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Investigating Dynamical Behavior with Higher Dimensions in Four Dimensional Hyperchaotic Systems

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Abstract

This paper considers the change of dynamical behavior with higher dimensions in four dimensional hyperchaotic systems. In particular, we focus on the shape and the complexity of chaotic attractors by changing the number of inductors and capacitors in the circuit. By means of the computer simulation and circuit experiment, the changes in the shape of chaotic attractors are investigated. Further, by means of poincaremap, the changes in the complexity of attractors are investigated. From the these results, it is shown that the system becomes more complex with the system becomes higher dimensions.

1. Introduction

Chaos is phenomena that exist close to us. For example, natural phenomena and neurons which build the brain and so on. From these viewpoints, the study of chaos has attracted a great attention from various fields such as natural science, biology and engineering. Particularly in the engineering field, confidential communications [1] which using the randomness of chaos, and a brain-type computer that incorporates brain information processing into a computer are expected and researched. Further in these studies, analysis of high dimensional systems that combine many elements is important. In recent years, the studies on the behavior in high dimensional systems and complex networks [2] - [3] that interconnect systems are conducted.

We focus on the systems that generate hyperchaos. Hyperchaos is generated from high dimensional systems with a minimum dimension of four [4] - [5]. In this study, we use the four dimensional hyperchaotic system proposed by Nishio *et al.* [6]. In this system, the results show that the resonant frequency governing the diode is related with generation of hyperchaos. Therefore, we change dimensions by increasing the number of inductors and capacitors and investigate the changes of dynamics.

2. Extreamely simple hyperchaos generators

Figure 1 shows the chaotic circuit which generates hyperchaos [6]. This circuit consists of a negative resistor, two inductors, two capacitors and one diode. This circuit is a system that can generate hyperchaos very simple because nonlinear element in the circuit is only one diode.

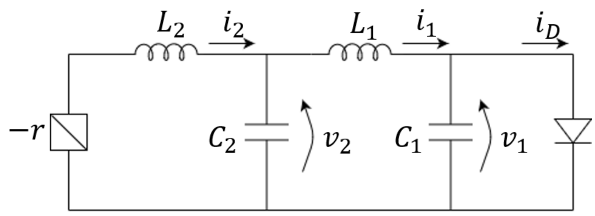


Figure 1: Chaotic circuit.

The circuit equations are given as follows:

$$\begin{cases} C_1 \dfrac{dv_1}{dt} &= i_1 - i_d \\ C_2 \dfrac{dv_2}{dt} &= i_2 - i_1 \\ L_1 \dfrac{di_1}{dt} &= v_2 - v_1 \\ L_2 \dfrac{di_2}{dt} &= -v_2 + ri_2. \end{cases} \tag{1}$$

where the $v - i$ characteristics of the diode are approximated as follows:

$$i_D = \frac{1}{2} G \left(|v_k - E| + v_k - E \right). \tag{2}$$

By using the parameters and the variables:

$$v_1 = Ex_1,\ v_2 = Ex_2,\ t = \sqrt{L_1 C_1}\tau$$

$$i_1 = \sqrt{\frac{L_1}{C_1}} Ex_3,\ i_2 = \sqrt{\frac{L_1}{C_{\ 1}}} Ex_4,\ \varepsilon = \frac{1}{G}\sqrt{\frac{C_1}{L_1}}$$

$$\gamma_C = \frac{C_1}{C_2},\ \gamma_L = \frac{L_1}{L_2},\ \alpha = R\frac{\sqrt{L_1 C_1}}{L_2}.$$

The normalized circuit equations are given as follows:

$$\begin{cases} \dot{x_1} &= x_3 - f(x_1) \\ \dot{x_2} &= \gamma_C(x_4 - x_3) \\ \dot{x_3} &= x_2 - x_1 \\ \dot{x_4} &= -\gamma_L x_2 + \alpha x_4. \end{cases} \tag{3}$$

where the parameter $f(x)$ is the equation for the diode and described as follows:

$$f(x) = \frac{1}{2}\varepsilon^{-1}\left(|x-1| + x - 1\right). \tag{4}$$

We set the values of parameters. Computer simulation for $\gamma_C = 0.47,\ \gamma_L = 0.4,\ \alpha = 0.16,\ \varepsilon = 0.01$. Circuit experiment for $L_1 = 20[mH],\ L_2 = 50[mH],\ C_1 = 0.022[\mu F],\ C_2 = 0.047[\mu F]$.

At this time, in the circuit of Fig. 1, the value of inductors and capacitors are set such that $L_2 > L_1,\ C_2 > C_1$.

3. System model

Figure 2 shows the chaotic circuit which one inductor and one capacitor are added to Fig. 1.

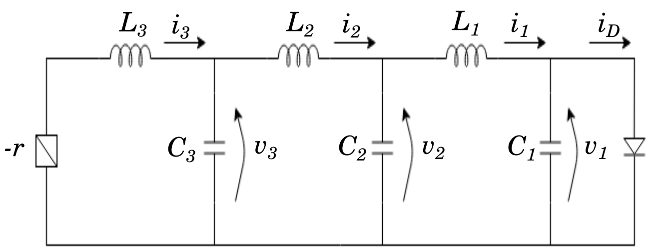


Figure 2: Proposed model 1.

By using the parameters and the variables:

$$v_1 = Ex_1,\ v_2 = Ex_2,\ v_3 = Ex_3,\ t = \sqrt{L_1 C_1}\tau$$

$$i_1 = \sqrt{\frac{L_1}{C_1}} Ex_4,\ i_2 = \sqrt{\frac{L_1}{C_{\ 1}}} Ex_5,\ i_3 = \sqrt{\frac{L_1}{C_{\ 1}}} Ex_6$$

$$\gamma_{C1} = \frac{C_1}{C_2},\ \gamma_{C2} = \frac{C_1}{C_3},\ \gamma_{L1} = \frac{L_1}{L_2},\ \gamma_{L2} = \frac{L_1}{L_3}$$

$$\alpha = R\frac{\sqrt{L_1 C_1}}{L_2},\ \varepsilon = \frac{1}{G}\sqrt{\frac{C_1}{L_1}}.$$

The normalized circuit equations are given as follows:

$$\begin{cases} \dot{x_1} &= x_4 - f(x_1) \\ \dot{x_2} &= \gamma_{C1}(x_5 - x_4) \\ \dot{x_3} &= \gamma_{C2}(x_6 - x_5) \\ \dot{x_4} &= x_2 - x_1 \\ \dot{x_5} &= \gamma_{L1}(x_3 - x_2) \\ \dot{x_6} &= -\gamma_{L2} x_3 + \alpha x_6. \end{cases} \tag{5}$$

We set the values of parameters. Computer simulation for $\gamma_{C1} = 0.45,\ \gamma_{C2} = 0.21,\ \gamma_{L1} = 0.5,\ \gamma_{L2} = 0.4,\ \alpha = 0.06,\ \varepsilon = 0.01$. Circuit experiment for $L_1 = 10[mH],\ L_2 = 20[mH],\ L_3 = 50[mH],\ C_1 = 0.010[\mu F],\ C_2 = 0.022[\mu F],\ C_3 = 0.047[\mu F]$.

At this time, in the circuit of Fig. 2, the value of inductors and capacitors are set such that $L_3 > L_2 > L_1,\ C_3 > C_2 > C_1$.

Figure 3 shows the chaotic circuit which two inductors and two capacitors are added to Fig. 1.

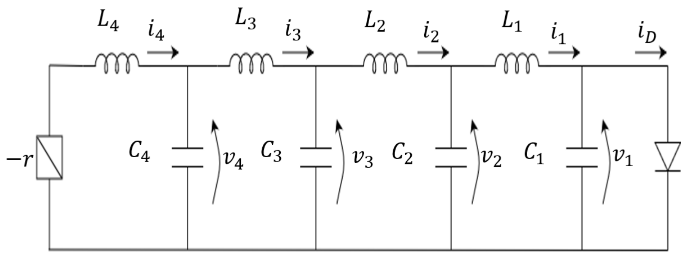


Figure 3: Proposed model 2.

By using the parameters and the variables:

$$v_1 = Ex_1,\ v_2 = Ex_2,\ v_3 = Ex_3,\ v_4 = Ex_4$$

$$i_1 = \sqrt{\frac{L_1}{C_1}}Ex_5,\ i_2 = \sqrt{\frac{L_1}{C_1}}Ex_6$$

$$i_3 = \sqrt{\frac{L_1}{C_1}}Ex_7,\ i_4 = \sqrt{\frac{L_1}{C_1}}Ex_8,\ \ t = \sqrt{L_1 C_1}\tau$$

$$\gamma_{C1} = \frac{C_1}{C_2},\ \gamma_{C2} = \frac{C_1}{C_3},\ \gamma_{C3} = \frac{C_1}{C_4}$$

$$\gamma_{L1} = \frac{L_1}{L_2},\ \gamma_{L2} = \frac{L_1}{L_3},\ \gamma_{L3} = \frac{L_1}{L_4}$$

$$\alpha = R\frac{\sqrt{L_1 C_1}}{L_2},\ \varepsilon = \frac{1}{G}\sqrt{\frac{C_1}{L_1}}.$$

The normalized circuit equations are given as follows:

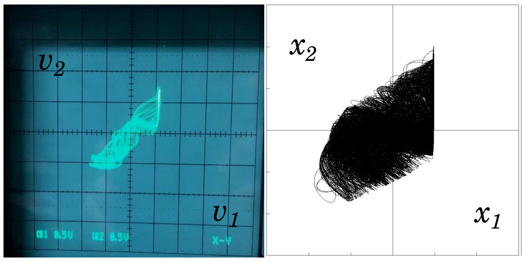
$$\left\{\begin{array}{lcl} \dot{x_1} & = & x_5 - f(x_1) \\ \dot{x_2} & = & \gamma_{C1}(x_6 - x_5) \\ \dot{x_3} & = & \gamma_{C2}(x_7 - x_6) \\ \dot{x_4} & = & \gamma_{C3}(x_8 - x_7) \\ \dot{x_5} & = & x_2 - x_1 \\ \dot{x_6} & = & \gamma_{L1}(x_3 - x_2) \\ \dot{x_7} & = & \gamma_{L2}(x_4 - x_3) \\ \dot{x_8} & = & -\gamma_{L3}x_4 + \alpha x_8. \end{array}\right. \tag{6}$$

We set the values of parameters. Computer simulation for $\gamma_{C1} = 0.47,\ \gamma_{C2} = 0.21,\ \gamma_{C3} = 0.1,\ \gamma_{L1} = 0.5,\ \gamma_{L2} = 0.25,\ \gamma_{L3} = 0.1,\ \alpha = 0.03,\ \varepsilon = 0.01$. Circuit experiment for $L_1 = 5[mH],\ L_2 = 10[mH],\ L_3 = 20[mH],\ L_4 = 50[mH],\ C_1 = 0.0047[\mu F],\ C_2 = 0.010[\mu F],\ C_3 = 0.022[\mu F],\ C_4 = 0.047[\mu F]$.

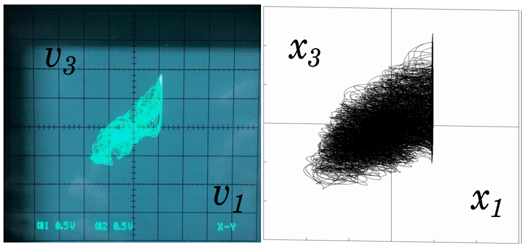
At this time, in the circuit of Fig. 3, the value of inductors and capacitors are set such that $L_4 > L_3 > L_2 > L_1,\ C_4 > C_3 > C_2 > C_1$.

4. Computer simulation and circuit experiment

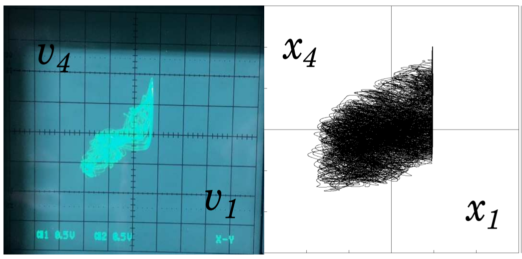
First, we compare the shape of attractors from the results obtained from computer simulations and circuit experiments. In this study, we measure the voltage values of two capacitors close to negative resistor and diode in each circuit and show to attractors. Figure 4(a) shows the attractors which obtained from the circuit of Fig. 1. Figure 4(b) shows the attractors which obtained from the circuit of Fig. 2. Figure 4(c) shows the attractors which obtained from the circuit of Fig. 3. Further the figures of left side show the results of circuit experiment and the figures of right side show the results of computer simulation in Fig. 4. In this results, we confirmed that the shape of attractors does not change so much. That is, it is confirmed that a slight increase in dimensions do not significantly affect the shape of the attractors.



(a) Attractors obtained from the circuit of Fig. 1.



(b) Attractors obtained from the circuit of Fig. 2. (Proposed model 1)

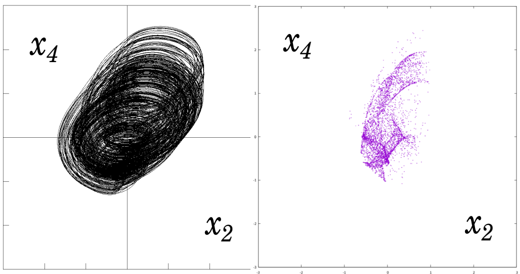


(c) Attractors obtained from the circuit of Fig. 3. (Proposed model 2)

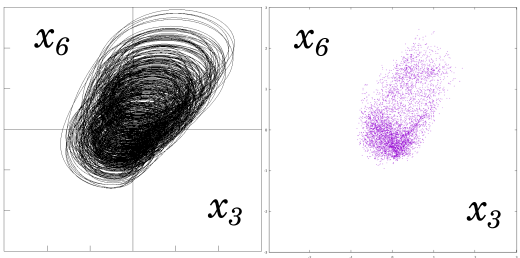
Figure 4: Attractors by the computer simulation and circuit experiment.

5. Poincaremap

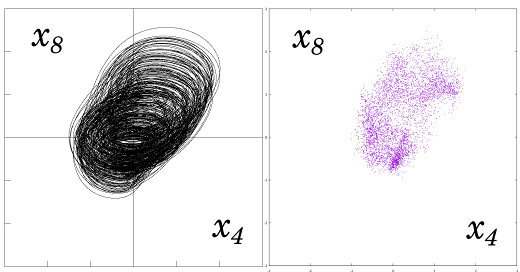
Second, we compare the complexity of attractors by using poincaremap. The poincaremap is obtained by setting a plane that crosses the trajectory of the attractor. Further it is used to analyze the local properties of attractors by plotting points when the trajectory intersects the plane. For example, when the attractor has one periodic trajectory, it is expected that the trajectory always intersects the plane at the same position. On the other hand, when the attractor has multiple trajectories, it is expected that the trajectories intersect the plane at various positions. That is, the attractor has a complex trajectory and many points are plotted on the plane. Focusing on this property, we evaluate the complexity of attractors using poincaremap. Figure 5(a) shows the attractors obtained from the circuit of Fig. 1. Figure 5(b) shows the attractors obtained from the circuit of Fig. 2. Figure 5(c) shows the attractors obtained from the circuit of Fig. 3. Further the figures of left side show the attractors and the figure of right side show the attractors of poincaremaps in Fig. 5. In this results, we confirmed that many points are plotted extensively in Fig. 5(b) and Fig. 5(c). Therefore, it is confirmed that the attractors which obtained from the circuits with increased dimensions become complex.



(a) Attractors obtained from the circuit of Fig. 1.



(b) Attractors obtained from the circuit of Fig. 2.
(Proposed model 1)



(c) Attractors obtained from the circuit of Fig. 3.
(Proposed model 2)

Figure 5: Attractors by the computer simulation.

6. Conclusion

This study investigated the change of behavior with higher dimensions in four dimensional hyperchaotic systems. In particular, we focus on the shape and the complexity of chaotic attractors by changing the number of inductors and capacitors. By means of the computer simulation and circuit experiment, the change in the shape of attractors are investigated. Further, by means of poincaremap, the change in the complexity of attractors are investigated. First, from the results of the computer simulation and circuit experiment, we confirmed that the shape of attractors did not change so much. Next, from the results of poincaremap, we confirmed that the complexity of attractors changed with increased dimensions.

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