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Amplitude Change of Coupled van der Pol Oscillators in Three-Dimensional Space

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Abstract—The study of Amplitude Death (AD) and amplitude change are important for understanding the control mechanisms and efficient regulators of a system's dynamics. Previously, the mechanism of AD observed from two-dimensional coupled polygonal oscillatory networks was made clear by using mathematical analysis. In this study, we extend the polygonal oscillatory networks to three-dimensional space. We investigate the occurrence of AD by increasing the number of oscillators.

1. Introduction

Synchronization phenomena in coupled oscillators are suitable models for analyzing a number of natural occurrences [1],[2]. Therefore, many researchers have proposed different coupled oscillatory networks, and some interesting synchronization phenomena have been discovered [3]-[6].

Oscillation quenching (oscillation and amplitude death), another fundamental emergent phenomenon in coupled nonlinear systems, can be caused by several factors [7],[8]. Amplitude death (AD) occurs in strongly coupled nonlinear oscillators when their interaction causes a pair of fixed points to become stable and attracting. Setou et al. reported AD in ring coupled oscillators when the frequencies of the coupled units differ [9].

We have investigated synchronization phenomena in coupled polygonal oscillatory networks that share branches [10], [11]. In this system, van der Pol oscillators are connected to every corner of each polygonal network. The first and the second oscillators, which are connected to both polygonal networks, are called "shared oscillators," and each polygonal network has an odd number of oscillators. We then observe N-phase synchronization. Through computer simulations and theoretical analysis, we confirmed that the coupled oscillators tended to synchronize to minimize the power consumption of the whole system. The phase difference of the shared oscillators was determined by finding the minimum value of the power consumption function. Additionally, we proposed a new polygonal circuit system that includes actual inductor models (with loss) at all ground parts [12]. Synchronization phenomena in coupled polygonal oscillatory networks with strong frustration are investigated. Strong frustration is realized using conflicting coupling organization in the network and by increasing the coupling strength.

We confirmed that the amplitude of the oscillators decreases as the value of the coupling strength increases, and that AD occurs in the polygonal oscillatory networks. If one of the polygonal networks is triangular, we observe global AD. However, for other types of networks, AD appears in a complicated way. First, AD occurs at the oscillators located farthest from the shared oscillators. Next, AD occurs simultaneously in all other oscillators as the coupling strength increases. We explained the mechanism by which AD occurs using a theoretical approach.

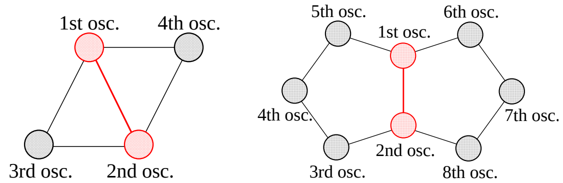
In this study, we investigate the occurrence of AD of coupled polygonal oscillatory networks in three-dimensional space. For considering the three-dimensional networks, the number of polygonal network is increased by three-dimensional way. By using computer simulations, we observe similar AD phenomena in the proposed three-dimensional coupled polygonal network with previous studies.

2. Coupled Oscillatory Networks in Two-Dimensional Space [12]

In our previous study, we investigated AD in two coupled oscillatory networks in two-dimensional space. Two polygonal oscillatory networks are coupled by sharing a branch. Examples of the network models used in the previous study are shown in Fig. 1. Here, the 3–3 and 5–5 coupling networks are symmetric models. In this circuit model, we consider the coupling method in which two adjacent oscillators tend to synchronize in the anti-phase state. The number of oscillators coupled to left and the right polygonal networks is set to odd numbers to produce anti-phase synchronization between adjacent oscillators. The first and second oscillators, which are connected to both sides of the polygonal networks, are called "shared oscillators."

Figure 2 shows the circuit model of the 3–3 coupling network. The earth resistances are inserted to the 3rd and 4th oscillators to model actual inductors and realize symmetry in the circuit network model. Tiny resistors (r_m) are inserted to avoid an L-loop in the computer simulations.

Next, we develop an expression for the circuit equations



(a) 3–3 coupling network. (b) 5–5 coupling network.

Figure 1: Two coupled oscillatory networks.

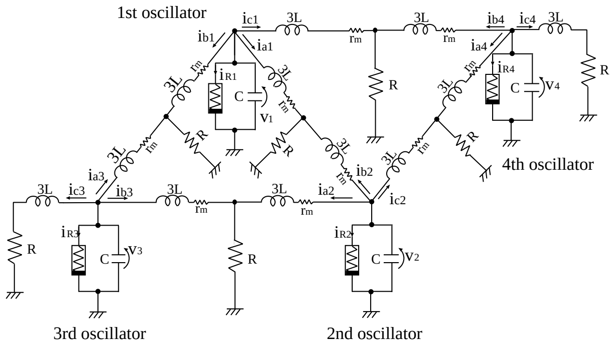


Figure 2: Coupling model (3–3 coupling network).

of the N–M coupling oscillatory network. The $v_k - i_{Rk}$ characteristics of the nonlinear resistor are approximated by the following third-order polynomial equation:

$$i_{Rk} = -g_1 v_k + g_3 {v_k}^3 \quad (g_1, g_3 > 0), \qquad (k = 1, 2, 3, ..., N + M - 2). \tag{1}$$

Using the variables and parameters

$$t = \sqrt{LC}\tau, \quad v_k = \sqrt{\frac{g_1}{3g_3}} x_k,$$

$$i_{ak} = \sqrt{\frac{g_1}{3g_3}} \sqrt{\frac{C}{L}} y_{ak}, \quad i_{bk} = \sqrt{\frac{g_1}{3g_3}} \sqrt{\frac{C}{L}} y_{bk},$$

$$i_{ck} = \sqrt{\frac{g_1}{3g_3}} \sqrt{\frac{C}{L}} y_{ck}, \quad i_n = \sqrt{\frac{g_1}{3g_3}} \sqrt{\frac{C}{L}} y_n,$$

$$\varepsilon = g_1 \sqrt{\frac{L}{C}}, \quad \gamma = R\sqrt{\frac{C}{L}}, \quad \eta = r_m \sqrt{\frac{C}{L}},$$

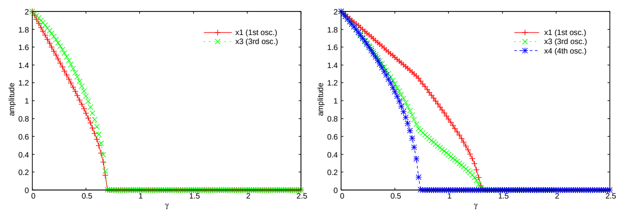
$$(k = 1, 2, 3, ..., N + M - 2),$$

the normalized circuit equations governing the circuit are expressed as:

$$\begin{cases} \dfrac{dx_k}{d\tau} = \varepsilon\left(1 - \dfrac{1}{3}{x_k}^2\right)x_k - (y_{ak} + y_{bk} + y_{ck}) \\ \dfrac{dy_{ak}}{d\tau} = \dfrac{1}{3}\left\{x_k - \eta y_{ak} - \gamma(y_{ak} + y_n)\right\} \\ \dfrac{dy_{bk}}{d\tau} = \dfrac{1}{3}\left\{x_k - \eta y_{bk} - \gamma(y_{bk} + y_n)\right\} \\ \dfrac{dy_{ck}}{d\tau} = \dfrac{1}{3}\left\{x_k - \eta y_{ck} - \gamma(y_{ck} + y_n)\right\} \\ \qquad\qquad (k = 1, 2, 3, ..., N + M - 2). \end{cases} \tag{2}$$

In this equation, γ is the coupling strength, ε denotes the nonlinearity of the oscillators, and y denotes the current of the inductor of the connected oscillator with the kth oscillator. For the computer simulations, we calculate Eq. (2) using the fourth-order Runge–Kutta method with step size $h = 0.005$. We set the parameters of this circuit model to $\varepsilon = 0.1$ and $\eta = 0.0001$. The coupling strength γ between the oscillators changes from a small to a large value.

Figure 3 shows the change in amplitude according to the size of the network. In the case of the 3–3 coupling network, global AD occurs at the same time, whereas in the 5–5 coupling network, AD first occurs in the oscillators located farthest from the shared oscillators, and then the other oscillators stop oscillating at the same time.

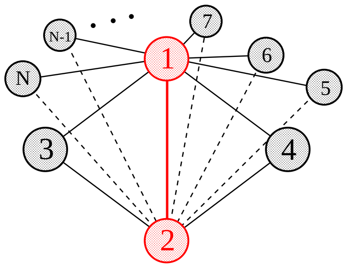


(a) 3–3 coupling network. (b) 5–5 coupling network.

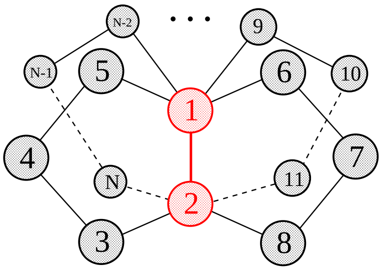
Figure 3: Amplitude change in two-dimensional coupled oscillatory networks.

3. Coupled Oscillatory Networks in Three-Dimensional Space

Here, we investigate the amplitude change of the oscillators when the system model is extended to three-dimensional space. Two types of network models are proposed as shown in Fig. 4. The first model is composed of a triangular network. We investigate the change in amplitude when the triangular network is increased in three dimensions around the shared oscillators as shown in Fig. 4 (a). The second model is composed of a pentagonal network (Fig. 4 (b)). Similar to the triangular network, the change in amplitude when the pentagonal network is increased three-dimensionally around the shared oscillators is investigated. In the proposed circuit model, N denotes the total number of oscillators.



(a) Triangular oscillatory network.

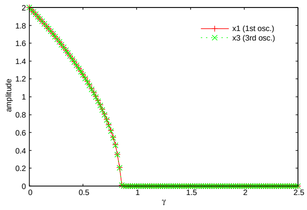


(b) Pentagonal oscillatory network.

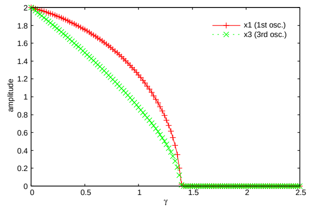
Figure 4: Coupled oscillatory networks in three-dimensional space.

Figure 5 shows the simulation results of amplitude change with the coupling strength. From these results, it was found that global AD was observed in the same way as in the case of a triangular oscillatory network connected two-dimensionally. When $N = 5$, the change in the amplitudes of the shear oscillator and other oscillators is almost equal. As the value of N increases, the difference between the sheared and other oscillators appears. The shared oscillator changes two-dimensionally, and the other oscillators decrease linearly. It can also be seen that as N increases, the coupling strength at which AD occurs increases. The coupling strength at which AD occurs with N is shown in Fig. 6. We confirm that the coupling strength almost changes linearly.

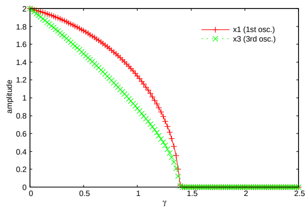
Next, Fig. 7 shows the simulation results of amplitude change of system model using pentagonal networks with the coupling strength. From these results, it was found that partial AD was observed in the same way as in the case of a pentagonal oscillatory network connected two-dimensionally. First, the oscillation death of the oscillators located farthest place from the shared oscillators is occurred. After that, the other oscillators stop to oscillate at same time. It can also be seen that as N increases, the coupling strength at which AD occurs increases. The coupling strength at which AD occurs with N is shown in Fig. 8. We confirm that the coupling strength almost changes linearly when N is smaller than 32. While, when N is larger than 35, two types of oscillators stop their oscillation at same time.



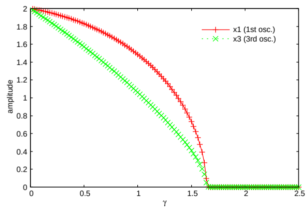
(a) N=5.



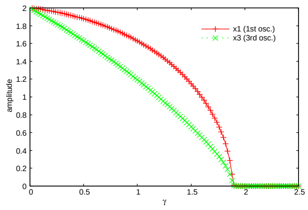
(b) N=7.



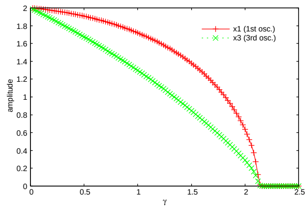
(c) N=9.



(d) N=11.



(e) N=13.



(f) N=15.

Figure 5: Amplitude change in three-dimensional coupled oscillatory networks (triangular oscillatory network).

4. Conclusions

In this study, we investigated the occurrence of AD observed from coupled oscillatory networks in three-dimensional space. Two types of network models are proposed. The first models is composed of a triangular network and the second model is composed of a pentagonal network. By using computer simulations, we observed similar AD phenomena in the coupled oscillatory network in three-dimensional space with two-dimensional space. In the case of a system model composed of a triangular net-

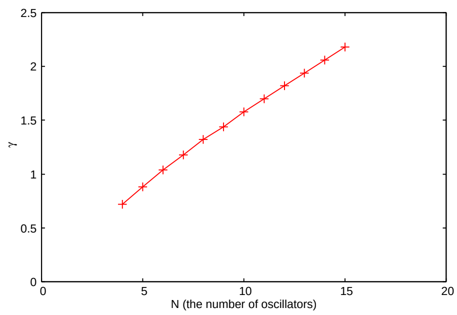
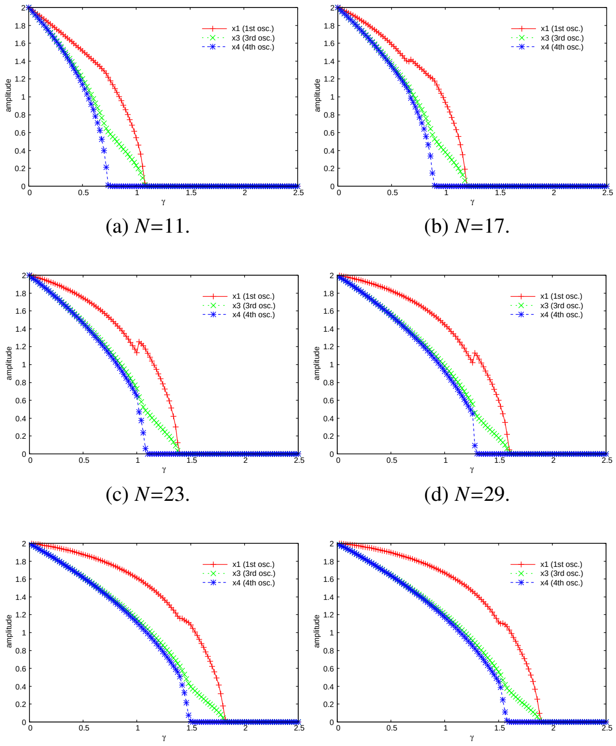


Figure 6: Coupling strength of AD occurrence (triangular oscillatory network).



(a) N=11. (b) N=17.

(c) N=23. (d) N=29.

(e) N=35. (f) N=41.

Figure 7: Amplitude change in three-dimensional coupled oscillatory networks (pentagonal oscillatory network).

work, a global AD is observed, and in the case of a system model composed of a pentagonal network, a partial AD is observed.

For the future works, we would like to make clear the mechanism of AD using theoretical analysis. Further investigation of AD for larger and more complex polygonal networks is one of the future issues.

References

[1] S. Boccaletti, J. Kurths, G. Osipov, D. Valladares and C. Zhou, "The Synchronization of Chaotic Systems" *Physics Reports*, 366, pp. 1-101, 2002.

[2] A. Arenas, A. Diaz-Guilera, J. Kurths, Y. Moreno and C. Zhou, "Phase Synchronization of Chaotic Oscillators" *Physics Reports*, 469, pp. 93-153, 2008.

[3] M. G. Rosenblum, A.S. Pikovsky and J. Kurths, "Phase Synchronization of Chaotic Oscillators" *Physical Review Letters*, vol.76, no.11, pp.1804-1807, Mar. 1996.

[4] W. Wang, I.Z. Kiss and J.L. Hudson, "Experiments on Arrays of Globally Coupled Chaotic Electrochemical Oscillators: Synchronization and Clustering" *Chaos*, vol.10, no.1, pp.248-256, Mar. 2000.

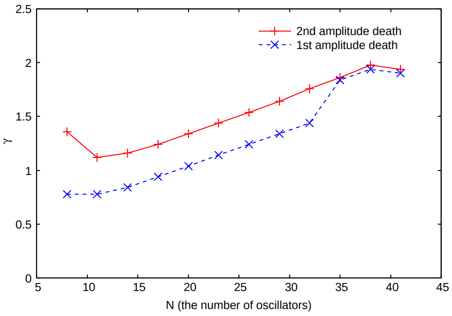


Figure 8: Coupling strength of AD occurrence (pentagonal oscillatory network).

[5] M. Yamauchi, Y. Nishio and A. Ushida, "Phase-waves in a Ladder of Oscillators" *IEICE Trans. Fundamentals*, vol.E86-A, no.4, pp.891-899, Apr. 2003.

[6] H.B. Fotsina and J. Daafouza, "Adaptive Synchronization of Uncertain Chaotic Colpitts Oscillators based on Parameter Identification" *Physics Letters A*, vol.339, pp.304-315, May 2005.

[7] G. Saxena, A. Prasad and R. Ramaswamy, "Amplitude Death: The Emergence of Stationarity in Coupled Nonlinear Systems," *Physics Reports*, 521, pp. 205-228, 2012.

[8] A. Koseska, E. Volkov and J. Kurths, "Oscillation Quenching Mechanisms: Amplitude vs. Oscillation Death," *Physics Reports*, 531, pp. 173-199, 2013.

[9] Y. Setou, Y. Nishio and A. Ushida, "Synchronization Phenomena in Many Oscillators Coupled by Resistors as a Ring," *Proc. of APCCAS'94*, pp. 570-575, Dec. 1994.

[10] Y. Uwate and Y. Nishio, "Synchronization in Several Types of Coupled Polygonal Oscillatory Networks," *IEEE Trans. Circuits Syst. I*, vol. 59, no. 5, pp. 1042-1050, May 2012.

[11] Y. Uwate and Y. Nishio, "Frustrated Synchronization in Two Coupled Polygonal Oscillatory Networks," *Proc. of ISCAS'13*, pp. 1781-1784, May 2013.

[12] Y. Uwate and Y. Nishio, "Amplitude Death in Strongly Coupled Polygonal Oscillatory Networks with Sharing Branch," *IEEE TNSE*, DOI: 10.1109/TNSE.2018.2801465, 2018.