Oscillation Quenching in Coupled van der Pol Oscillators with Different Frequencies

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***Abstract*—In this study, we investigate oscillation quenching in coupled van der Pol oscillators with different frequencies. The network topology is inspired from real brain network. Here, we consider the network which consists of two modules including connector and provincial hubs. We confirm oscillation quenching from the proposed system.**

***Keywords*—oscillation quenching; coupled oscillatory network**

I. INTRODUCTION

In our research group, we have focused on synchronization phenomena observed from nonlinear oscillatory networks [1], [2]. This is because the results of synchronization in complex networks are useful deeper understanding of control methods in power networks, communication systems and so on. Also, they can be used as an alternative approach, apart from existing ones, for describing mode-locking phenomena in biological networks. Therefore, we consider the schematic brain networks as one of complex biological networks. Because, we would like to propose modeling of synchronization in brain by using coupled electrical oscillatory circuits, in order to make clear the mechanism of functional operation in brain.

Recently, the relationship between structural and functional network in biological neural network has attracted their attention from many researchers. Hartelt et al. has discovered the network topology in the pre-BotC which consists of densely connected clusters with rare inter-cluster links. And, the hubs of the network behave quiescent output [3].

We apply this phenomenon for the coupled oscillatory systems using electrical oscillator. Namely, one node is set to lower frequency than the others. We also investigate the influence of location of oscillator with different frequencies.

II. PROPOSED SYSTEM

A. Network Model

A network model composed of 13 nodes is shown in Fig. 1 [4]. There are two important hubs in this network, "Connector hub" and "Provincial hub". The both hubs are high-degree nodes. "Connector hub" shows a diverse connectivity by connecting two sub-networks. "Provincial hub" primarily connects nodes in the same sub-network. There are two modules and the node is expressed by van der Pol oscillator as shown in Fig. 2. The van der Pol oscillators are coupled by a resistor.

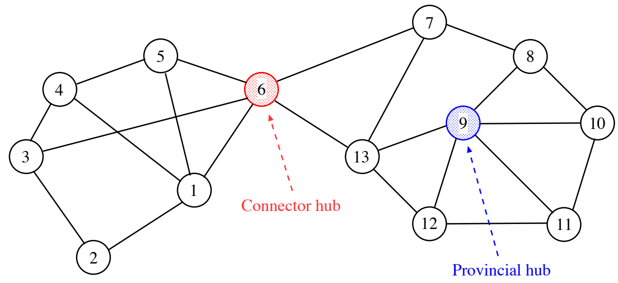


Fig. 1. Network model.

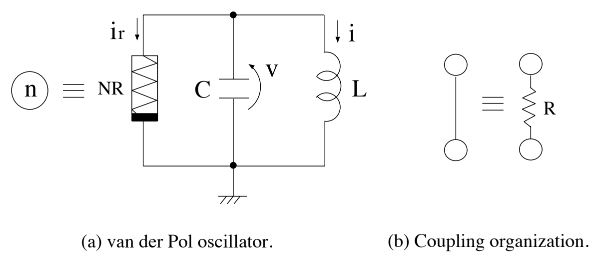


Fig. 2. Ocillator and coupling organization.

Next, we develop the expression for the circuit equation of the network model. The normalized circuit equations governing the circuit are expressed as

$$\begin{cases} \dfrac{dx_k}{d\tau} = \varepsilon\Big(1 - \dfrac{1}{3}{x_k}^2\Big)x_k - y_k - \gamma \displaystyle\sum_{n \in S_k} (y_k - y_n) \\ \dfrac{dy_k}{d\tau} = x_k \\ \qquad\qquad\qquad\qquad (k = 1, 2, \ldots 13). \end{cases} \tag{1}$$

In these equations, γ is the coupling strength, ε denotes the nonlinearity of the oscillators. For the computer simulations, we calculate circuit equations using the fourth-order Runge-Kutta method with the step size h =0.005. The parameter of the nonlinearity of oscillator is fexed as ε =0.1.

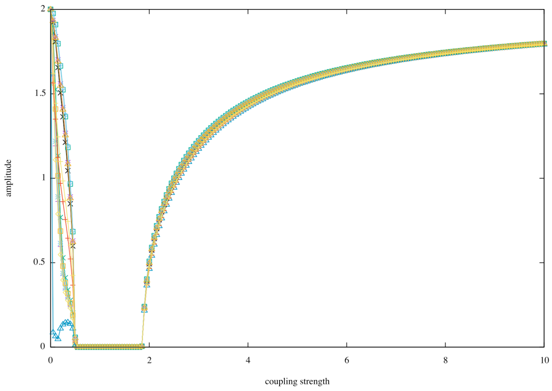
B. Adding Different Frecuency

In this simulation, one node of the network has lower frequency than the others. The standard frequency is ω=1.0 and the lower frequency is set to ω =0.47. In order to investigate influence of location of oscillator with different frequencies, we focus on several nodes as follows.

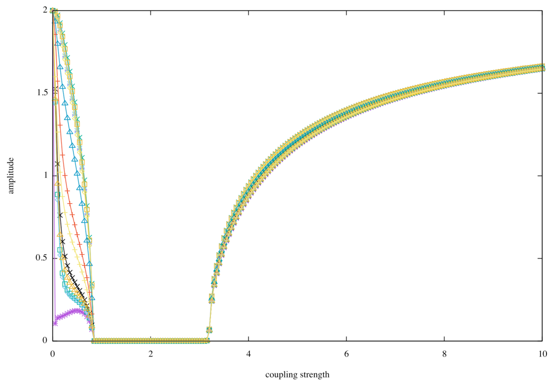
Case-1) Connector hub (oscillator no. 6)
Case-2) Provincial hub (oscillator no. 9)
Case-3) Other node (oscillator no. 2)
Case-4) Other node (oscillator no. 10)

III. OSCILLATION QUENCHING

Figure 4 shows the simulation result of amplitude change with the coupling strength. From these results, we confirm amplitude death at certain range. Furthermore, the range of amplitude death of connector hub is smaller than the provincial hub.



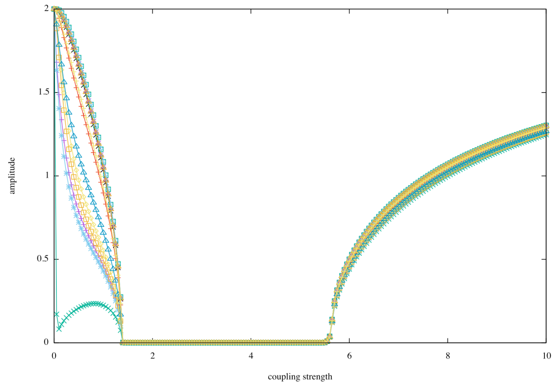
(a) Case-1: Connector hub (oscillator no. 6).



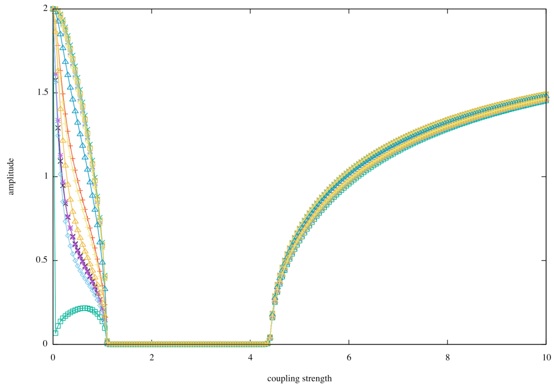
(b) Case-2: Provincial hub (oscillator no. 9).

Fig. 4. Amplitude of connector and provincial hubs.

Next, the simulation results of the amplitude change in Cases 3 and 4 are shown in Fig. 5. By increasing the coupling strength, we also observe the amplitude death in the both cases. We can see that the range of amplitude death of Cases 3 and 4 is longer than Cases 1 and 2.



(a) Case-3: Oscillator no. 2.



(b) Case-4: Oscillator no. 10.

Fig. 5. Amplitude of other nodes.

IV. CONCLUSIONS

In this study, we have investigated oscillation quenching observed in coupled oscillatory system with different frequencies. We have confirmed amplitude death at certain range with the coupling strength from the proposed system. The range of amplitude death is smallest when the lower frequency is set to the connector hub. In the future work, we would like to investigate the relationship between the amplitude death and importance of the nodes.

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