Amplitude Death in Coupled Oscillatory Systems Inspired by Brain Networks with Different Frequency

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Abstract— **In this study, we investigate amplitude change and amplitude death in coupled van der Pol oscillators with different oscillation frequency. The network topology is inspired by the concept of a real brain network. We observe the amplitude death of all oscillators when the coupling strength reaches to the certain value. We also confirm that the characteristics of amplitude death depends on the position of oscillator with different oscillation frequency.**

I. Introduction

The synchronization phenomena observed from coupled oscillators are suitable model to analyze the natural phenomena [1], [2]. Therefore, many researchers have proposed different coupled oscillatory networks and have discovered many interesting synchronization phenomena [3]-[6].

In our research group, we have focused on synchronization phenomena observed from nonlinear oscillatory networks. This is because the results of synchronization in complex networks are useful a deeper understanding of control methods in power networks, communication systems and so on. Also, they can be used as an alternative approach, apart from existing ones, for describing mode-locking phenomena in biological networks. Because we would like to propose a modeling of synchronization in brain by using coupled electrical oscillatory circuits, in order to make clear the mechanism of functional operation in the brain.

On the other hand, amplitude death (AD) occurs in strongly coupled nonlinear oscillators when their interaction causes a pair of fixed points to become stable and attracting. Examples of AD include coupled laser systems in which a constant output is required and fluctuations should be regulated, and the suppression of pathological rhythms in an ensemble of coupled neurons related to some specific neuronal disorders [7], [8]. These different requirements suggest that the study of an AD is important for understanding the control mechanisms and efficient regulators of a system's dynamics.

Recently, the relationship between structural and functional network in biological neural network has attracted their attention from many researchers [9]. Hartelt et al. have discovered the network topology in the pre-BotC which consists of densely connected clusters with rare inter-cluster links. And, the hub of the network behave quiescently output [10], [11]. We apply this phenomenon to the coupled oscillatory systems using electrical oscillator. Namely, one node is set to lower frequency than the others. We also investigate the influence of the location of the oscillator with different frequencies. By using computer simulations, we observe amplitude death by setting the approximate different oscillation frequency and the coupling strength. We confirm that the position of the oscillator with the different frequency is important for producing clustering patterns.

II. Network Model

A network model composed of 13 nodes and 22 edges is shown in Fig. 1. There are two important hubs in this network, "Connector hub" and "Provincial hub". Both hubs are high-degree nodes. "Connector hub" shows a diverse connectivity by connecting two sub-networks. "Provincial hub" primarily connects nodes in the same sub-network.

In this study, the node is expressed by van der Pol oscillator as shown in Fig. 2(a). The oscillators are coupled by a resistor (see Fig. 2(b)). Namely, two coupled oscillators tend to synchronize with an in-phase state. In the system, only one oscillator has different oscillation frequency. Only one oscillator is set to lower frequency than the others. The position of the oscillator with the different frequency is defined as P_{odf}. We investigate the influence of P_{odf} for the amplitude death.

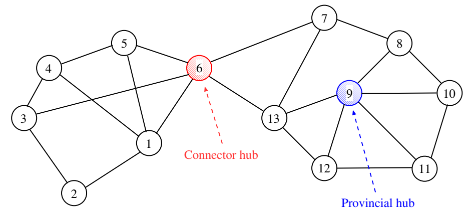

Fig. 1. Network model.

Next, we develop the expression for the circuit equations of the network model. The $v_k - i_{Rk}$ characteristics of the

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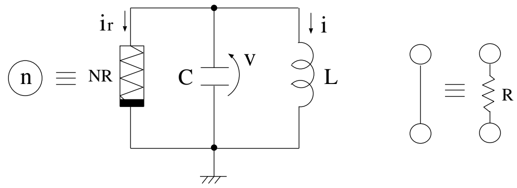


(a) van der Pol oscillator. (b) Coupling method.

Fig. 2. Circuit model.

nonlinear resistor are approximated by the following third order polynomial equation,

$$i_{Rk} = -g_1 v_k + g_3 {v_k}^3 \quad (g_1, g_3 > 0), \qquad (1)$$
$$(k = 1, 2, ...13).$$

The normalized circuit equations governing the circuit are expressed as
[kth oscillator]

$$\begin{cases} \dfrac{dx_k}{d\tau} = \varepsilon\left(1 - \dfrac{1}{3}{x_k}^2\right)x_k - y_k) - \gamma \displaystyle\sum_{n \in S_k}(y_k - y_n) \\ \dfrac{dy_k}{d\tau} = x_k \\ \qquad\qquad\qquad\qquad (k = 1, 2, ...13). \end{cases} \qquad (2)$$

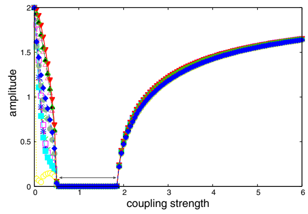
The normalized circuit equations of the oscillator with different frequency are expressed as
[Oscillator with different frequency]

$$\begin{cases} \dfrac{dx_k}{d\tau} = \omega^2\varepsilon\left(1 - \dfrac{1}{3}{x_k}^2\right)x_k - y_k) - \omega^2\gamma \displaystyle\sum_{n \in S_k}(y_k - y_n) \\ \dfrac{dy_k}{d\tau} = x_k \\ \qquad\qquad\qquad\qquad (k = 1, 2, ...13). \end{cases} \qquad (3)$$

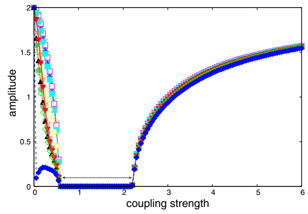
In these equations, γ is the coupling strength, ε denotes the nonlinearity of the oscillators and y_n denotes the current of neighbor oscillator on the coupling resistor. ω denotes the different oscillation frequency. For the computer simulations, we calculate Eqs. (2) and (3) using the fourth-order Runge-Kutta method with the step size $h = 0.005$. The parameters of this circuit model are fixed as $\varepsilon = 0.1$.

III. Simulation Results

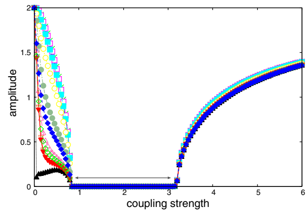
First, we investigate the amplitude change of all oscillators in the proposed network when the coupling strength is changed. We confirm that AD occurs when the different oscillation frequency is smaller than 0.5. In the simulations, the different oscillation frequency is fixed with ω=0.47. The simulation results which are characteristics cases are shown in Fig. 3. In all cases, we observe the AD at a certain range of the coupling strength. The occurrence timing and range of AD are the difference which are depending on the position of oscillator with different frequency (P_{odf}).



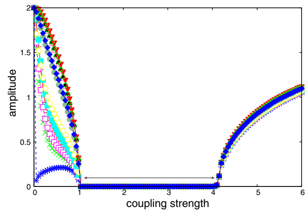
(a) P_{odf}: Oscillator 6.



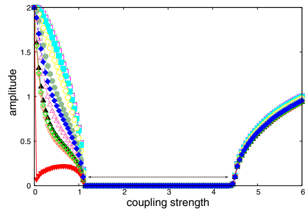
(b) P_{odf}: Oscillator 13.



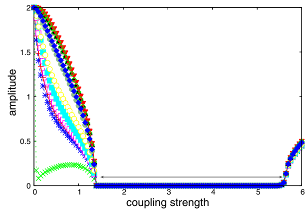
(c) P_{odf}: Oscillator 9.



(d) P_{odf}: Oscillator 3.



(e) P_{odf}: Oscillator 11.



(f) P_{odf}: Oscillator 2.

Fig. 3. Amplitude (ω=0.47).

If the different frequency is set to the connector and provincial hubs, AD occurs with small coupling strength and the range of AD is short as shown in Fig. 3 (a)-(c). While, if the different frequency is set to other nodes which have low connections, AD occurs with large coupling strength and the range of AD is wide as shown in Fig. 3 (d)-(f).

The occurrence timing and range of AD are summarized in Fig 4. In the case of when the oscillators 7 and 13 are added different frequency, AD occurs with small coupling strength and the range of AD is short. These two oscillators are connected with both connector and provincial hubs. Namely, we assume that the oscillators 7 and 13 are also important nodes in the network. Figure 5 shows the category of oscillators depending on the range of AD.

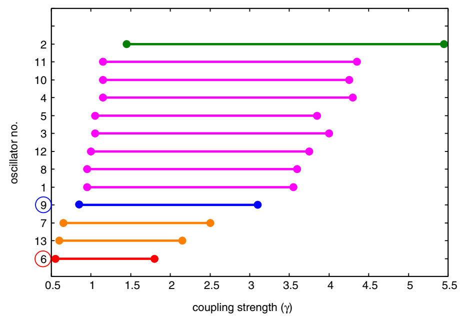


Fig. 4. Range of AD.

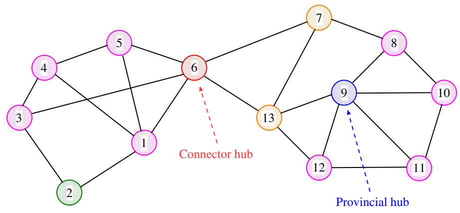
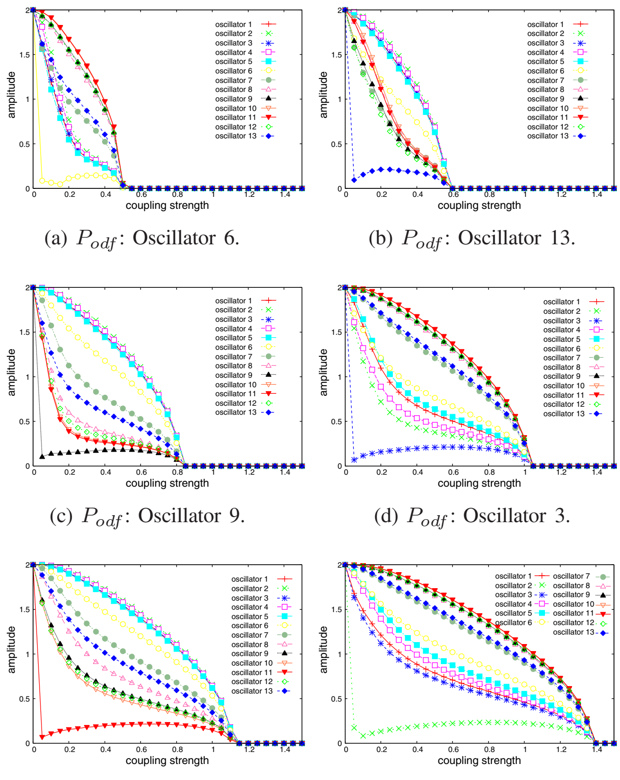


Fig. 5. Category of oscillators depending on the range of AD.

Next, we focus on how to decrease the amplitude in more detail. Figure 6 shows the enlarged figure of Fig. 3, when the coupling strength is the small value. In the case of that the different frequency is added to oscillator 6 (connector hub) and 13, there are several groups of curves of amplitude (Fig. 6 (a), (b)). However, in other cases, each curve does not overlap (Fig. 6 (c)-(f)). We consider that clustering phenomena can occur when the different frequency is added to the important nodes such as the connector hub.



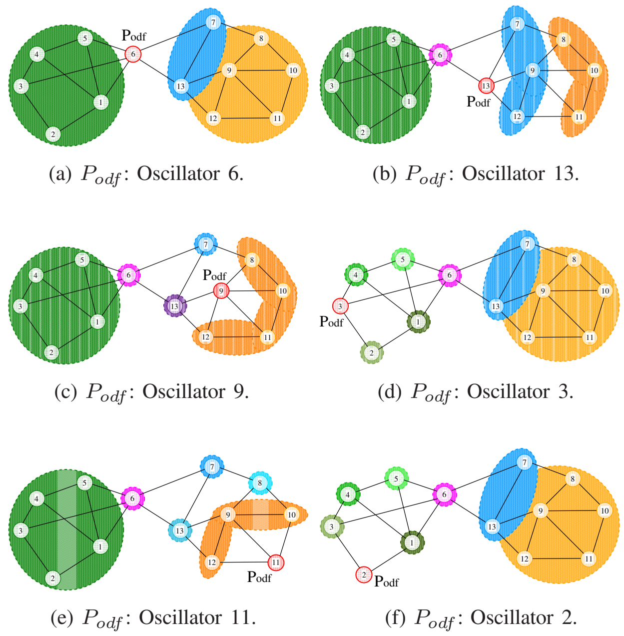
(a) P_{odf}: Oscillator 6. (b) P_{odf}: Oscillator 13.

(c) P_{odf}: Oscillator 9. (d) P_{odf}: Oscillator 3.

(e) P_{odf}: Oscillator 11. (f) P_{odf}: Oscillator 2.

Fig. 6. Amplitude (ω=0.47).

Figure 7 shows the clustering patterns depending on the amplitude change. We observe several types of clustering patterns. In the same circle, the oscillators are synchronized with perfect synchronization. The synchronization with certain phase difference can be observed between the oscillators located in a different circle. When the position of the different frequency is set to the connector hub, there are three clustering. While, in the case of that P_{odf} is set to oscillator 2, there are seven clusters. Namely, by adding different frequency to the important node, the network is divided into several groups and all nodes collaborate to synchronize within the group.



(a) P_{odf}: Oscillator 6. (b) P_{odf}: Oscillator 13.

(c) P_{odf}: Oscillator 9. (d) P_{odf}: Oscillator 3.

(e) P_{odf}: Oscillator 11. (f) P_{odf}: Oscillator 2.

Fig. 7. Clustering phenomena.

IV. Extension to Large Network

In this section, we investigate amplitude changes and AD in more large network. Figure 8 shows the second network model which is composed of 25 oscillators and 41 edges. There are four connector hubs and four modules.

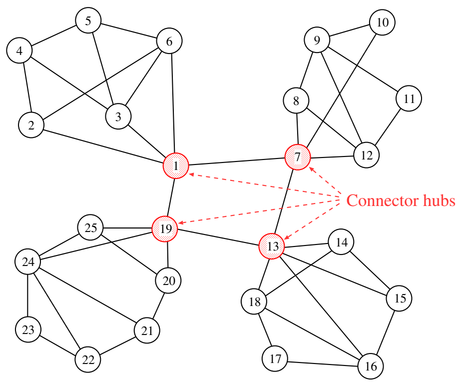
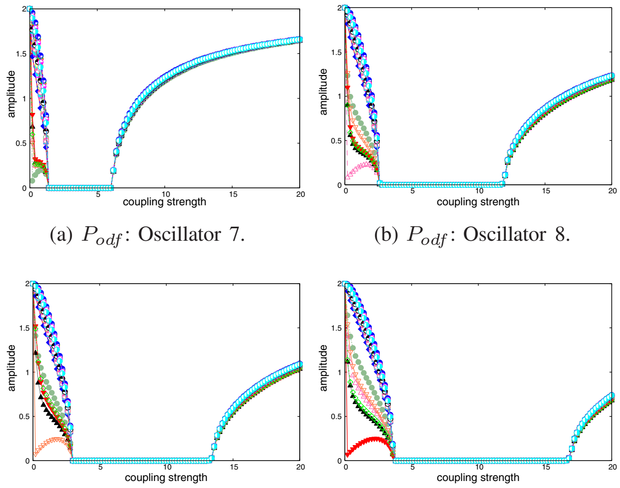


Fig. 8. Network model (N=25).

We confirm that AD occurs when the different oscillation frequency ω is smaller than 0.4. In the simulations, the different oscillation frequency is fixed with ω=0.35. We also investigate the amplitude change of all oscillators in the proposed network when the coupling strength is changed. The simulation results which are characteristics cases are shown in Fig. 9.

We obtain a similar result with the previous network model. In all cases, we observe the AD at a certain range of the coupling strength. The occurrence timing and range of AD are the difference which are depending on the position of oscillator with different frequency (P_{odf}). If the different frequency is set to the connector and provincial hubs, AD occurs with small coupling strength and the range of AD is short as shown in Fig. 9 (a). While, if the different frequency is set to other nodes which have low connections, AD occurs with large coupling strength and the range of AD is wide as shown in Fig. 9 (b)-(d).



(a) P_{odf}: Oscillator 7. (b) P_{odf}: Oscillator 8.

(c) P_{odf}: Oscillator 10. (d) P_{odf}: Oscillator 11.

Fig. 9. Amplitude for N=25 (ω=0.35).

The occurrence timing and range of AD are summarized in Fig 10. In the case of when the connector hubs are added different frequency, AD occurs with small coupling strength and the range of AD is short.

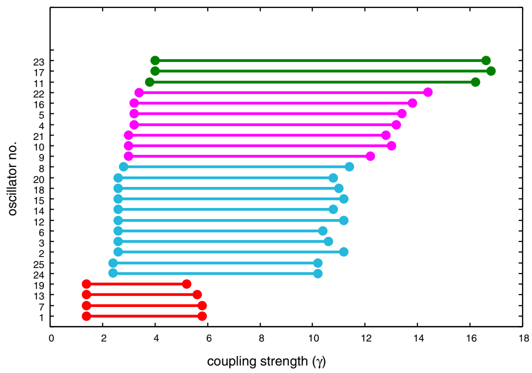


Fig. 10. Range of AD for N=25.

Next, we focus on how to decrease the amplitude in more detail. Figure 10 shows the enlarged figure of Fig. 9, when the coupling strength is the small value. In the case of that the different frequency is added to oscillator 7 (connector hub), there are several groups of curves of amplitude.

Figure 11 shows the clustering patterns depending on the amplitude change. We consider that clustering phenomena can occur when the different frequency is added to the important nodes such as the connector hub.

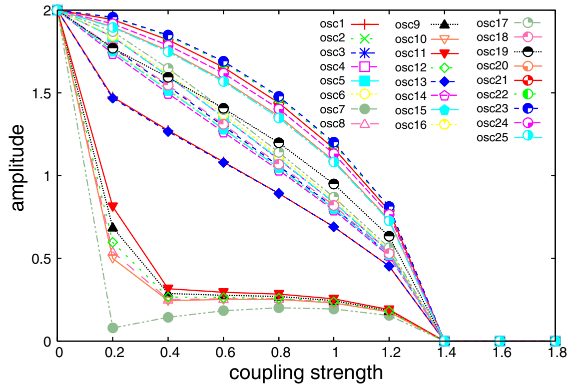


Fig. 11. Amplitude for N=25.

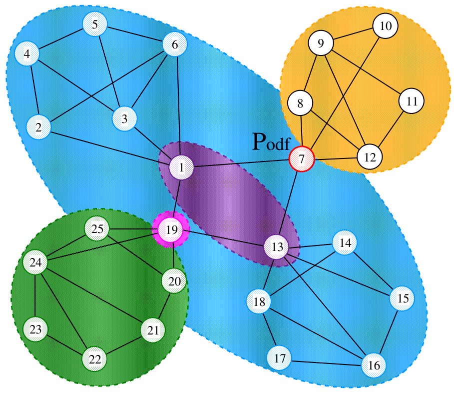


Fig. 12. Clustering phenomena for N=25.

V. Conclusion

In this study, we have investigated amplitude change and AD in coupled van der Pol oscillators with different frequencies. The network topology is inspired by the concept of the real brain network. We observed amplitude death of all oscillators when the coupling strength reaches to the certain value. We confirm that the position of the oscillator with the different frequency is important for producing clustering patterns.

In future work, we intend to investigate whether such AD occurs in other network patterns. We would also like to apply the proposed circuits system to larger-scale networks to model complex biological networks.

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