Analysis of Chaotic Circuit Networks with One-Way Coupling

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Abstract— **In this study, we investigate the synchronization phenomena of the complex network in which the chaotic circuits are coupled with one direction. We observe the synchronization by using the voltage difference between the nodes expressed by the chaos circuit. Moreover, we also observe the synchronization phenomena when the coupling strength is changed.**

Keywords; Complex network; Synchronization Phenomena; Chaotic Circuit.

I. INTRODUCTION

Synchronization phenomena are observed everywhere. For example, flashing firefly, crowing tree frogs, beating rhythm of the heart. Chaos is applied to biology, medical science and engineering. Recently, many scientists study synchronization phenomena of chaotic circuits. Especially, synchronization of chaos is very interesting phenomena. Additionally, coupled systems of chaotic elements produce many kinds of complexity phenomena such as clustering, chaos propagation and so on. Studying synchronization of chaos is expected useful in a variety of fields.

In this study, we investigate synchronization phenomena of coupled chaotic circuits with one-way coupling. Each node is applied chaotic circuit and connected with resistance via buffer. By using computer simulations, we observe interesting synchronization phenomena in the proposed chaotic circuits network.

II. NETORK MODEL

Figure 1 shows basic chaotic circuit. In this study, chaotic circuits are applied to nodes of the network. A proposed network model is shown in Fig. 2. The network consists of a chaotic circuit connected in one direction.

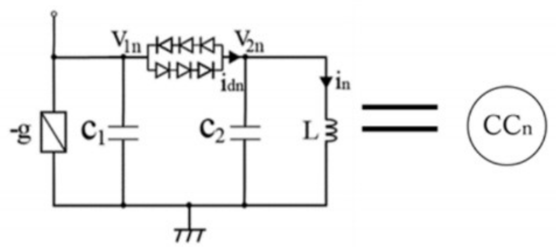

Figure 1: Circuit model.

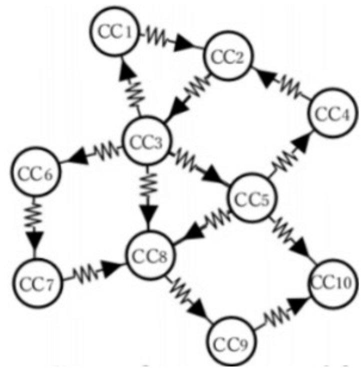

Figure 2: Network model (n=10).

First, the circuit equations are given as follows:

$$\begin{cases} L\dfrac{di_n}{dt} = v_{2n} \\ C_1\dfrac{dv_{1n}}{dt} = gv_{1n} - i_{dn} - \dfrac{1}{R}\sum\limits_{CC_n}(v_{1n} - v_{1k}) \\ C_2\dfrac{dv_{2n}}{dt} = -i_n + i_{dn} \end{cases}$$

$$(n = 1,2,3,\ldots,10),$$

we approximate the $i - v$ characteristics of the nonlinear resistor consisting of the diodes,

$$i_{dn} = \begin{cases} G_d(v_{1n} - v_{2n} - a) & (v_{1n} - v_{2n} > a) \\ 0 & (|v_{1n} - v_{2n}| \le a) \\ G_d(v_{1n} - v_{2n} + a) & (v_{1n} - v_{2n} < -a). \end{cases}$$

By using the parameters and variables as follows:

$$i_n = \sqrt{\frac{C_2}{L}}ax_n, v_{1n} = ay_{n,} \; v_{2n} = az_n$$

$$t = \sqrt{LC_2}\tau, " \cdot " = \frac{d}{d\tau}, \alpha = \frac{C_2}{C_1}$$

$$\beta = \sqrt{\frac{L}{C_2}}G_d, \gamma = \sqrt{\frac{L}{C_2}}g, \delta = \frac{1}{R}\sqrt{\frac{L}{C_2}}.$$

The normalized circuit equations are given as follows:

$$\begin{cases} \dot{x} = z_n \\ \dot{y} = \alpha\gamma y_n - \alpha\beta f(y_n - z_n) - \alpha\delta f(y_k - y_n) \\ \dot{z} = \beta f(y_n - z_n) - x_n, \end{cases}$$

where the nonlinear function corresponding to the characteristics of the nonlinear resistor of the diodes and are described as follows:

$$f(y_n - z_n) = \begin{cases} y_n - z_n - 1 & (y_n - z_n > 1) \\ 0 & (|y_n - z_n| \leq 1) \\ y_n - z_n + 1 & (y_n - z_n < -1). \end{cases}$$

By computer simulations, we observe the synchronization by calculating the voltage and observing the voltage difference.

III. Simulation Result

In this study, we fix parameters as α = 0.5, β = 20, γ = 0.5 and δ = 0.22 on all circuits. The simulation results are shown in Figs. 3 to 5. Figures 3 and 4 show the phase difference.

We observe completely in-phase state, in CC1-CC2 to CC5 (Fig. 3). However, we did not observe in-phase state in CC1-CC6 to CC10 (Fig. 4). We can say that there are two groups in the chaotic network depending on the synchronization state.

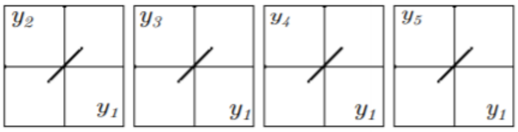

Figure 3: Phase difference (CC1-CC2, CC1-CC3, CC1-CC4, and CC1-CC5). δ = 0.22.

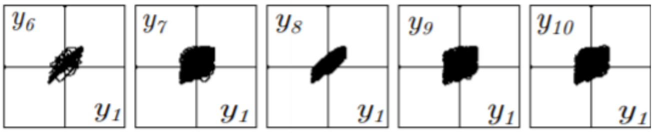

Figure 4: Phase difference (CC1-CC6, CC1-CC7, CC1-CC8, CC1-CC9, and CC1-CC10). δ = 0.22.

Figure 5 shows the voltage difference. We did not observe difference of voltage in CC1-CC2 to CC5. However, we observed difference of voltage in CC1-CC6 to CC10. We observed that the voltage difference is smaller in CC1-CC6 than in CC1-CC9 to CC10.

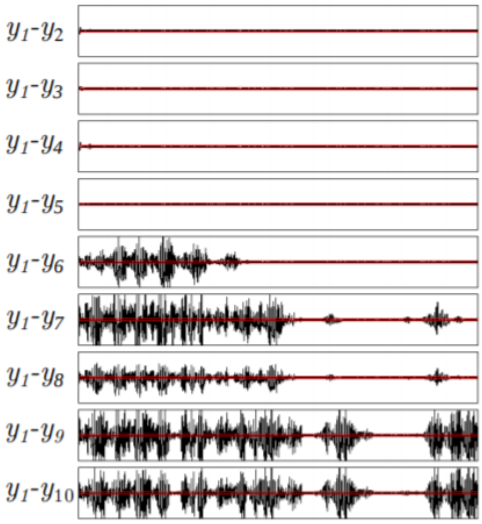

Figure 5: Voltage difference. δ = 0.22.

IV. Conclusion

We studied synchronization phenomena of coupled chaotic circuit with one-way coupling. Synchronization phenomena was observed certain part of the circuit with completely in-phase state. In this study, we used ten chaotic circuits.

For the future works, we will use more number of chaotic circuits. By using more chaotic circuits, we expect to observe complicated phenomena such as clustering. Also we expect to observe chaos synchronization. Also, it is expected that differences in characteristics will be observed by changing network connection.

Acknowledgment

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