such as engineering, biology, and sociology. The network also has characteristics with different topologies. Therefore, it is important to investigate the dynamics due to the difference in network structure, and research to analyze each topology is underway [1].

On the other hand, synchronization in coupled chaotic systems is a suitable model for describing various high-dimensional nonlinear phenomena. Especially, in recent years, many studies of chaotic phenomena using coupled chaotic circuits have been conducted. Circuit experiments and computer simulations of chaotic circuits with simple configurations are considered to be suitable due to their high reproducibility. Investigation of nonlinear phenomena related to chaotic phenomena will be an important issue in future engineering. Networks using chaotic circuits are expected to be applied to modeling of the natural world and social networks [2].

Previously, our research group investigated the synchronization phenomena observed in ladder coupled systems including ring structures. We confirmed perfect synchronization and chaotic synchronization in different network topologies [3]. In this study, we investigate the synchronization phenomena of a larger scale than the previous network model. We particularly focus on the bridge positions that connect the ladder and ring structures. Each coupled chaotic circuit sets parameters for generating a chaotic solution or a periodic solution. By computer simulation, we investigate the synchronization phenomenon in each part by changing the bifurcation parameter in the chaotic network. We also observe the switching effect of synchronous stability in each part of the network.

2. Experimental setup

In this study, we use chaotic circuits. The model of chaotic circuit is shown in Fig. 1. This circuit consists of a negative resistor, two inductors, a capacitor and dual-directional diodes. This chaotic circuit is called Nishio-Inaba circuit [4]. The normalized circuit equations are given as follows:

$$\begin{cases} \frac{dx}{dt} = \alpha x + z \\ \frac{dy}{dt} = z - f(y) \\ \frac{dz}{dt} = -x - \beta y \end{cases} \quad (1)$$

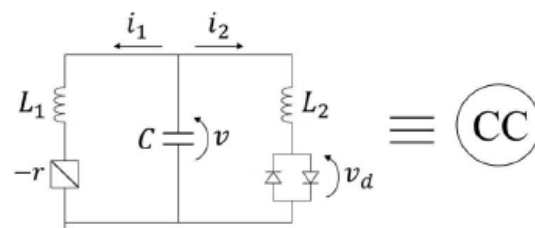


Fig. 1. Chaotic circuit.

In this study, we propose a network model with ladder-coupled chaotic network including ring structure. Each chaotic circuit is coupled by resistor. Figure 2 shows the proposed network model.

We set the parameters that the ladder position composed of CC1 to CC5 generates a chaotic solution, and the five ring positions composed of CC6 to CC20 generate periodic solutions.

The normalized circuit equations of the systems are given as follows:

$$\begin{cases} \frac{dx_i}{dt} = \alpha x_i + z_i \\ \frac{dy_i}{dt} = z - f(y) \\ \frac{dz_i}{dt} = -x_i - \beta y_i - \sum_{i,j=1}^N \gamma_{ij} (z_i - z_j) \end{cases} \quad (2)$$

($i, j = 1, 2, \dots, N$)

In Eq. (2), N means the number of coupling circuits, γ is the coupling strength between circuits, and α is parameter indicating nonlinear degree. In this study, we set the parameters of the system as $\beta = 3.0$ and $\delta = 470.0$. For the parameter α , the ladder part is set as $\alpha = 0.430$ and the ring part is set as $\alpha = 0.412$. The coupling strength is defined as a bifurcation parameter. The coupling strength at the ladder and ring positions is set as $\gamma = 0.2$. In this research, to investigation based on the combination of two topological types of ladder structure and ring structure, we simulate the synchronization by changing the positions of bridge that couple between the two structures (for example, the resistance that couples CC1 and CC6).

3. Results and discussion

We investigate the synchronization state by the computer simulation. In this study, we simulate the synchronization of chaotic network by dividing the coupling strength of the bridge part of $\gamma = 0.01$.

Voltage difference waveform between each circuit is used as method for confirming the synchronization state. When the voltage difference is small, it is considered that each circuit is performing the same output and synchronization is considered to occur. When the voltage difference is large, it is considered that each circuit outputs a different output and the state is regarded as asynchronous.

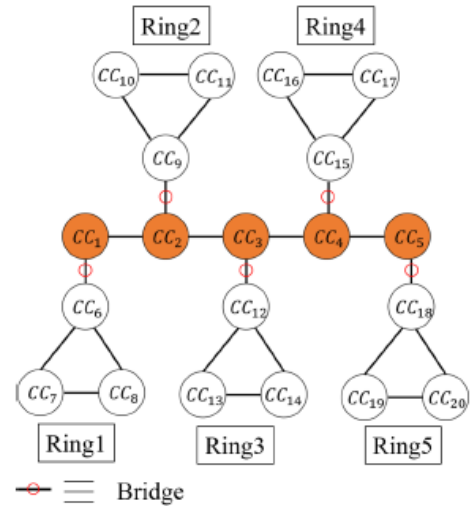
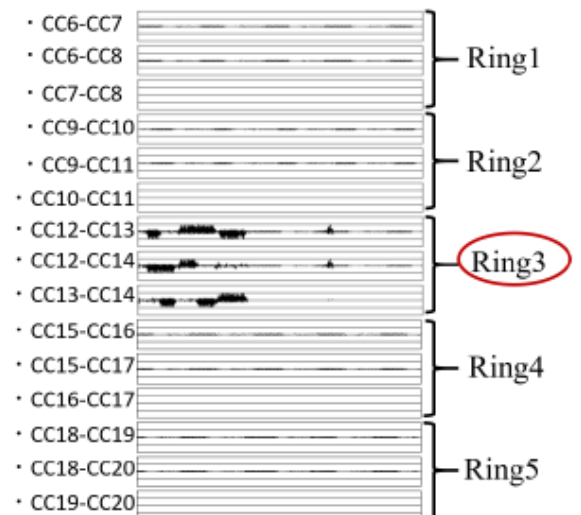


Fig. 2. Network model.



(a) Time advanced $\tau = 50000-100000$ in rings.

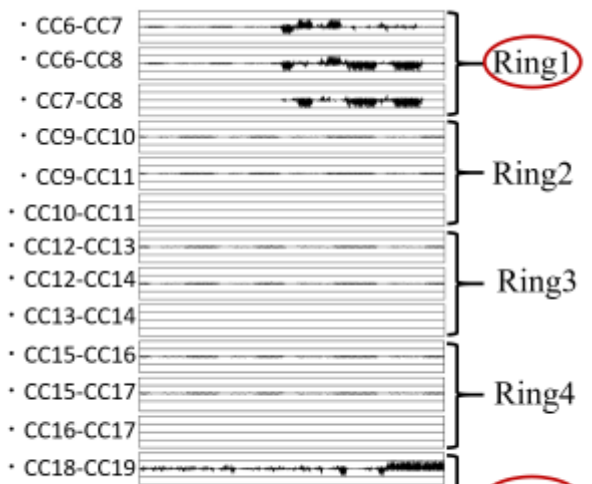
Figure 3 shows the results when the value of coupling strength at the bridge is $\gamma = 0.01$, which is much weaker than other coupling strength. In this case, as shown in Fig. 3(a), we obtain the difference of each ring. As time advanced, the position of synchronization or asynchronous state changes to another place. On another front, Fig. 3(b) shows that the bridge position is constantly asynchronous. The state is initially asynchronous in Ring3, but we confirm that the asynchronous position moved to Ring1 and Ring5 over time. Subsequently, the effect of switching the synchronization state on the rings is repeated.

4. Conclusion

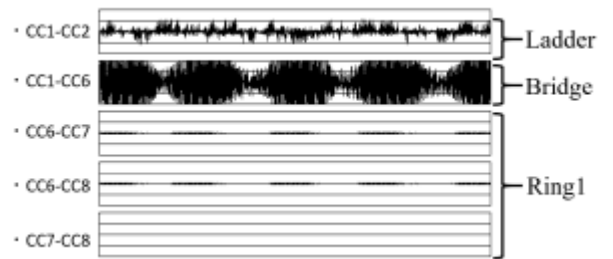
In this study, we investigated the synchronization phenomena in a network composed of coupled chaotic circuits. The network topology is a ladder coupled system including a ring structure.

As a result of investigation by computer simulation, we observed synchronization states such as chaotic synchronization and perfect synchronization by changing the coupling strength of the bridge part. In the case of $\gamma = 0.01$, the ladder position and ring position are asynchronous.

In our future works, we analyze the dynamics of synchronization in more intricate networks



(b) Time advanced $\tau = 150000-200000$ in rings.



(c) Time advanced $\tau = 50000-100000$ in rings.

Fig. 3. Voltage difference waveform.

References

- [1] D. Malagarriga, A. E. P. Villa, J. Garca-Ojalvo and A. J. Pons, "Consistency of heterogeneous synchronization patterns in complex weighted networks" *Chaos* 27, 031102, 2017.
- [2] T. Nishiumi, Y. Uwate and Y. Nishio, "Synchronization Phenomena of Chaotic Circuits with Stochastically-Changed Network Topology", *Proceedings of International Symposium on Nonlinear Theory and its Applications (NOLTA'14)*, pp. 811-814, Sep. 2014.
- [3] K. Nakabai, Y. Uwate and Y. Nishio, "Synchronization in Ladder-Coupled Chaotic Circuits Including Ring Structure" *Proceedings of IEEE International Symposium on Circuits and Systems (ISCAS'19)*, DOI:10.1109/ISCAS.2019.8702350, May 2019.
- [4] Y. Nishio, N. Inaba, S. Mori and T. Saito, "Rigorous Analyses of Windows in a Symmetric Circuits" *IEEE Transactions on Circuits and Systems*, vol. 37, no. 4, pp. 473-487, Apr. 1990.