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Clustering Phenomena in Network of Coupled Chaotic Circuits Distributed in 3-Dimensional Space

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Abstract—In this study, we investigate synchronization phenomena in coupled chaotic circuits which are connected by resistance. In addition, we investigate the difference of synchronization phenomena by distance information of coupled chaotic circuits and the difference of synchronization phenomena by increasing the number of coupled chaotic circuits. We confirm that the coupled chaotic circuits located in the near distance are synchronized at in-phase state, and the coupled chaotic circuits located in the far distance are not synchronized. Therefore, the clustering phenomena of coupled chaotic circuits are observed in two-dimensional space and three-dimensional space.

1. Introduction

Synchronization phenomena are the most familiar phenomena that exist in nature and they have been studied in various fields. Synchronization phenomena can be observed everywhere in our life. For example, we can confirm flashing firefly lights, metronome, gate patterns of four-leg animals, beating rhythm of the heart and so on. Especially, synchronization phenomena of oscillatory network are interesting. In addition, complex networks attract attention from various fields. The feature of networks is the degree distribution, the path length and the clustering coefficient. Therefore, we focus on the clustering phenomena in this research.

In addition, clustering phenomena are one of interesting nonlinear phenomena observed from coupled chaotic circuits. The clustering phenomena is to divide the set to be classified into subsets. Previously, many of the studies for clustering have been carried out for discrete time model, for example Coupled Map Lattics (CML) and Self Organization Map (SOM) [1]-[2] and so on. However, analysis of using a continuous time model has not almost studied. Therefore, we focus on research on clustering phenomena using real electronic circuits in continuous time model.

On the other hand, coupled chaotic circuits that are real electronic circuits can be observed various amusing phenomena. In recent years, many methods are studied to apply to clustering and synchronization phenomena observed in coupled chaotic circuits for natural sciences. At the same time, synchronization phenomena and clustering have been studied associated with the chaos phenomena [3]-[4].

In this study, we focus on the clustering phenomena in the network of coupled chaotic circuits. For this investigation, the coupling strength reflected the distance information when the chaotic circuits are placed in two-dimensional and three-dimensional space. Moreover, we investigate from two clusters. From there, we increase the number of clusters. Finally, we consider the case of complex networks to extend to large scale networks and apply to the real world data for clustering.

2. Circuit Model

Figure 1 shows the circuit model which is called Shinriki-Mori circuit. This circuit consists of a negative resistor, an inductor, two capacitors and dual-directional diodes.

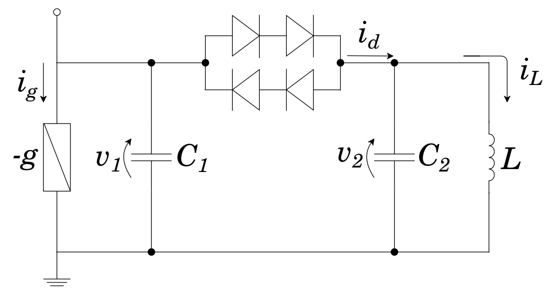

Figure 1: Circuit model.

The circuit equation of this circuit are given as follows:

$$\begin{cases} L\dfrac{di_L}{dt} = v_2 \\ C_1\dfrac{dv_1}{dt} = gv_1 - i_{dn} \\ C_2\dfrac{dv_2}{dt} = i_{dn} - i_L. \end{cases} \tag{1}$$

The nonlinear function i_{dn} corresponds to the $i - v$ characteristics of the nonlinear resistors consisting of the diodes and are given as follows:

$$i_{dn} = \begin{cases} G_d(v_1 - v_2 - V) & (v_1 - v_2 > V) \\ 0 & (|v_1 - v_2| \leq V) \\ G_d(v_1 - v_2 + V) & (v_1 - v_2 < V). \end{cases} \tag{2}$$

By changing the variables and parameters such that

$$
\begin{gathered}
i_L = \sqrt{\frac{C_2}{L}} V x, \;\; v_1 = V y, \;\; v_2 = V z \\
\alpha = \frac{C_2}{C_1}, \;\; \beta = G_d \sqrt{\frac{L}{C_2}}, \;\; \gamma = g \sqrt{\frac{L}{C_2}} \\
t = \sqrt{L C_2}\tau, \;\; \text{"}\cdot\text{"} = \frac{d}{d\tau}
\end{gathered}
$$

The normalized equations of chaos circuit are given as follows:

$$
\begin{cases}
\dot{x} = z \\
\dot{y} = \alpha\gamma y - \alpha\beta f \\
\dot{z} = \beta f - x.
\end{cases} \tag{3}
$$

The nonlinear function f corresponds to the characterisitics of the nonlinear resistor consisting of the diodes and described as follows:

$$
f = \begin{cases}
y - z - 1 & (y - z > 1) \\
0 & (|y - z| \leq 1) \\
y - z + 1 & (y - z < 1).
\end{cases} \tag{4}
$$

For the computer simulation, we set the parameters as $\alpha = 0.50, \beta = 20.00$ and $\gamma = 0.50$.

3. Simulation Result

3.1. Network of seven chaotic circuits in two-dimensional space

First, we investigate the synchronization phenomena and clustering phenomena when seven chaotic circuits are coupled in two-dimensional space. The location of seven chaotic circuits is shown in Fig. 2 and Tab. 1.

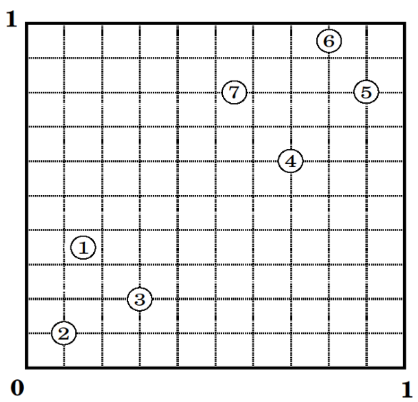

Figure 2: Location of seven chaotic circuits in two-dimensional space.

Table 1: The location of chaotic circuits in the two-dimensional space.

location	x	y
1	0.15	0.35
2	0.10	0.10
3	0.30	0.20
4	0.70	0.60
5	0.90	0.80
6	0.80	0.95
7	0.55	0.80

All circuits connected each other by resistors. Figure 3 shows coupling method of the first chaotic circuit as an example.

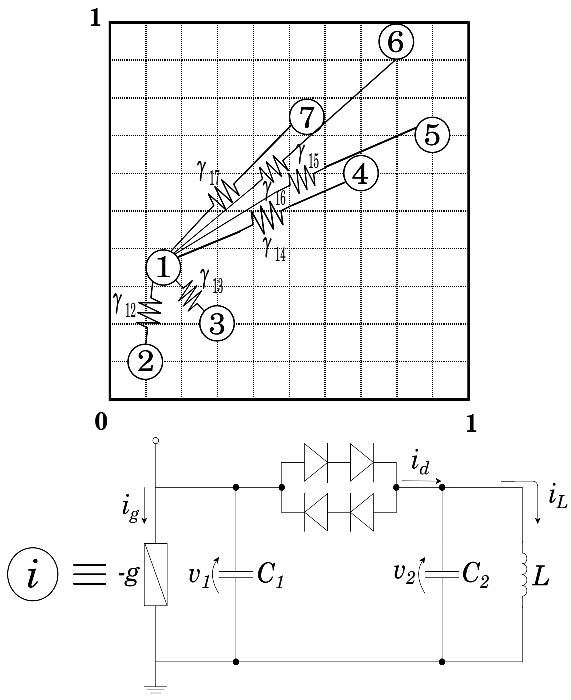

Figure 3: Coupling between the first chaotic circuit and others.

We consider the coupled chaotic circuits:

$$
\begin{cases}
\dfrac{dx_i}{d\tau} = z_i \\
\dfrac{dy_i}{d\tau} = \alpha\gamma y_i - \alpha\beta f - \alpha \displaystyle\sum_{i,j=1}^{N} r_{i,j}(y_i - y_j) \\
\dfrac{dz_i}{d\tau} = \beta f - x_i.
\end{cases} \tag{5}
$$

The nonlinear function f corresponds to the i-v characteristics of the nonlinear resistors consisting of the diodes and are given as follows:

$$f = \begin{cases} y_i - z_i - 1 & (y_i - z_i > 1) \\ 0 & (|y_i - z_i| \leq 1) \\ y_i - z_i + 1 & (y_i - z_i < 1). \end{cases} \tag{6}$$

where, i in the equation represents the circuit itself, and j is the coupling with other circuits. The parameter r represents the coupling strength between the circuits. In this simulation, we set the coupling parameter value $r_{i,j}$ to correspond the distance between the circuits by the following equation:

$$r_{i,j} = \frac{q}{(d_{i,j})^2}. \tag{7}$$

$d_{i,j}$ represents the Euclidean distance between the $i-th$ and the $j-th$ circuits. Further, the parameter q is the weight parameter that determines the coupling strengths. In this case, we set parameter $q = 0.01$.

Figure 4 shows the computer simulation results obtained from the seven chaotic circuits located as shown in Fig. 2. From these results, we confirm that the first, second and third chaotic circuits are synchronized at in-phase state, and also the fourth, fifth, sixth and seventh chaotic circuits are synchronized at in-phase state. However, the first and the fourth chaotic circuits are not synchronized. From these results, the circuits can form two clusters defined by chaotic synchronization as shown in Fig. 5.

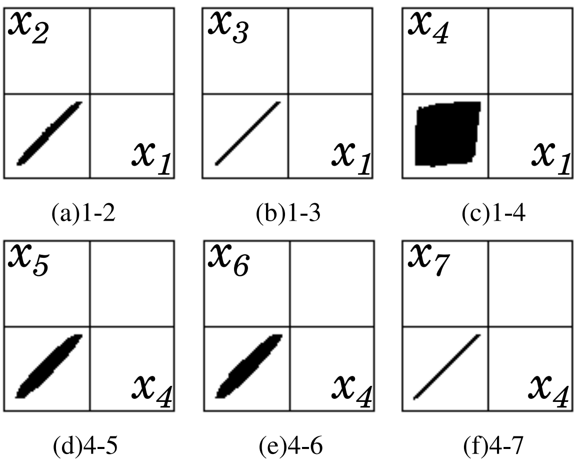

Figure 4: Phase difference between seven circuits in two-dimensional space.

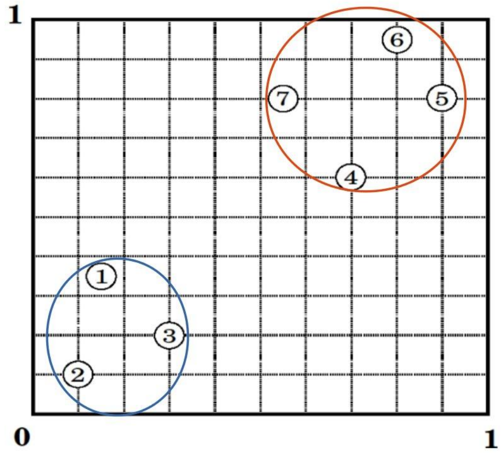

Figure 5: The clustering result of seven chaotic circuits.

3.2. Network of thirty chaotic circuits in three-dimensional space

Next, we investigate the case of three-dimensional networks. Thirty chaotic circuits are located in three-dimensional, including the positional information. The location of thirty chaotic circuits is shown in Fig. 6. Also, Table 2 shows the location of the chaotic circuits. Similarly, all the chaotic circuits are connected each other by resistors, and the coupling strength between each circuits is determined by Eq. (7). In this case, we put the parameter $q = 0.004724$.

Table 2: The location of chaotic circuits in the three-dimensional space.

location	x	y	z	location	x	y	z
1	0.15	0.05	0.15	16	0.80	0.20	0.15
2	0.20	0.25	0.30	17	0.85	0.15	0.05
3	0.35	0.35	0.25	18	0.70	0.60	0.95
4	0.25	0.25	0.05	19	0.90	0.80	0.85
5	0.30	0.15	0.05	20	0.80	0.95	0.75
6	0.05	0.20	0.10	21	0.75	0.85	0.70
7	0.15	0.30	0.15	22	0.85	0.80	0.85
8	0.05	0.05	0.25	23	0.60	0.60	0.60
9	0.20	0.05	0.35	24	0.80	0.65	0.90
10	0.25	0.10	0.20	25	0.65	0.80	0.80
11	0.35	0.05	0.25	26	0.65	0.65	0.85
12	0.25	0.05	0.35	27	0.95	0.95	0.95
13	0.75	0.15	0.25	28	0.90	0.65	0.75
14	0.80	0.25	0.10	29	0.70	0.85	0.65
15	0.95	0.30	0.35	30	0.75	0.80	0.85

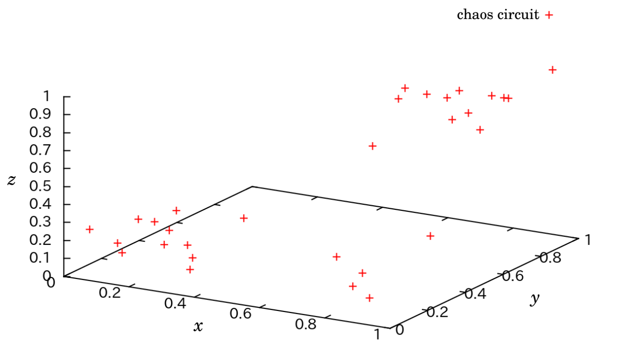

Figure 6: Location of thirty chaotic circuits in three-dimensional space.

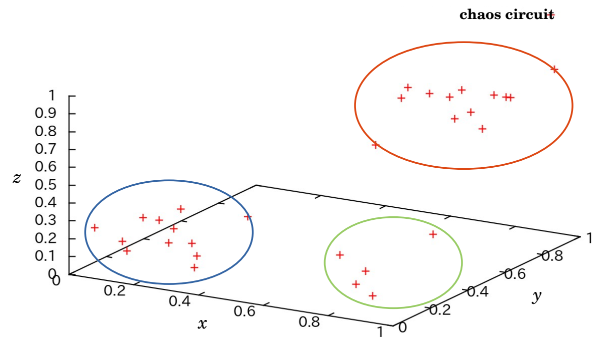

Figure 8: The clustering result of thirty chaotic circuits.

Figure 7 shows the computer simulation results obtained from the thirty chaotic circuits located as shown in Fig. 6. From these results, we confirm that the first and the second circuits are synchronized at in-phase state. However, the group of first chaotic circuit and the group of thirteenth chaotic circuit are not synchronized. Also the group of first chaotic circuit and the group of eighteenth chaotic circuit are not synchronized. Similarly, between the group of thirteenth and fourteenth chaotic circuit are synchronized, between the group of eighteenth and nineteenth chaotic circuit are synchronized. However, between the group of thirteenth and eighteenth chaotic circuit are not synchronized. From these results, the circuits can form three clusters defined by chaotic synchronization as shown in Fig. 8.

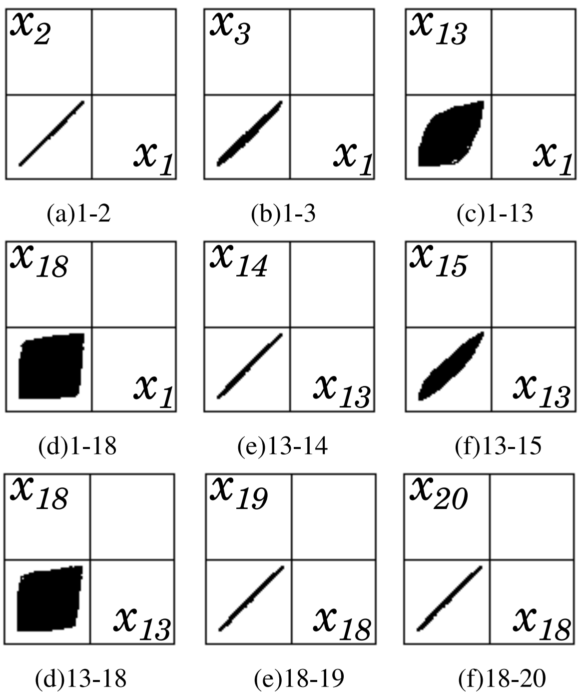

Figure 7: Phase difference between thirty circuits in three-dimensional space.

4. Conclusion

In this study, we investigated synchronization phenomena when the chaotic circuits are located in two-dimensional and three-dimensional space. Synchronization phenomena were seen between circuits at near distance, and synchronization phenomena could not be seen between circuits at far distance. With these results, it was confirmed that the chaotic circuits were different from synchronization phenomena by distance information and the clustering phenomena were observed.

In the future works, we would like to increase the number of chaotic circuits and the cluster. Moreover, we consider that we would like to change of the dimensional space.

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