# Synchronization in Ladder-Coupled Chaotic Circuits Including Ring Structures

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*Abstract*—**In this study, we investigate synchronization phenomena in ladder-coupled chaotic circuits including ring structures. We focus on the effect of the position of the ring structures in the network to the synchronization of the whole network. In our proposed network model, chaotic circuits are coupled by resistors. We set the parameters of the circuits to generate periodic solutions or chaotic solutions. Computer simulations confirm that the various synchronization phenomena occur in different situations.**

## I. Introduction

Synchronization in a network is one of the interesting topics in scientific disciplines. Investigation of this is important work in order to clarify nonlinear phenomena in the natural world. It is observed over the various fields, for example, engineering, biology, sociology and so on [1]-[3]. Incidentally, the networks have different topology and characters in each even if they are similar type [4]-[6]. Therefore, the investigation of dynamics in a different network is significant and research has proceeded to analyze the dynamics of synchronization on each topology.

On the other hand, synchronization of a coupled chaotic system is interesting models to describe various higher-dimensional nonlinear phenomena [7], [8]. Recently, studying chaotic phenomena by using coupled chaotic circuits increases. On the chaotic circuit with simple elements, circuit experiment and computer simulation are suitable for studying in terms of that experiment time is short and repeatability of experiments is high. Networks by using coupled chaotic circuits can be modeled natural world or social networks. Moreover, it is important for future engineering to investigate nonlinear phenomena like chaotic synchronization. As these reasons, a chaotic-generate circuit is counted on application, for instance, chaotic communication systems, Internet of Things, modeling neurons and so on.

In modern society, the agenda of vegetation exists about environment or food. The study of plant bioelectric potential has attracted rising attention recently from the viewpoint of plant growth [9]. Bioelectric potential is known that it changes depending on the surrounding environment. Therefore, we consider the bioelectric potential as the voltage phase difference of chaotic circuits and we model the plants by chaotic circuits.

In the previous study, we have investigated synchronization phenomena in our proposed chaotic model with hybrid topology by ladder and rings modeling sapling of the plant [10]. In this study, we increase the types of network and investigate the synchronization. In our proposed network model, chaotic circuits are coupled by resistors. These chaotic circuits are little difference of each other by the bifurcation parameter. We set the parameters of the circuits to generate periodic solutions or chaotic solutions. The number of rings is changed to each model. And we observe the synchronization phenomena in different network models.

## II. Network model

In this study, we use chaotic circuits. The model of chaotic circuit is shown in Fig. 1. This circuit consists of a negative resistor, two inductors, a capacitor and dual-directional diodes. This chaotic circuit is called Nishio-Inaba circuit [11].

We propose ladder-coupled chaotic circuits including ring structures. In this case of Fig. 2, we set five chaotic circuits in ladder structure and one ring structure coupling to CC1 by resistors. Likewise, we propose three network models that coupled ring structure to CC1, CC2 or CC3. We set the parameters of the system as $\beta = 3.0$ and $\delta = 470.0$. Next, we change the condition in terms of the parameter $\alpha$. $\alpha$ denotes a degree of chaos. The network models are divided into two types by the value of $\alpha$. One pattern is to unify the value of all $\alpha$ (unified system) like shown in Fig. 2, and the other is to change the value of all $\alpha$ (ununified system) like shown in Fig. 3. In the case of Fig. 2, the value of $\alpha$ is unified 0.412. And in the case of Fig. 3, the value of $\alpha$ is not unified. The parameters $\alpha$ are set from 0.411 to 0.415 with step size 0.001 from CC1 to CC5. The $\alpha$ of CC6 and CC7 are added 0.001 from the $\alpha$ of coupled circuit.

Subsequently, we augment the number of rings in the case of one ring, two rings or three rings. Likewise in ununified systems, the $\alpha$ of CC6, CC7, CC8 and CC9 are added 0.001 from the $\alpha$ of coupled circuit. We show the models from Fig. 3 to Fig. 7. Figure 4 and 6 are unified systems with two and three rings and Fig. 5 and 7 are ununified systems with two and three rings.

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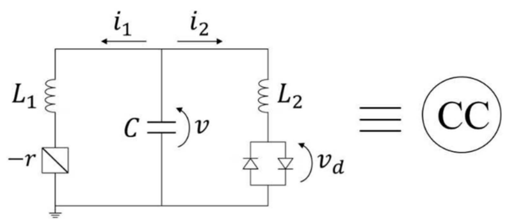



Fig. 1: Chaotic circuit.

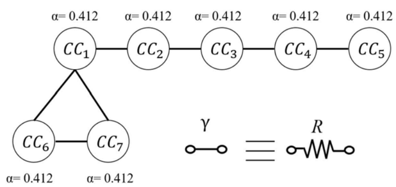



Fig. 2: Unified system with one ring.

By changing the variables and parameters,

$$\begin{cases} i_1 = \sqrt{\dfrac{C}{L_1}} V x_n, \quad i_2 = \dfrac{\sqrt{L_1 C}}{L_2} V y_n, \quad v = V z_n \\ \alpha = r\sqrt{\dfrac{C}{L_1}}, \quad \beta = \dfrac{L_1}{L_2}, \quad \delta = r_d \dfrac{\sqrt{L_1 C}}{L_2}, \\ \gamma = \dfrac{1}{R}, \quad t = \sqrt{L_1 C_2}\tau. \end{cases} \tag{1}$$

$R$ is the resistor and each chaotic circuit is coupled by the resistor. $R$ is defined as coupling strength for one of the bifurcation parameters.

The normalized circuit equations of the systems are given as follows:

$$\begin{cases} \dfrac{dx_i}{d\tau} = \alpha x_i + z_i \\ \dfrac{dy_i}{d\tau} = z_i - f(y) \\ \dfrac{dz_i}{d\tau} = -x_i - \beta y_i - \displaystyle\sum_{i,j=1}^{N} \gamma_{ij}(z_i - z_j) \\ \qquad\qquad\qquad\qquad (i, j = 1, 2, \cdots, N). \end{cases} \tag{2}$$

In Eq. (2), $N$ is the number of coupled chaotic circuits and $\gamma$ is the coupling strength as $\gamma = 0.008$ in this study. We set the parameters of the circuits to generate periodic solutions or chaotic solutions. When we set the parameter as $\alpha = 0.411$ to $\alpha = 0.415$, the circuits generate three-periodic solutions. And when the parameter is more than $\alpha = 0.416$, the circuits generate chaotic solutions.

## III. Simulation Results

We investigate synchronization phenomena in ladder-coupled chaotic circuits including ring structures. In this time, we use attractors and Lissajous figures to research synchronization. Lissajous figures show the phase difference

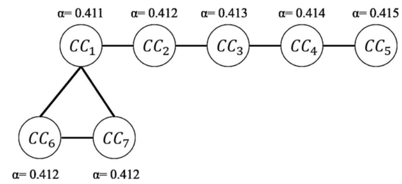



Fig. 3: Ununified system with one ring.

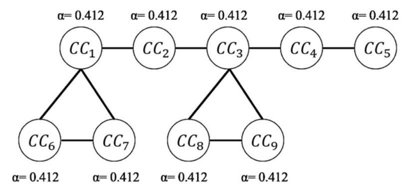



Fig. 4: Unified system with two rings.

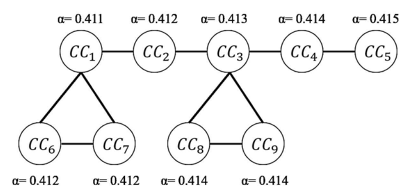



Fig. 5: Ununified system with two rings.

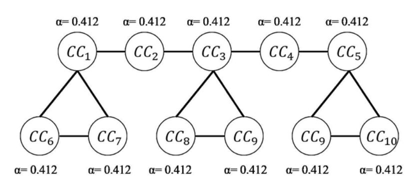



Fig. 6: Unified system with three rings.

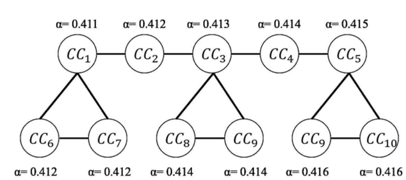
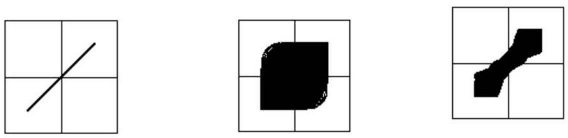



Fig. 7: Ununified system with three rings.

between each chaotic circuit. When the phase difference is close to 0, the Lissajous figures become like Fig. 8(a). In this case, the system causes phase synchronization (perfect synchronization) and this condition is stable. And when the phase differential increases, the Lissajous figures approach the shape of Fig. 8(b). This shows that the system is asynchronous and this condition is unstable. Moreover, the phase difference is to some extent, the Lissajous figure becomes like Fig. 8(c). In this case, the system causes chaotic synchronization. We compare the synchronization with these three patterns.



(a) Synchronization. (b) Asynchronous. (c) Chaotic synchronization.

Fig. 8: Lissajous figure.

First, we simulate the circuit model with one ring, and the result is shown in Fig. 9, 12 and 13. In this case, despite of that the circuits are set three periodic attractors, both unified system and ununified system cause chaotic synchronization and observe chaotic attractors by coupling ring to CC1 or CC2. On the other hand, when we make the ring structure CC3, all chaotic circuits are generated three periodic attractors and cause perfect synchronization in both cases.

Second, we simulate the model with two rings, and the result is shown in Fig. 10, 14 and 15. In this case, the position of occurred chaotic synchronization increases more than the situation of one ring. When we make the ring structure on CC1 and CC3 in the unified system and couple rings to CC1 and CC5 in the ununified system, many positions in the systems are asynchronous. Though, when we make the ring structures on CC2 and CC3 or CC2 and CC4, synchronized state of both systems are difference. The circuits of the ununified system occur chaotic synchronization and the circuits of the ununified system occur perfect synchronization. Furthermore, in the case of CC2 and CC5 in the ununified system, despite CC8 and CC9 has chaotic parameter as $\alpha = 0.416$, CC8 and CC9 generate three periodic solutions and perfect synchronization occur on CC8-CC9.

Finally, we simulate the model with three rings, and the result is shown in Fig. 11, 16 and 17. In this case, on both systems, the positions of the bridge to the ladder and ring tend to be asynchronous. However, in the case of coupling to CC1, CC3 and CC5, both systems obtain synchronization on bridge. Furthermore, in the case of coupling rings to CC2, CC3 and CC4, synchronization is difference in both systems. The ununified system has asynchronous however unified system obtains many perfect synchronizations.

From these results, we confirm that when the number of rings increases, synchronization phenomena are easy to occur in the ununified system. We consider the reason why the tendency exists is that by changing the characteristic, the system has multiformity and can adapt to the surrounding dynamics.

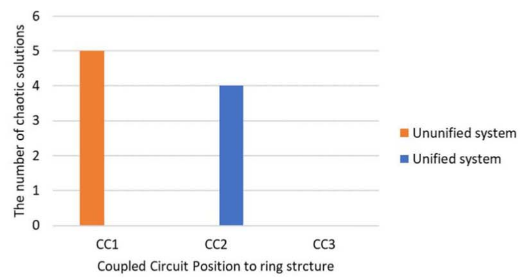



Fig. 9: The number of chaotic attractors in case of one ring.

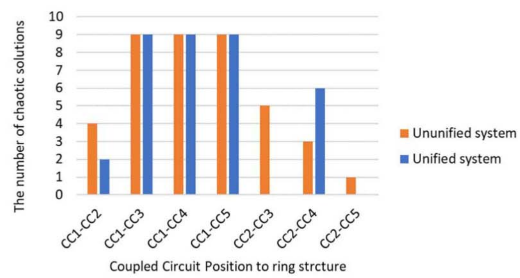



Fig. 10: The number of chaotic attractors in case of two rings.

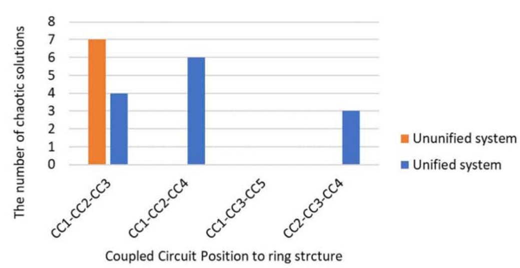



Fig. 11: The number of chaotic attractors in case of three rings.

## IV. Conclusions

In this study, we have investigated synchronization phenomena in ladder-coupled chaotic circuits including ring structures modeling sapling of the plant. The models are two types named unified system and ununified system. We have set the parameters of the circuits to generate periodic solutions or chaotic solutions. And we have proposed that we change the number of ring structure. By the computer simulations, we have confirmed that when we changed the number of rings, we can obtain various synchronization.

In the case of one ring, the large difference of dynamics does not exist, however, the positions of synchronization are the difference. In the case of two rings, the unified systems tend to have occurred chaotic synchronization and ununified systems tend to have occurred perfect synchronization. In the case of three rings, a synchronization phenomenon is easier to have occurred in the ununified system than the unified system.

In our future works, we investigate synchronization phenomena by using $\alpha$ as the random value in the ununified system. Furthermore, we carry out circuit experiment and confirm the reproducibility.

| Between circuits | Coupled Circuit Position to ring strcture | | |
|---|---|---|---|
| | CC1 | CC2 | CC3 |
| CC1-CC2 | Perfect sync. | Asynchronous | Perfect sync. |
| CC2-CC3 | Perfect sync. | Chaotic sync. | Perfect sync. |
| CC3-CC4 | Perfect sync. | Chaotic sync. | Perfect sync. |
| CC4-CC5 | Perfect sync. | Chaotic sync. | Perfect sync. |
| CC1-CC6 | Perfect sync. | | |
| CC1-CC7 | Perfect sync. | | |
| CC2-CC6 | | Chaotic sync. | |
| CC2-CC7 | | Perfect sync. | |
| CC3-CC6 | | | Perfect sync. |
| CC3-CC7 | | | Perfect sync. |
| CC6-CC7 | | | Perfect sync. |

Fig. 12: Synchronized pattern in unified system with one ring.

| Between circuits | Coupled Circuit Position to ring strcture | | |
|---|---|---|---|
| | CC1 | CC2 | CC3 |
| CC1-CC2 | Chaotic sync. | Perfect sync. | Perfect sync. |
| CC2-CC3 | Chaotic sync. | Perfect sync. | Perfect sync. |
| CC3-CC4 | Chaotic sync. | Perfect sync. | Perfect sync. |
| CC4-CC5 | Chaotic sync. | Perfect sync. | Perfect sync. |
| CC1-CC6 | Chaotic sync. | | |
| CC1-CC7 | Chaotic sync. | | |
| CC2-CC6 | | Perfect sync. | |
| CC2-CC7 | | Perfect sync. | |
| CC3-CC6 | | | Perfect sync. |
| CC3-CC7 | | | Perfect sync. |
| CC6-CC7 | Perfect sync. | Perfect sync. | Perfect sync. |

Fig. 13: Synchronized pattern in ununified system with one ring.

| Between circuits | Coupled Circuit Position to ring strcture | | | | | | |
|---|---|---|---|---|---|---|---|
| | CC1,CC2 | CC1,CC3 | CC1,CC4 | CC1,CC5 | CC2,CC3 | CC2,CC4 | CC2,CC5 |
| CC1-CC2 | Chaotic sync. | Asynchronous | Chaotic sync. | Chaotic sync. | Chaotic sync. | Chaotic sync. | Chaotic sync. |
| CC2-CC3 | Chaotic sync. | Asynchronous | Chaotic sync. | Chaotic sync. | Chaotic sync. | Chaotic sync. | Chaotic sync. |
| CC3-CC4 | Chaotic sync. | Asynchronous | Chaotic sync. | Chaotic sync. | Chaotic sync. | Chaotic sync. | Chaotic sync. |
| CC4-CC5 | Chaotic sync. | Asynchronous | Chaotic sync. | Chaotic sync. | Chaotic sync. | Chaotic sync. | Chaotic sync. |
| CC1-CC6 | Chaotic sync. | Chaotic sync. | Chaotic sync. | Asynchronous | | | |
| CC1-CC7 | Chaotic sync. | Chaotic sync. | Chaotic sync. | Chaotic sync. | | | |
| CC2-CC6 | | | | | Chaotic sync. | Chaotic sync. | Chaotic sync. |
| CC2-CC7 | | | | | Chaotic sync. | Chaotic sync. | Chaotic sync. |
| CC2-CC8 | Chaotic sync. | | | | | | |
| CC2-CC9 | Chaotic sync. | | | | | | |
| CC3-CC8 | | Chaotic sync. | | | Chaotic sync. | | |
| CC3-CC9 | | Chaotic sync. | | | Chaotic sync. | | |
| CC4-CC8 | | | Chaotic sync. | | | Chaotic sync. | |
| CC4-CC9 | | | Chaotic sync. | | | Chaotic sync. | |
| CC5-CC8 | | | | Chaotic sync. | | | Chaotic sync. |
| CC5-CC9 | | | | Chaotic sync. | | | Chaotic sync. |
| CC6-CC7 | Chaotic sync. | Chaotic sync. | Chaotic sync. | Chaotic sync. | Perfect sync. | Perfect sync. | Perfect sync. |
| CC8-CC9 | Perfect sync. | Chaotic sync. | Chaotic sync. | Chaotic sync. | Perfect sync. | Perfect sync. | Perfect sync. |

Fig. 14: Synchronized pattern in unified system with two rings.

| Between circuits | Coupled Circuit Position to ring strcture | | | | | | |
|---|---|---|---|---|---|---|---|
| | CC1,CC2 | CC1,CC3 | CC1,CC4 | CC1,CC5 | CC2,CC3 | CC2,CC4 | CC2,CC5 |
| CC1-CC2 | Chaotic sync. | Chaotic sync. | Chaotic sync. | Asynchronous | Perfect sync. | Chaotic sync. | Perfect sync. |
| CC2-CC3 | Chaotic sync. | Chaotic sync. | Chaotic sync. | Asynchronous | Perfect sync. | Chaotic sync. | Perfect sync. |
| CC3-CC4 | Chaotic sync. | Chaotic sync. | Chaotic sync. | Asynchronous | Perfect sync. | Chaotic sync. | Perfect sync. |
| CC4-CC5 | Asynchronous | Asynchronous | Chaotic sync. | Asynchronous | Perfect sync. | Chaotic sync. | Perfect sync. |
| CC1-CC6 | Chaotic sync. | Chaotic sync. | Chaotic sync. | Asynchronous | | | |
| CC1-CC7 | Chaotic sync. | Chaotic sync. | Chaotic sync. | Asynchronous | | | |
| CC2-CC6 | | | | | Perfect sync. | Chaotic sync. | Perfect sync. |
| CC2-CC7 | | | | | Perfect sync. | Chaotic sync. | Perfect sync. |
| CC2-CC8 | Chaotic sync. | | | | | | |
| CC2-CC9 | Chaotic sync. | | | | | | |
| CC3-CC8 | | Chaotic sync. | | | Perfect sync. | | |
| CC3-CC9 | | Chaotic sync. | | | Perfect sync. | | |
| CC4-CC8 | | | Chaotic sync. | | | Chaotic sync. | |
| CC4-CC9 | | | Chaotic sync. | | | Chaotic sync. | |
| CC5-CC8 | | | | Asynchronous | | | Perfect sync. |
| CC5-CC9 | | | | Asynchronous | | | Perfect sync. |
| CC6-CC7 | Perfect sync. | Chaotic sync. | Chaotic sync. | Chaotic sync. | Perfect sync. | Perfect sync. | Perfect sync. |
| CC8-CC9 | Perfect sync. | Chaotic sync. | Chaotic sync. | Chaotic sync. | Perfect sync. | Chaotic sync. | Perfect sync. |

Fig. 15: Synchronized pattern in ununified system with two rings.

| Between circuits | Coupled Circuit Position to ring strcture | | | |
|---|---|---|---|---|
| | CC1,CC2,CC3 | CC1,CC2,CC4 | CC1,CC3,CC5 | CC2,CC3,CC4 |
| CC1-CC2 | Chaotic sync. | Chaotic sync. | Perfect sync. | Asynchronous |
| CC2-CC3 | Chaotic sync. | Chaotic sync. | Perfect sync. | Chaotic sync. |
| CC3-CC4 | Chaotic sync. | Chaotic sync. | Perfect sync. | Chaotic sync. |
| CC4-CC5 | Asynchronous | Asynchronous | Perfect sync. | Asynchronous |
| CC1-CC6 | Asynchronous | Asynchronous | | |
| CC1-CC7 | Asynchronous | Asynchronous | | |
| CC2-CC6 | | | | Asynchronous |
| CC2-CC7 | | | | Asynchronous |
| CC3-CC6 | | | | |
| CC3-CC7 | | | | |
| CC2-CC8 | Asynchronous | Chaotic sync. | | |
| CC2-CC9 | Asynchronous | Chaotic sync. | | |
| CC3-CC8 | | | Perfect sync. | Asynchronous |
| CC3-CC9 | | | Perfect sync. | Asynchronous |
| CC3-CC10 | Asynchronous | | | |
| CC3-CC11 | Asynchronous | | | |
| CC4-CC10 | | Asynchronous | | Asynchronous |
| CC4-CC11 | | Asynchronous | | Asynchronous |
| CC5-CC10 | | | Perfect sync. | |
| CC5-CC11 | | | Perfect sync. | |
| CC6-CC7 | Chaotic sync. | Perfect sync. | Perfect sync. | Perfect sync. |
| CC8-CC9 | Perfect sync. | Chaotic sync. | Perfect sync. | Perfect sync. |
| CC10-CC11 | Perfect sync. | Perfect sync. | Perfect sync. | Perfect sync. |

Fig. 16: Synchronized pattern in unified system with three rings.

| Between circuits | Coupled Circuit Position to ring strcture | | | |
|---|---|---|---|---|
| | CC1,CC2,CC3 | CC1,CC2,CC4 | CC1,CC3,CC5 | CC2,CC3,CC4 |
| CC1-CC2 | Chaotic sync. | Chaotic sync. | Perfect sync. | Perfect sync. |
| CC2-CC3 | Chaotic sync. | Chaotic sync. | Perfect sync. | Perfect sync. |
| CC3-CC4 | Chaotic sync. | Chaotic sync. | Perfect sync. | Perfect sync. |
| CC4-CC5 | Chaotic sync. | Chaotic sync. | Perfect sync. | Perfect sync. |
| CC1-CC6 | Asynchronous | Asynchronous | | |
| CC1-CC7 | Asynchronous | Asynchronous | | |
| CC2-CC6 | | | | Perfect sync. |
| CC2-CC7 | | | | Perfect sync. |
| CC3-CC6 | | | | |
| CC3-CC7 | | | | |
| CC2-CC8 | Asynchronous | Asynchronous | | |
| CC2-CC9 | Asynchronous | Asynchronous | | |
| CC3-CC8 | | | Perfect sync. | Perfect sync. |
| CC3-CC9 | | | Perfect sync. | Perfect sync. |
| CC3-CC10 | Chaotic sync. | | | |
| CC3-CC11 | Chaotic sync. | | | |
| CC4-CC10 | | Chaotic sync. | | Chaotic sync. |
| CC4-CC11 | | Chaotic sync. | | Chaotic sync. |
| CC5-CC10 | | | Chaotic sync. | |
| CC5-CC11 | | | Chaotic sync. | |
| CC6-CC7 | Perfect sync. | Perfect sync. | Perfect sync. | Perfect sync. |
| CC8-CC9 | Perfect sync. | Perfect sync. | Perfect sync. | Perfect sync. |
| CC10-CC11 | Chaotic sync. | Chaotic sync. | Perfect sync. | Perfect sync. |

Fig. 17: Synchronized pattern in ununified system with three rings.

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