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Evaluation of Complexity of Chaos in Many Degrees of Freedom Chaotic Circuits Using Poincare Map

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Abstract

This paper considers comparison of the complexity of chaos generated in many degrees of freedom chaotic circuits. We increase the number of connected subcircuits from two to three in order to produce more complex chaos. By means of the circuit experiment and computer simulation, we show chaotic attractors and Poincare maps. From the results, we confirmed that more complex chaos is generated in the each circuit.

1. Introduction

Chaos has two major features. They are initial value sensitivity and long-term unpredictability. It is difficult to predict long-term weather forecasts [1] due to initial values such as temperature, atmospheric pressure, and wind speed, etc.

Chaotic circuit is used to considers nonlinear phenomena such as natural phenomena. It is faster and easier to experiment than actual natural phenomena. Strictness analysis is difficult for chaotic phenomena generated in high-dimensional systems [2]. Therefore, getting closer to high-dimensional chaos that exists in nature leads to an effect that is useful for the real world from a new perspective. For example, it may be possible to improve the confidentiality of chaotic communications [3] or to be closer to human judgment in brain-type computers [4]. In addition, when we use chaos for engineering applications, it is expected to be smaller type, faster speed, and larger scale [5].

In this study, we investigate comparison of the complexity of chaos generated in many degrees of freedom chaotic circuits. In the previous study, it was proposed the circuit consisting of two Inaba's circuits coupled by one linear negative resister [6]. At this time, the circuit is set a different parameter values, especially natural frequencies for each circuit. Then we compare the two circuits in series with the three. We show chaotic attractors generated in the circuit experiment and computer simulation. In addition, we compare the complexity with Poincare maps created by computer simulation.

2. System model

In the previous study, we use the circuit model of two degrees of freedom chaotic circuit in Fig. 1. This circuit is expanded from the Inaba's circuit to two degrees of freedom chaotic circuit. In this circuit, we investigate the case which the natural frequency of the lower subcircuit is higher than the natural frequency of the upper subcircuit.

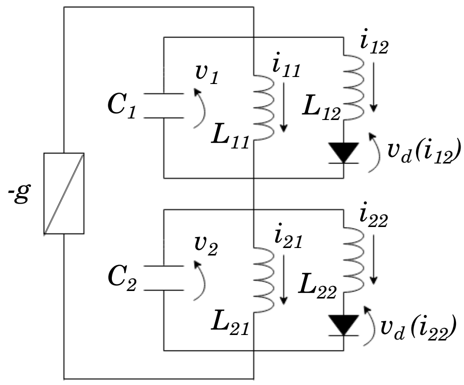


Fig.1 Circuit model of two degrees of freedom chaotic circuit.

The parameters are described as follows:

$$t = \sqrt{L_{11}C_1}\tau, \text{"}\cdot\text{"} = \frac{d}{d\tau}, \alpha = g\sqrt{\frac{L_{11}}{C_1}},$$

$$\beta_1 = \frac{L_{11}}{L_{12}}, \beta_2 = \frac{L_{11}}{L_{21}}, \beta_3 = \frac{L_{11}}{L_{22}},$$

$$\gamma = \frac{C_1}{C_2}, \varepsilon = \frac{1}{r_d}\sqrt{\frac{L_{11}}{C_1}},$$

$$v_1 = Ex_1, i_{11} = E\sqrt{\frac{C_1}{L_{11}}}x_2, i_{12} = E\sqrt{\frac{C_1}{L_{11}}}x_3,$$

$$v_2 = Ex_4, i_{21} = E\sqrt{\frac{C_1}{L_{11}}}x_5, i_{22} = E\sqrt{\frac{C_1}{L_{11}}}x_6.$$

The normalized circuit equations are described as follows:

$$\begin{cases} \dot{x_1} = \alpha(x_1 + x_4) - (x_2 + x_3) \\ \dot{x_2} = x_1 \\ \dot{x_3} = \beta_1(x_1 - f(x_3)) \\ \dot{x_4} = \alpha\gamma(x_1 + x_4) - \gamma(x_5 + x_6) \\ \dot{x_5} = \beta_2 x_4 \\ \dot{x_6} = \beta_3(x_4 - f(x_6)). \end{cases} \tag{1}$$

The characteristic equation for the diode is described as follows:

$$f(x) = \tfrac{1}{2\varepsilon}(x + \varepsilon - |x - \varepsilon|). \tag{2}$$

3. Proposed system

In this study, we use the circuit model of three degrees of freedom chaotic circuit in Fig. 2. This circuit consists of two degrees of freedom chaotic circuit and another Inaba's circuit connected in series in order to produce more complex chaos.

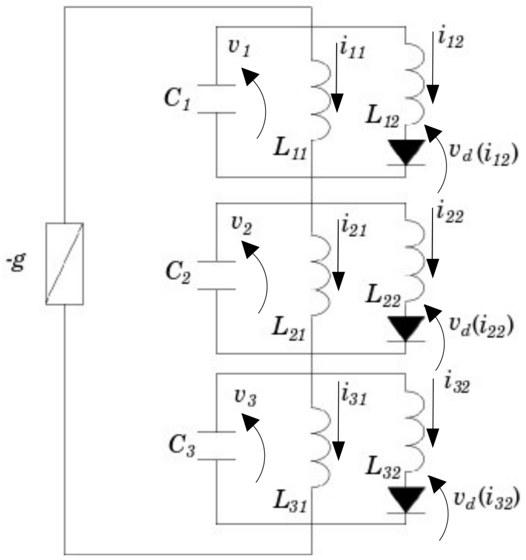


Fig.2 Circuit model of three degrees of freedom chaotic circuit.

The parameters are described as follows:

$t = \sqrt{L_{11}C_1}\tau$, " $\cdot$ " $= \frac{d}{d\tau}$, $\alpha = g\sqrt{\frac{L_{11}}{C_1}}$,

$\beta_1 = \frac{L_{11}}{L_{12}}, \beta_2 = \frac{L_{11}}{L_{21}}, \beta_3 = \frac{L_{11}}{L_{22}}, \beta_4 = \frac{L_{11}}{L_{31}}, \beta_5 = \frac{L_{11}}{L_{32}}$,

$\gamma_1 = \frac{C_1}{C_2}, \gamma_2 = \frac{C_1}{C_3}, \varepsilon = \frac{1}{r_d}\sqrt{\frac{L_{11}}{C_1}}$,

$v_1 = Ex_1, i_{11} = E\sqrt{\frac{C_1}{L_{11}}}x_2, i_{12} = E\sqrt{\frac{C_1}{L_{11}}}x_3$,

$\mathrm{v}_2 = Ex_4, i_{21} = E\sqrt{\frac{C_1}{L_{11}}}x_5, i_{22} = E\sqrt{\frac{C_1}{L_{11}}}x_6$,

$\mathrm{v}_3 = Ex_7, i_{31} = E\sqrt{\frac{C_1}{L_{11}}}x_8, i_{32} = E\sqrt{\frac{C_1}{L_{11}}}x_9$.

The normalized circuit equations are described as follows:

$$\begin{cases} \dot{x_1} = \alpha(x_1 + x_4 + x_7) - (x_2 + x_3) \\ \dot{x_2} = x_1 \\ \dot{x_3} = \beta_1(x_1 - f(x_3)) \\ \dot{x_4} = \alpha\gamma_1(x_1 + x_4 + x_7) - \gamma_1(x_5 + x_6) \\ \dot{x_5} = \beta_2 x_4 \\ \dot{x_6} = \beta_3(x_4 - f(x_6)) \\ \dot{x_7} = \alpha\gamma_2(x_1 + x_4 + x_7) - \gamma_2(x_8 + x_9) \\ \dot{x_8} = \beta_4 x_7 \\ \dot{x_9} = \beta_5(x_8 - f(x_9)). \end{cases} \tag{3}$$

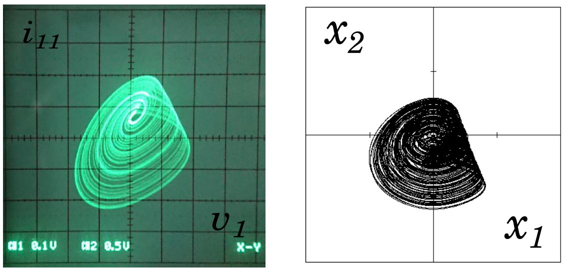
The characteristic equation for the diode is described as follows:

$$f(x) = \tfrac{1}{2\varepsilon}(x + \varepsilon - |x - \varepsilon|). \tag{4}$$

4. Results

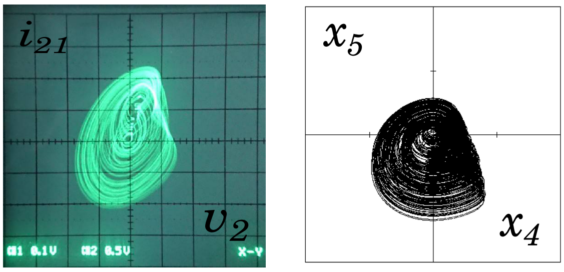
4.1. Two degrees of freedom chaotic circuit

We show the experimental and computer simulation results of two degrees of freedom chaotic circuit in Fig. 1. Figure 3 (a) is the result of the top circuit from circuit experiment. Figure 3 (b) is the result of the top circuit from computer simulation. Figure 4 (a) is the result of the bottom circuit from circuit experiment. Figure 4 (b) is the result of the bottom circuit from computer simulation.



(a) (b)

Fig. 3 Chaotic attractors in the top circuit.



(a) (b)

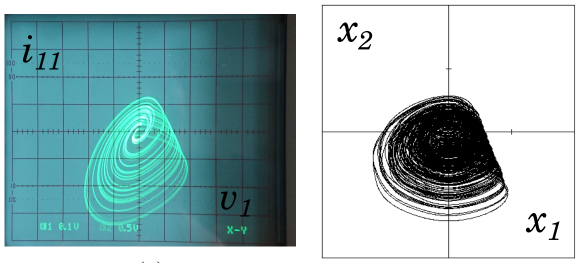
Fig. 4 Chaotic attractors in the bottom circuit.

In the circuit experiment, the circuit parameters are chosen as $C_1 = 15[nF]$, $L_{11} = 300[mH]$, $L_{12} = 30[mH]$, $C_2 = 7.5[nF]$, $L_{21} = 150[mH]$, and $L_{22} = 15[mH]$. In the computer simulation, the circuit parameters are chosen as $\alpha = 0.3$, $\beta_1 = 10.0$, $\beta_2 = 2.0$, $\beta_3 = 20.0$, $\gamma = 2.0$, and $\varepsilon = 0.01$.

As a result, the same attractors were observed qualitatively in each circuit in circuit experiment and computer simulation.

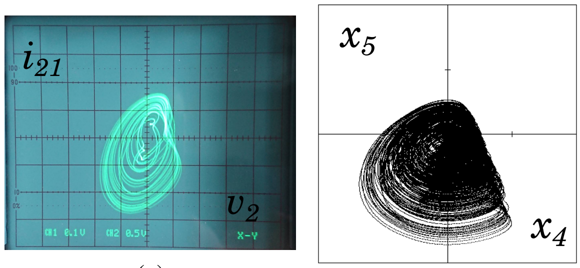
4.2. Three degrees of freedom chaotic circuit

In this study, we change the number of Inaba's circuit connected from two to three. We show the experimental and computer simulation results of three degrees of freedom chaotic circuit. Figure 5 (a) is the result of the top circuit from circuit experiment. Figure 5 (b) is the result of the top circuit from computer simulation. Figure 6 (a) is the result of the middle circuit from circuit experiment. Figure 6 (b) is the result of the middle circuit from computer simulation. Figure 7 (a) is the result of the bottom circuit from circuit experiment. Figure 7 (b) is the result of the bottom circuit from computer simulation.



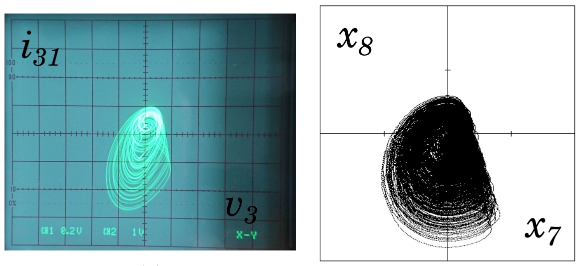
(a) (b)

Fig. 5 Chaotic attractors in the top circuit.



(a) (b)

Fig. 6 Chaotic attractors in the middle circuit.



(a) (b)

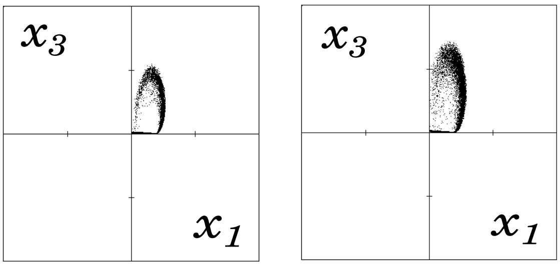
Fig. 7 Chaotic attractors in the bottom circuit.

In the circuit experiment, the circuit parameters are chosen as $C_1 = 15[nF]$, $L_{11} = 300[mH]$, $L_{12} = 30[mH]$, $C_2 = 7.5[nF]$, $L_{21} = 150[mH]$, $L_{22} = 15[mH]$, $C_3 = 5[nF]$, $L_{31} = 100[mH]$, and $L_{32} = 10[mH]$. In the computer simulation, the circuit parameters are chosen as $\alpha = 0.3$, $\beta_1 = 10.0$, $\beta_2 = 2.0$, $\beta_3 = 20.0$, $\beta_4 = 3.0$, $\beta_5 = 30.0$, $\gamma_1 = 2.0$, $\gamma_2 = 3.0$, and $\varepsilon = 0.01$.

Comparing the results of two degrees and three, attractors are similar in shape, and become more complex.

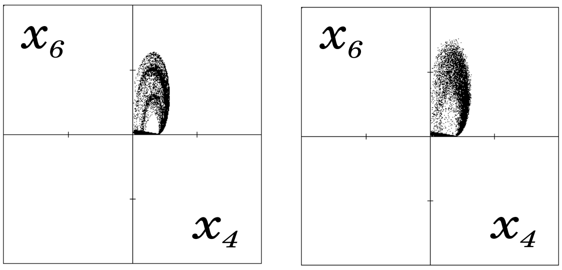
4.3. Poincare map

We show simple Poincare maps in Figs. 8 - 10. We examine the behavior of the point when the periodic trajectory repeatedly passes through the cut surface in the computer simulation. Figure 8 (a) is the result of the top circuit in two degrees of freedom chaotic circuit. Figure 8 (b) is the result of the top circuit in three degrees of freedom chaotic circuit. Figure 9 (a) is the result of the middle circuit in two degrees of freedom chaotic circuit. Figure 9 (b) is the result of the middle circuit in three degrees of freedom chaotic circuit. Figure 10 (b) is the result of the bottom circuit in three degrees of freedom chaotic circuit.



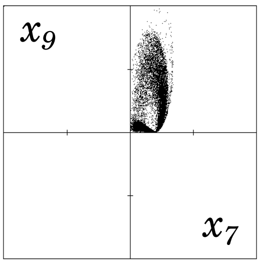
(a) (b)

Fig. 8 Poincare map in the top circuit.



(a) (b)

Fig. 9 Poincare map in the middle circuit.



(b)

Fig. 10 Poincare map in the bottom circuit.

In two degrees of freedom chaotic circuit, the circuit parameters are chosen as $\alpha = 0.3$, $\beta_1 = 10.0$, $\beta_2 = 2.0$, $\beta_3 = 20.0$, $\gamma_1 = 2.0$, and $\varepsilon = 0.01$. In three degrees of freedom chaotic circuit, the circuit parameters are chosen as $\alpha = 0.3$, $\beta_1 = 10.0$, $\beta_2 = 2.0$, $\beta_3 = 20.0$, $\beta_4 = 3.0$, $\beta_5 = 30.0$, $\gamma_1 = 2.0$, $\gamma_2 = 3.0$, and $\varepsilon = 0.01$.

Looking at the attractor, the result was similar, but there was a difference by creating Poincare map. Comparing the figures, it can be seen that when the number of circuits is increased, the order is especially broken in the top circuit.

5. Conclusion

In this study, we have investigated comparison of the complexity of chaos generated in multiple Inaba's circuit in series. We have increased the number of connected subcircuits from two to three in order to produce more complex chaos. We have evaluated the complexity using attractors obtained by circuit experiment and computer simulation, and Poincare maps obtained by computer simulation. Focusing on the chaotic attractor, more complex chaos is generated in the new connected circuit. However, when comparing the same circuits, it seems that there is not much difference. Next, when the Poincare map is observed, it can be seen that the complexity is high because the order is broken, especially in the top circuit. From the results, we confirmed that the chaos generated in some circuits are more complex.

As our future work, we will conduct rigorous analysis of the chaos generated in chaotic circuits and research on various network types. In a rigorous analysis, we will show the bifurcation phenomenon in a diagram and aim to visualize the route from the periodic solution to the chaotic solution [7]. In research on various network types, we will investigate interactions between systems [8].

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