Nonlinear Time Series Analysis of Coupled Bursting Neuron Model Depending on Coupling Strength

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Abstract— **It is well known that burst patterns within neurons may have some important role for operating information processing in a brain. However, the analysis of judgement of synchronization and correlation between burst patterns is not so easy. Therefore, we have proposed a visualization method of bursting patterns of whole neural network using nonlinear time series analysis. Additionally, we consider that it is required to construct a model using mathematical neuron model producing burst patterns, because it is difficult to obtain real biological neuron data. In this study, we propose a visualization method of network characteristics of FitzHu-Nagumo (FHN) neuron model using nonlinear time-series analysis.**

Keywords; nonlinear time-seris analysis, attractor reconstruction, coupled bursint neuron model

I. INTRODUCTION

Burst patterns within neurons may have some important role for operating information processing in a brain. Therefore, to develop detecting and analyze burst pattern are investigated in various fields [1]. Although it is important to study burst pattern in order to make clear correlation and communication process between neurons, unveiling a structure of the whole neuronal network is also required. While, nonlinear time-series analysis is a useful tool for characterizing about the dynamics behind observed time-series data [2]. The neuronal data obtained from living neurons should be high-dimensional and dynamic data. Then nonlinear time-series analysis matches well to characterize the neuronal data. Therefore, we have proposed a visualization method of bursting patterns of whole neural network using nonlinear time series analysis [3].

However, it is difficult obtain amount of real biological neuron data. We consider that it is required to construct a model using mathematical neuron model producing burst patterns. In this study, we propose a visualization method of network characteristics of burst patterns obtained from FitzHu-Nagumo (FHN) neuron model using nonlinear time series analysis. As a first step, we build up the system model which is that ten neurons are coupled randomly. In order to mimic a real system, the small mismatch of oscillation frequency is added to all neurons. We calculate the spike timing of the bursting patterns and obtain the raster plot using the threshold. Next, we calculate the spike rate at a certain range, then time-series data is obtained. The attractor reconstruction of nonlinear time-series is applied to sum of time-series data. From the simulation results, we confirm that the attractor has different shape depending on the coupling strength.

II. COUPLED BURSING NEURON MODEL

The model of gap junction coupled FHN neurons [4], [5] is described by the following equation.

$$\begin{cases} \dfrac{dx_i}{dt} = x_i(x_i-1)(1-rx_i) - y_i - g\displaystyle\sum_{k\in S_n}(x_i - x_k) + I_o \\ \dfrac{dy_i}{dt} = bx_i \end{cases} \qquad (1)$$

where x, y are sate variables, provided that orbit is close enough to the basin of attraction, g denotes the coupling strength of gap junction, and Io is the external stimulation expressed by cosine function. Sn is set of nodes which are connected to i-th neurons. In the coupled neuron system, small frequency error is added to every neuron.

Figure 1 shows the bursting patterns of all neurons when the number of neurons is set to N=10. The coupling strength is fixed with g=0.1.

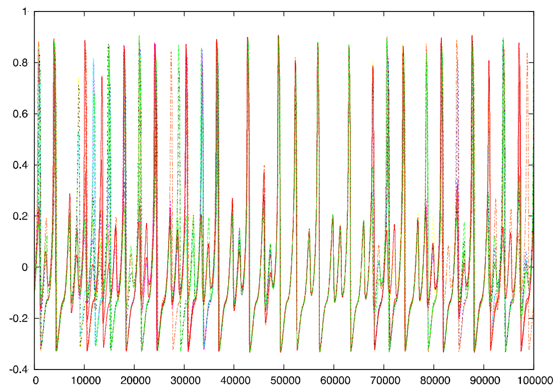

Fig. 1. Bursting patterns of all neurons (N=10).

III. SIMULATION RESULTS

A. Raster Plot

Figure 2 shows the raster plot obtained from all neurons, by checking the peak points of the bursting spike data. However, it is difficult to understand the network characteristics from this figure. Next, we calculate the spike rate at a certain range, then time-series data is obtained as shown in Fig. 3. From this figure, we can see that the time series has periodicity.

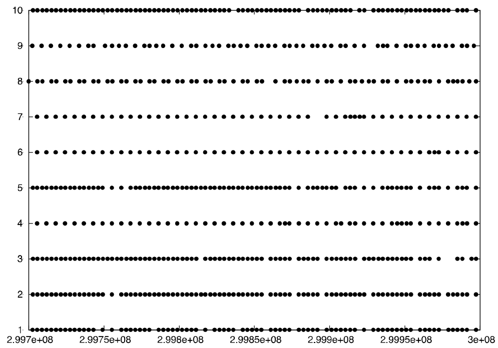

Fig. 2. Raster plot of coupled 10 neurons.

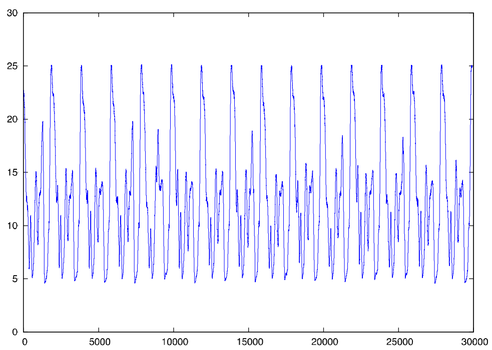

Fig. 3. Time-series data.

B. Attractor Reconstruction

The attractor of dynamical systems can be reconstructed topologically in the embedding space from Takens' theorem [6]. The state vectors in the reconstructed m-dimensional embedding space are defined by

$$y(t) = \{x(t), x(t+\tau), , x(t+(m-1)\tau)\}$$

where x(t) means a scalar time series and tau is the delay time.

Figure 4 shows the simulation results when neuronal time-series data (Fig. 3) is embedded in 3-dimensional space by setting the delay time tau=30. From the figure, we confirm that the neuron has some sort of structure because the orbit draws in certain range, does not move about randomly.

At my presentation, the other attractor reconstruction results are shown, when the coupling strength in the network are changed.

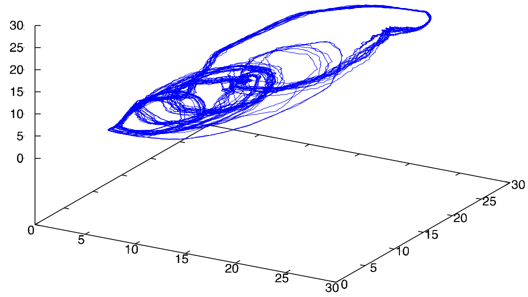

Fig. 4. Simulation result of attractor reconstruction.

IV. CONCLUSIONS

In this study, we have proposed a visualization method of network characteristics of neuron data using mathematical model. By calculating the computer simulations, we obtained the attractors in 3-dimensioanl space. From observed results, we can see that the proposed system has possibility to show the network characteristics of biological neuron data.

In the future works, we would like to use larger networks, and different types of network structures. Applying the proposed system to real biological neuron data is also our future work.

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