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Whale Optimization Algorithm with Increased Number of Agents for Global Search

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Abstract

In this study, we focuse on global search of Whale Optimization Algorithm (WOA). In the conventional WOA, the number of agents for global search is reduced by the probability determined by the number of iterations. In the proposed method, there are always certain number of agents for global search until number of iteration exceeds half of the maximum number of iterations. The proposed method and the conventional WOA are compared with searching for optimal value of the benchmark functions. Their minimum values, average values and standard deviations are compared. The proposed method shows better results than the conventional WOA.

1. Introduction

Swarm intelligence (SI) is an artificial intelligence that simulates the behavior of creatures that form swarms. Examples of animals simulated are ants, bees, fishes, etc. The feature of SI is each individual can only take simple movements, and the whole group shows complicated movements. SI is important because simple controls are more efficient than complex controls. SI is used for driving automation and for analysis of human movement during evacuation and traffic jams.

Recently, optimization problems have attracted attention. Optimization derive the best result under the condition or deriving the method. Examples of optimization problems are traveling salesman problems and knapsack problems. Optimization problems are often nonlinear, it is difficult to derive an optimal value. Metaheuristic algorithms are metaheuristic and are used to seeking the optimal solution. Optimization by SI is an effective optimization method. Examples are Ant Colony Optimization (ACO) [1], Artificial Bee Colony Algorithm (ABC) [2], Particle Swarm Optimization (PSO) [3], and Whale Optimization Algorithm (WOA) [4].

In this study WOA is used. WOA have insufficient global search. We propose WOA with a certain number of whales that perform global search, when number of iteration is less than half of the maximum iteration. The search ability of the proposed method is better than WOA.

2. Whale optimization algorithm

Whale Optimization Algorithm (WOA) is proposed by Mirjalili and Lewis in 2016. WOA is nature-inspired metaheuristic optimization algorithm which simulate the feeding behavior of humpback whales. The feeding behavior is performed by bubble net method. Whales take three types of actions: Encircling prey, Bubble-net attacking method and Search for prey in WOA.

(a) Encircling prey

Humpback whales find prey and surround prey. There is a prey at the position of the best agent in WOA, and that prey is the optimal solution. After the best agent is determined, the other agents update their position towards the best agent. The movement is represented by Eqs. (1) and (2).

$$\overrightarrow{D} = |\overrightarrow{C}.\overrightarrow{X^*}(t) - \overrightarrow{X}(t)| \tag{1}$$

$$\overrightarrow{X^*}(t+1) = \overrightarrow{X}(t) - \overrightarrow{A}.\overrightarrow{D}, \tag{2}$$

where t indicates the current iteration, $\overrightarrow{A}$ and $\overrightarrow{C}$ are coefficient vectors, $\overrightarrow{X}$ is the position vector, $\overrightarrow{X^*}$ is the position vector of the best solution obtained, . is an element-by-element multiplication. $\overrightarrow{A}$ and $\overrightarrow{C}$ are determined by Eqs. (3) and (4).

$$\overrightarrow{A} = 2\vec{a}.\vec{r_1} - \vec{a} \tag{3}$$

$$\overrightarrow{C} = 2\vec{r_2}, \tag{4}$$

where $\vec{a}$ is linearly decreased from 2 to 0 over the course of iterations, $\vec{r_1}$ and $\vec{r_2}$ are random vector in $[0, 1]$.

(b) Bubble-net attacking method

In the bubble-net attacking method, the humpback whale moves along the spiral path toward the prey. Equations (5) and (6) represent the relationship between whale position and prey position for simulating spiral movement.

$$\overrightarrow{D'} = |\overrightarrow{X^*}(t) - \overrightarrow{X}(t)| \tag{5}$$

$$\overrightarrow{X}(t+1) = \overrightarrow{D'}e^{bl}\cos(2\pi l) + \overrightarrow{X^*}(t), \tag{6}$$

where $\overrightarrow{D'}$ is vector of the distance between the whale and the best agent, b is a constant for defining the shape of the logarithmic helix, l is a random number in $[-1, 1]$. The probability of choosing either Eq. (2) or (6) to update the whale position is 50%. The equation of movement is selected by Eq. (7).

$$\begin{cases} \overrightarrow{X}(t+1) = \overrightarrow{X}(t) - \overrightarrow{A}.\overrightarrow{D} & (if\ p < 0.5) \\ \overrightarrow{X}(t+1) = \overrightarrow{D'}e^{bl}\cos(2\pi l) + \overrightarrow{X^*}(t), & (if\ p \geq 0.5) \end{cases} \tag{7}$$

where p is random number in $[0, 1]$.

(c) Search for prey

Besides the bubble-net attacking method, humpback whales randomly search for prey. The position of the whale is updated with value of $\overrightarrow{A}$ greater than 1 or less than -1 to move the agent away from the chosen whale. The update of location plays a global search role in WOA. The movement during search for prey is represented by Eqs. (8) and (9).

$$\overrightarrow{D_{rand}} = |\overrightarrow{X_{rand}} - \overrightarrow{X}(t)| \tag{8}$$

$$\overrightarrow{X}(t+1) = \overrightarrow{X_{rand}} - \overrightarrow{A}.\overrightarrow{D_{rand}} \tag{9}$$

$\overrightarrow{X_{rand}}$ is position vector of whales randomly selected from the population.

(d) Procedure of WOA

The procedure for WOA to search for solution is shown below.

Step 1. Initialize the whales population n and the maximum number of iterations t_{max}.

Step 2. Set the position of the whale $\overrightarrow{X_i}$. i is the number of the whale. $\overrightarrow{X_i}$ is determined randomly within the range.

Step 3. Calculate the fitness value f_i and update the best whale position $\overrightarrow{X^*}$.

Step 4. Update $\vec{a}$, $\vec{r_1}$, $\vec{r_2}$, $\overrightarrow{A}$, $\overrightarrow{C}$, l and p.

Step 5. Update the position of the whale by Eq. (2),(6) or (9). Equations are selected by $|\overrightarrow{A}|$ and p.

Step 6. Repeat Step 3 to Step 5 until all search agents are done.

Step 7. Check if any search agent goes beyond the search space and amend it.

Step 8. Calculate f_i of each search agent.

Step 9. Update X* if there is a better solution.

Step 10. Repeat Step 3 to Step 9 t_{max} times.

3. Proposed method

WOA performs a global search using Eqs. (8) and (9). However, Eqs. (8) and (9) represents a linear movement, and Eqs. (8) and (9) cannot perform sufficient global search. Each number of whales performing local search and global search is determined by a. The value of a decreases, then the number of whales in the global search decreases. If a is less than 1, there are no whales in the global search. The number of iterations in this case is more than half of t_{max}. We propose WOA with a certain number of whales that perform global search when a is 1 or more. The number of whales constantly searching globally is three, four or five.

4. Simulation result

We find the minimum values of the benchmark functions using the proposed method and WOA [5]-[8]. Five of the benchmark functions are unimodal and seven are multimodal. Table 1 shows the formula, range, and optimal value (OV) of unimodal benchmark functions. x is variable in the function. x_i denotes i-dimensional x.

Table 1: unimodal benchmark functions.

Name	Formula	Range	OV
f_1	$\sum_{i=1}^{30} x_i^2$	[-100,100]	0
f_2	$\sum_{i=1}^{30} \|x_i\| + \prod \|x_i\|$	[-10,10]	0
f_3	$\max \{\|x_i\|, 1 \leq i \leq 30\}$	[-100,100]	0
f_4	$\sum_{i=1}^{30} [100(x_{i+1} - x_i^2)^2 + (x_i - 1)^2]$	[-30,30]	0
f_5	$\sum_{i=1}^{30} (x_i + 0.5)^2$	[-100,100]	0

Table 2 shows the formula, range, and optimal value of multimodal benchmark functions.

Table 2: multimodal benchmark functions.

Name	Formula	Range	OV
f_6	$\sum_{i=1}^{30} -x_i \sin(\sqrt{\lvert x_i \rvert})$	[-500,500]	-12569
f_7	$\sum_{i=1}^{30} [x_i^2 - 10\cos(2\pi x_i) + 10]$	[-5.12,5.12]	0
f_8	$-20\exp(-0.2\sqrt{\frac{1}{30}\sum_{i=1}^{30} X_i^2})$ $-\exp(\frac{1}{30}\sum_{i=1}^{30}\cos(2\pi x_i)) + 20 + e$	[-32,32]	0
f_9	$\frac{1}{4000}\sum_{i=1}^{30} x_i^2 - \prod_{i=1}^{30}\cos(\frac{x_i}{\sqrt{i}}) + 1$	[-600,600]	0
f_{10}	$4x_1^2 - 2.1x_1^4 + \frac{1}{3}x_1^6 + x_1x_2 - 4x_2^2 + 4x_2^4$	[-5,5]	1.0316
f_{11}	$(x_2 - \frac{5.1}{4\pi^2}x_1^2 + \frac{5}{\pi}x_1 - 6)^2$ $+10(1-\frac{1}{8\pi})\cos x_1 + 10$	[-5,5]	0.398
f_{12}	$[1 + (x_1 + x_2 + 1)^2(19 - 14x_1 + 3x_1^2$ $-14x_2 + 6x_1x_2 + 3x_2^2)] \times [30 + (2x_1 + x_2 + 1)^2$ $\times(18 - 32x_1 + 12x_1^2 + 48x_2 - 36x_1x_2 + 27x_2^2)]$	[-2,2]	3

In each test function, we define $t_{max} = 500$, $n = 30$. We repeated the search 500 times and found the minimum values, average values and standard deviations of the search results with the conventional WOA and the proposed methods. The minimum values, average values and standard deviations are shown in Table 3. The numbers in the second column are the population of whales for global search constantly. If it is 0, the conventional WOA is used.

According to Table 3, the minimum value obtained by the proposed method is much smaller than the conventional WOA in f_1 and f_3. The best values of average value and standard deviations are found by the proposed method in all unimodal functions except f_1. The proposed method with three agents for global search found the best average value and standard deviations in two types of unimodal functions, and found better average value and standard deviations than WOA in all unimodal functions except f_1.

In multimodal function, the minimum values, average values and standard deviations are shown in Table 4. According to Table 4, the conventional WOA and the proposed method find the same minimum value in all multimodal functions. All best values of average values and standard deviations are found by the proposed methods. In particular, when there are always three agents for global search, average value and standard deviations are smaller than the conventional WOA in all functions. From the above, WOA with three agents for global search has the best search ability in the proposed methods.

Table 3: Results in unimodal functions.

name		MIN	AVE	STD
f_1	0(WOA)	6.9753×10^{-84}	$\mathbf{6.0423 \times 10^{-84}}$	$\mathbf{7.1808 \times 10^{-62}}$
	3	$\mathbf{4.7185 \times 10^{-278}}$	1.6108×10^{-61}	1.6433×10^{-60}
	4	1.4505×10^{-79}	2.0660×10^{-60}	3.7556×10^{-59}
	5	1.4185×10^{-57}	7.3986×10^{-60}	9.2180×10^{-59}
f_2	0(WOA)	1.3178×10^{-50}	3.4737×10^{-40}	6.1151×10^{-39}
	3	$\mathbf{1.8813 \times 10^{-52}}$	$\mathbf{2.1338 \times 10^{-40}}$	$\mathbf{1.5445 \times 10^{-39}}$
	4	3.2534×10^{-49}	1.3163×10^{-39}	1.2485×10^{-38}
	5	3.4654×10^{-49}	2.1338×10^{-39}	1.5445×10^{-38}
f_3	0(WOA)	7.9001×10^{-13}	1.0573×10^{-5}	4.5413×10^{-5}
	3	$\mathbf{9.1663 \times 10^{-145}}$	$\mathbf{8.9641 \times 10^{-6}}$	$\mathbf{3.0281 \times 10^{-5}}$
	4	8.1488×10^{-19}	1.0425×10^{-5}	3.3772×10^{-5}
	5	3.4718×10^{-139}	1.9876×10^{-5}	9.8165×10^{-5}
f_4	0(WOA)	$\mathbf{9.7889 \times 10^{-8}}$	1.0943×10^{1}	1.2743×10^{1}
	3	2.1159×10^{-7}	1.0439×10^{1}	1.2394×10^{1}
	4	3.4053×10^{-7}	$\mathbf{9.8581 \times 10^{0}}$	$\mathbf{1.2165 \times 10^{1}}$
	5	1.2294×10^{-7}	1.0047×10^{1}	1.2445×10^{1}
f_5	0(WOA)	1.2743×10^{-11}	4.8566×10^{-3}	7.1133×10^{-3}
	3	5.6239×10^{-14}	4.6502×10^{-3}	5.9021×10^{-3}
	4	5.6076×10^{-13}	4.4180×10^{-3}	$\mathbf{5.8768 \times 10^{-3}}$
	5	$\mathbf{1.9573 \times 10^{-15}}$	$\mathbf{4.2435 \times 10^{-3}}$	6.3817×10^{-3}

5. Conclusion

WOA cannot perform a sufficient global search. In the proposed method, there are always certain number of agents for global search until half of the maximum number of iterations. The number of the agents are three, four or five. We compared the search performance of the proposed method and conventional WOA with five kinds of unimodal benchmark functions and seven kinds of multimodal benchmark functions. In unimodal function, the proposed method find best value of average and standard deviations. The minimum value obtained by the proposed method is much smaller than the conventional WOA in two types of unimodal functions. The proposed method performed best in all multimodal functions. WOA with three whale for global search has the best search capabilities in the proposed methods.

Table 4: Results in multimodal functions.

name		MIN	AVE	STD
f_6	0(WOA)	-1.2569×10^4	-1.2531×10^4	1.8893×10^2
	3	-1.2569×10^4	-1.2559×10^4	6.5279×10^1
	4	-1.2569×10^4	-1.2553×10^4	$\mathbf{1.1135 \times 10^2}$
	5	-1.2569×10^4	$\mathbf{-1.2564 \times 10^4}$	4.7501×10^1
f_7	0(WOA)	0	6.0396×10^{-17}	3.4097×10^{-16}
	3	0	5.6843×10^{-17}	3.1263×10^{-16}
	4	0	$\mathbf{3.5527 \times 10^{-17}}$	$\mathbf{2.9511 \times 10^{-16}}$
	5	0	6.7502×10^{-17}	3.3963×10^{-16}
f_8	0(WOA)	4.4409×10^{-16}	4.5018×10^{-15}	2.6610×10^{-15}
	3	4.4409×10^{-16}	$\mathbf{4.4160 \times 10^{-15}}$	$\mathbf{2.5423 \times 10^{-15}}$
	4	4.4409×10^{-16}	4.5623×10^{-15}	2.6831×10^{-15}
	5	4.4409×10^{-16}	4.5581×10^{-15}	2.6987×10^{-15}
f_9	0(WOA)	0	6.200×10^{-3}	6.9374×10^{-2}
	3	0	1.4273×10^{-3}	2.2831×10^{-2}
	4	0	$\mathbf{1.2602 \times 10^{-3}}$	$\mathbf{2.1420 \times 10^{-2}}$
	5	0	2.6824×10^{-3}	3.5339×10^{-2}
f_{10}	0(WOA)	-1.0316	-1.0289	4.7233×10^{-2}
	3	-1.0316	$\mathbf{-1.0295}$	4.0684×10^{-2}
	4	-1.0316	-1.0293	4.6685×10^{-2}
	5	-1.0316	-1.0294	$\mathbf{3.9914 \times 10^{-2}}$
f_{11}	0(WOA)	3.9789×10^{-1}	4.0553×10^{-1}	1.6005×10^{-2}
	3	3.9789×10^{-1}	4.0475×10^{-1}	1.2734×10^{-2}
	4	3.9789×10^{-1}	4.0475×10^{-1}	1.3225×10^{-2}
	5	3.9789×10^{-1}	$\mathbf{4.0330 \times 10^{-1}}$	$\mathbf{1.2482 \times 10^{-2}}$
f_{12}	0(WOA)	3.0000	3.1284	0.23827
	3	3.0000	**3.1052**	**0.19148**
	4	3.0000	3.1309	0.26735
	5	3.0000	3.1132	0.21927

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