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Investigation of Coupled Chaotic Circuits with Ladder and Ring Structures

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Abstract

In this study, we investigate synchronization phenomena in ladder-coupled chaotic circuits including ring structures modeling sapling of plant. We focus on the effect of the position of the ring structures in the network to the synchronization of the whole network. In our proposed network model, chaotic circuits are coupled by resistors. We set the parameters of the circuits to generate periodic solutions or chaotic solutions. Computer simulations confirm that the various synchronization phenomena occur in different situations.

1. Introduction

Synchronization in a network is one of the interesting topics in scientific disciplines. Investigation of this is important work in order to clarify nonlinear phenomena in the natural world. It is observed over the various fields, for example, engineering, biology, sociology and so on [1]. Incidentally, the networks have different topology and characters in each even if they are the similar type [2], [3]. Therefore, the investigation of dynamics in different of the network is significant and research has proceeded to analysis the dynamics of synchronization on each topology.

On the other hand, synchronization of coupled chaotic system is interesting models to describe various higher-dimensional nonlinear phenomena [4], [5]. Recently, studying chaotic phenomena by using coupled chaotic circuits increases. On the chaotic circuit with simple elements, circuit experiment and computer simulation are suitable for studying in terms of that experiment time is short and repeatability of experiments is high. Networks by using coupled chaotic circuits can be modeled natural world or social networks. Moreover, it is important for future engineering to investigate nonlinear phenomena like chaotic synchronization.

In modern society, agenda of vegetation exist about environment or food. The study of plant bioelectric potential has attracted rising attention in recently from the viewpoint of plant growth [6]. Bioelectric potential is known that it changes depending on surrounding environment. Therefore, we deem the bioelectric potential to the voltage phase difference of chaotic circuit and we model the plants by chaotic circuit.

In this study, we investigate the stability in our proposed model by ladder and rings modeling sapling of the plant by synchronization phenomena. In our proposed network model, chaotic circuits are coupled by resistors. These chaotic circuits are little difference of each other by the bifurcation parameter. We set the parameters of the circuits to generate periodic solutions or chaotic solutions. The number of rings changes to each model. And we observe the synchronization phenomena in different network models.

2. System model

In this study, we use chaotic circuits. The model of chaotic circuit is shown in Fig. 1. This circuit consists of a negative resistor, two inductors, a capacitor and dual-directional diodes. This chaotic circuit is called Nishio-Inaba circuit [7].

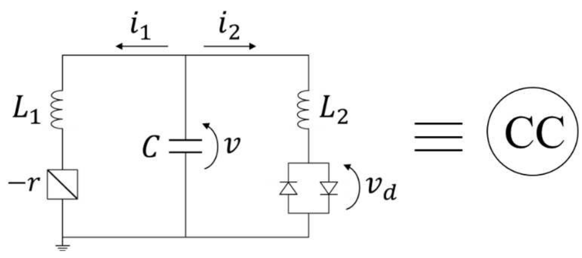


Figure 1: Chaotic circuit.

We propose ladder-coupled chaotic circuits including ring structures. Figure 2 shows examples of the network models. In this case of Fig. 2(a), we set five chaotic circuits in ladder structure and one ring structure coupling to CC1 by resisters. Likewise, we propose three network models that coupled ring structure to CC1, CC2 or CC3. We set the parameters of the system as $\beta = 3.0$ and $\delta = 470.0$. Next, we change the condition in terms of the parameter α. α denotes degree of chaos. The network models are divided into two types by the value of α. One pattern is to unify the value of all α (unified system) like shown in Fig. 2(a), and the other is to change the value of all α (ununified system) like shown in Fig. 2(b) . In case of Fig. 2(a), the value of α is unified 0.412. And in case of Fig. 2(b), the value of α is not unified. The parameters α are set from 0.411 to 0.415 with step size 0.001 from CC1 to CC5. The α of CC6 and CC7 are added 0.001 from the α of coupled circuit.

Subsequently, we augment the number of rings in the case of one ring and three rings. Likewise in ununified systems, the α of CC6, CC7, CC8, CC9, CC10 and CC11 are added 0.001 from the α of coupled circuit. We show the models from Fig. 2(c) and Fig. 2(d). Figure 2(c) is unified systems with three rings and Fig. 2(d) is ununified systems with three rings.

By changing the variables and parameters,

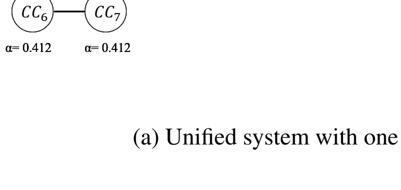
$$\begin{cases} i_1 = \sqrt{\dfrac{C}{L_1}} V x_n, \ \ i_2 = \dfrac{\sqrt{L_1 C}}{L_2} V y_n, \ \ v = V z_n \\ \alpha = r\sqrt{\dfrac{C}{L_1}}, \ \ \beta = \dfrac{L_1}{L_2}, \ \ \delta = r_d \dfrac{\sqrt{L_1 C}}{L_2}, \\ \gamma = \dfrac{1}{R}, \ \ t = \sqrt{L_1 C_2}\tau. \end{cases} \tag{1}$$

R is the resistor that couples each chaotic circuit.

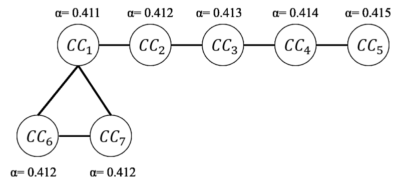
The normalized circuit equations of the systems are given as follows:

$$\begin{cases} \dfrac{dx_i}{d\tau} = \alpha x_i + z_i \\ \dfrac{dy_i}{d\tau} = z_i - f(y) \\ \dfrac{dz_i}{d\tau} = -x_i - \beta y_i - \displaystyle\sum_{i,j=1}^{N} \gamma_{ij}(z_i - z_j) \\ \qquad\qquad (i, j = 1, 2, \cdots, N). \end{cases} \tag{2}$$

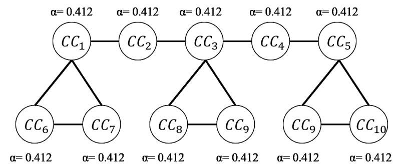
In Eq. (2), N is the number of coupled chaotic circuits and γ is the coupling strength as $\gamma = 0.008$ in this study. We set the parameters of the circuits to generate periodic solutions or chaotic solutions. When we set the parameter as $\alpha = 0.411$ to $\alpha = 0.415$, the circuits generate three-periodic solutions. And when the parameter is more than $\alpha = 0.416$, the circuits generate chaotic solutions. The attractors are shown in Fig. 3.



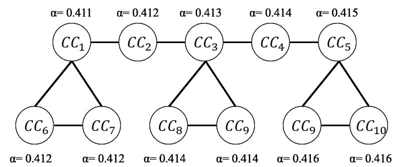
(a) Unified system with one ring.



(b) Ununified system with one ring.



(c) Unified system with three rings.



(d) Ununified system with three rings.

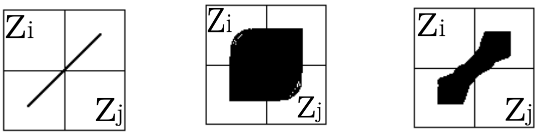
Figure 2: Network models.

(a) Three periodic attractor. (b) Chaotic attractor.

Figure 3: Attractors.

3. Simulation results

We investigate synchronization phenomena in ladder-coupled chaotic circuits including ring structures. In this time, we use attractors and Lissajous figures to research synchronization. Lissajous figures show the phase difference between each chaotic circuit. When the phase difference is close to 0, the Lissajous figures become like Fig. 4(a). In this case, the system causes phase synchronization (perfect synchronization) and this condition is stable. And when the phase differential increases, the Lissajous figures approach the shape of Fig. 4(b). This shows that the system is asynchronous and this condition is unstable. Moreover, the phase difference is to some extent, the Lissajous figure become like Fig. 4(c). In this case, the system causes chaotic synchronization. We compare the synchronization with these three patterns.



(a) Synchronization. (b) Asynchronous. (c) Chaotic synchronization.

Figure 4: Lissajous figures.

First, we simulate the circuit model with one ring, and the result is shown in Fig. 5 and Tables 1 and 2. In this case, despite of that the circuits are set three periodic attractors, the both unified system and ununified system cause chaotic synchronization and observe chaotic attractors by coupling ring to CC1 or CC2. On the other hand, when we make the ring structure with CC3, all chaotic circuits are generated three periodic attractors and cause perfect synchronization in both cases.

Tables 3 and 4 show the percentage of each synchronized state in these models. We obtain the result that network model with one ring in ununified system is no asynchronous point and it indicates a stable state.

Next, we simulate the model with three rings, and the result is shown in Fig. 6 and Tables 5 and 6. In this case, on the both systems, the positions of bridge on the ladder and ring tend to be asynchronous. However, in the case of coupling to CC1, CC3 and CC5, both systems are occured synchronization on the bridge. Furthermore, in the case of coupling rings to CC2, CC3 and CC4, synchronization is difference in both systems.

Tables 7 and 8 show the percentage of each synchronization state in these models. In case of coupling to CC1, CC2 and CC3, and in case of coupling to CC2, CC3 and CC4, we obtain the result that the percentage of asynchronous decreases. Though, in case of coupling to CC1, CC2 and CC4, the percentage of asynchronous increases.

From these results, we consider that in ununified system, the percentage of asynchronous tend to decrease.

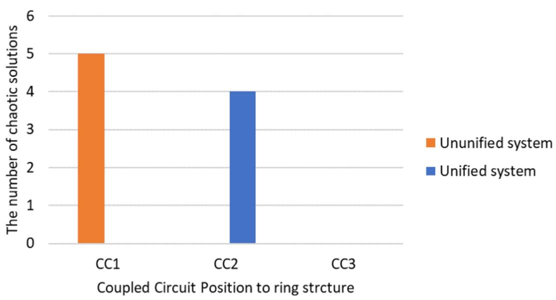


Figure 5: The number of chaotic attractors in case of one ring.

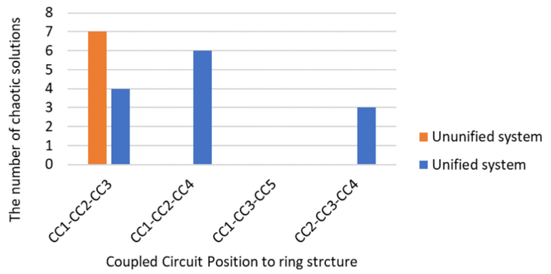


Figure 6: The number of chaotic attractors in case of three rings.

Table 1: Coupling one ring in unified system.

Between circuits	Coupled Circuit Position to ring strcture		
	CC1	CC2	CC3
CC1-CC2	Perfect sync.	Asynchronous	Perfect sync.
CC2-CC3	Perfect sync.	Chaotic sync.	Perfect sync.
CC3-CC4	Perfect sync.	Chaotic sync.	Perfect sync.
CC4-CC5	Perfect sync.	Chaotic sync.	Perfect sync.
CC1-CC6	Perfect sync.		
CC1-CC7	Perfect sync.		
CC2-CC6		Chaotic sync.	
CC2-CC7		Perfect sync.	
CC3-CC6			Perfect sync.
CC3-CC7			Perfect sync.
CC6-CC7			Perfect sync.

Table 2: Coupling one ring in ununified system.

Between circuits	Coupled Circuit Position to ring strcture		
	CC1	CC2	CC3
CC1-CC2	Chaotic sync.	Perfect sync.	Perfect sync.
CC2-CC3	Chaotic sync.	Perfect sync.	Perfect sync.
CC3-CC4	Chaotic sync.	Perfect sync.	Perfect sync.
CC4-CC5	Chaotic sync.	Perfect sync.	Perfect sync.
CC1-CC6	Chaotic sync.		
CC1-CC7	Chaotic sync.		
CC2-CC6		Perfect sync.	
CC2-CC7		Perfect sync.	
CC3-CC6			Perfect sync.
CC3-CC7			Perfect sync.
CC6-CC7	Perfect sync.	Perfect sync.	Perfect sync.

Table 3: Percentage of synchronization state in unified system with one ring.

Percentage of synchronization	Coupled circuit position to ring strcture		
	CC1	CC2	CC3
Perfect sync.	100.00	28.57	100.00
Chaotic sync.	0.00	57.14	0.00
Asynchronous	0.00	14.29	0.00

Table 4: Percentage of synchronization state in ununified system with one ring.

Percentage of synchronization	Coupled circuit position to ring strcture		
	CC1	CC2	CC3
Perfect sync.	14.29	100.00	100.00
Chaotic sync.	85.71	0.00	0.00
Asynchronous	0.00	0.00	0.00

Table 5: Coupling three rings in unified system.

Between circuits	Coupled Circuit Position to ring strcture			
	CC1,CC2,CC3	CC1,CC2,CC4	CC1,CC3,CC5	CC2,CC3,CC4
CC1-CC2	Chaotic sync.	Chaotic sync.	Perfect sync.	Asynchronous
CC2-CC3	Chaotic sync.	Chaotic sync.	Perfect sync.	Chaotic sync.
CC3-CC4	Chaotic sync.	Chaotic sync.	Perfect sync.	Chaotic sync.
CC4-CC5	Asynchronous	Asynchronous	Perfect sync.	Asynchronous
CC1-CC6	Asynchronous	Asynchronous		
CC1-CC7	Asynchronous	Asynchronous		
CC2-CC6				Asynchronous
CC2-CC7				Asynchronous
CC3-CC6				
CC3-CC7				
CC2-CC8	Asynchronous	Chaotic sync.		
CC2-CC9	Asynchronous	Chaotic sync.		
CC3-CC8			Perfect sync.	Asynchronous
CC3-CC9			Perfect sync.	Asynchronous
CC3-CC10	Asynchronous			
CC3-CC11	Asynchronous			
CC4-CC10		Asynchronous		Asynchronous
CC4-CC11		Asynchronous		Asynchronous
CC5-CC10			Perfect sync.	
CC5-CC11			Perfect sync.	
CC6-CC7	Chaotic sync.	Perfect sync.	Perfect sync.	Perfect sync.
CC8-CC9	Perfect sync.	Chaotic sync.	Perfect sync.	Perfect sync.
CC10-CC11	Perfect sync.	Perfect sync.	Perfect sync.	Perfect sync.

Table 6: Coupling three rings in ununified system.

Between circuits	Coupled Circuit Position to ring strcture			
	CC1,CC2,CC3	CC1,CC2,CC4	CC1,CC3,CC5	CC2,CC3,CC4
CC1-CC2	Chaotic sync.	Chaotic sync.	Perfect sync.	Perfect sync.
CC2-CC3	Chaotic sync.	Chaotic sync.	Perfect sync.	Perfect sync.
CC3-CC4	Chaotic sync.	Chaotic sync.	Perfect sync.	Perfect sync.
CC4-CC5	Chaotic sync.	Chaotic sync.	Perfect sync.	Perfect sync.
CC1-CC6	Asynchronous	Asynchronous		
CC1-CC7	Asynchronous	Asynchronous		
CC2-CC6				Perfect sync.
CC2-CC7				Perfect sync.
CC3-CC6				
CC3-CC7				
CC2-CC8	Asynchronous	Asynchronous		
CC2-CC9	Asynchronous	Asynchronous		
CC3-CC8			Perfect sync.	Perfect sync.
CC3-CC9			Perfect sync.	Perfect sync.
CC3-CC10	Chaotic sync.			
CC3-CC11	Chaotic sync.			
CC4-CC10		Chaotic sync.		Chaotic sync.
CC4-CC11		Chaotic sync.		Chaotic sync.
CC5-CC10			Chaotic sync.	
CC5-CC11			Chaotic sync.	
CC6-CC7	Perfect sync.	Perfect sync.	Perfect sync.	Perfect sync.
CC8-CC9	Perfect sync.	Perfect sync.	Perfect sync.	Perfect sync.
CC10-CC11	Chaotic sync.	Chaotic sync.	Perfect sync.	Perfect sync.

Table 7: Percentage of synchronization state in unified system with three rings.

Percentage of synchronization	Coupled circuit position to ring strcture			
	CC1,CC2,CC3	CC1,CC2,CC4	CC1,CC3,CC5	CC2,CC3,CC4
Perfect sync.	15.38	15.38	100.00	23.08
Chaotic sync.	30.77	46.15	0.00	15.38
Asynchronous	53.85	38.47	0.00	61.54

Table 8: Percentage of synchronization state in ununified system with three rings.

Percentage of synchronization	Coupled circuit position to ring strcture			
	CC1,CC2,CC3	CC1,CC2,CC4	CC1,CC3,CC5	CC2,CC3,CC4
Perfect sync.	15.38	15.38	81.82	84.62
Chaotic sync.	53.85	38.47	18.18	15.38
Asynchronous	30.77	46.15	0.00	0.00

4. Conclusions

In this study, we have investigated synchronization phenomena in ladder-coupled chaotic circuits including ring structures modeling sapling of the plant. The models are two types named unified system and ununified system. We have set the parameters of the circuits to generate periodic solutions or chaotic solutions. And we have proposed that we change the number of ring structure. We set the parameters of the circuits to generate periodic solutions or chaotic solutions. By the computer simulations, we have confirmed that when we changed the number of rings, we can obtain various synchronization.

In case of one ring, the large difference of dynamics does not exist, however the positions of synchronization are different. In case of three rings, synchronization phenomenon is easier to occur in ununified system than unified system. Furthermore, we have considered that in ununified system, the percentage of asynchronous tend to decrease.

In our future works, we investigate synchronization phenomena by using α as the random value in ununified system. Furthermore, we carry out circuit experiment and confirm the reproducibility.

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