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Synchronization of Coupled van der Pol Oscillators with Distributed Hub Nodes

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Abstract

In this study, we focus on network structure with a hub which is consisted of Scale-free network. We consider the synchronization of coupled van der Pol oscillator network whose hub nodes are split two nodes. We investigate synchronization phenomena of network which a hub node replace by two distributed nodes connected by a negative resistor. Furthermore, we compare synchronization phenomena of this network of coupled van der Pol oscillators and chaotic circuits.

1. Introduction

Synchronization phenomena can be observed everywhere in our life. For example, we can confirm metronome, flashing firefly lights, beating rhythm of the heart and so on. Especially, synchronization phenomena of oscillatory network are interesting. In addition, complex networks attract attention from various fields. The feature of networks is characterized by the degree distribution, the path length and the clustering coefficient.

Numerous researches on complex networks have been made so far [1]-[7]. Especially, there are many studies related with hub nodes and these are very interesting. Because, in the real world, large-scale complex networks often include huge nodes with a large number of edges, and such "hub" nodes play key roles in the networks.

We pay our attention on the behavior of the networks whose hub nodes are split into several nodes. Namely, when such a hub node is split into several nodes, how other nodes connected to the hub node change their behavior? Furthermore, when such split hub nodes agree with or oppose to each other, how the whole network behaves to cooperate or to compete? In our previous study, we investigated synchronization phenomena of complex networks by using coupled chaotic circuits and evaluate it by using synchronization rate between a pair of the chaotic circuits [8]. We used the model which we replace a hub node with two distributed hub nodes connected by a negative resistor in previous study. In this study, we investigate synchronization phenomena by using same network model and change the circuit from chaotic circuit to van der Pol oscillator. That is because chaotic circuit behave very complex, so we need to research with a simpler oscillator. Furthermore, we compare synchronization phenomena of coupled van der Pol oscillators and chaotic circuits.

2. Circuit model

Figure 1 shows van der Pol oscillator. This circuit consists of a nonlinear resister, an inductor and a capacitors. In this study, we use a network in which each node is replaced by a van der Pol oscillator. The circuit equation is shown in Eq. (1).

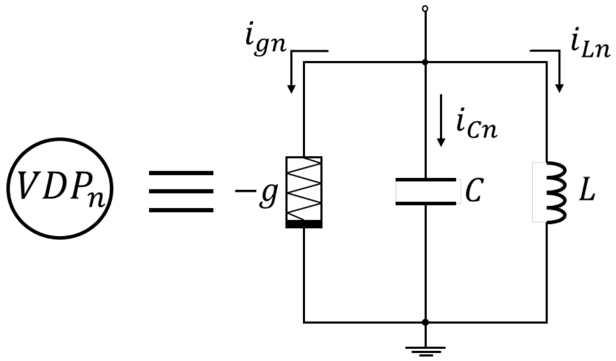


Figure 1: Circuit model.

$$\begin{cases} L\dfrac{di_{Ln}}{dt} = v_n \\ C_1\dfrac{dv_{Cn}}{dt} = \dfrac{1}{R}\displaystyle\sum_{k\in S_n}(v_n - v_k) - i_g n - i_{Ln} \end{cases} \quad (1)$$

where $n, k = 1, 2, 3, \cdots$ and S_n is the set of nodes which are directly connected to the node n. Nonlinear resistor defined as follows:

$$i_{gn} = -g_1 v_n + g_3 {v_3}^3. \quad (2)$$

By changing the variables and parameters,

$$\left\{\begin{array}{l} v_n = \sqrt{\dfrac{g_1}{3g_3}}x_n,\ \ i_{Ln} = \sqrt{\dfrac{g_1 C}{3g_3 L}}yn,\ \ \epsilon = g_1\sqrt{\dfrac{L}{C}}, \\ \delta = \dfrac{1}{R}\sqrt{\dfrac{L}{C}},\ \ t = \sqrt{LC}\tau,\ \ \text{''}\cdot\text{''} = \dfrac{d}{dt} \end{array}\right. \tag{3}$$

the normalized equations of this circuit are given as follows:

$$\left\{\begin{array}{l} \dot{x}_n = \epsilon(x_n - {x_n}^3) - y_n - \delta \displaystyle\sum_{k\in S_n}(x_n - x_k) \\ \dot{y}_n = x_n. \end{array}\right. \tag{4}$$

The parameter δ corresponds the coupling strength between the circuits. The parameter ϵ is the strength of nonlinear.

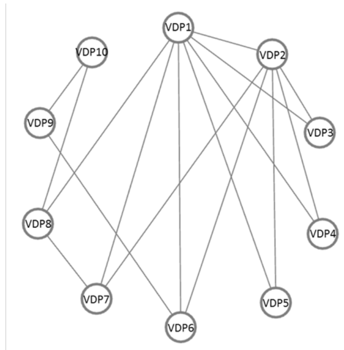
3. Network model

We made the underlying network which is 10 van der Pol oscillators and each circuit connected by a resistor. This network model and the degree distribution of this network which imitate Scale-free network are shown in Fig. 2.

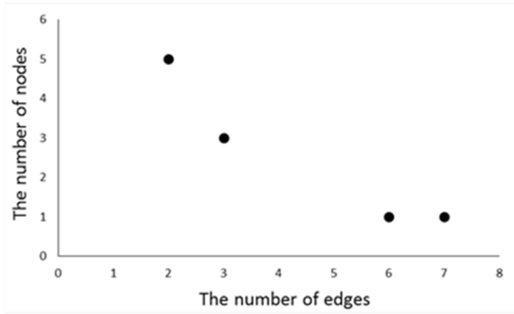
In this study, we proposed network model which a hub node replaced with two distributed nodes. The example of splitting hub is shown in Fig. 3. Right network model in Fig. 4 is the network that applied method shown in Fig. 3 to VDP1 as a hub of the basic network. Further, distributed hub nodes are connected by a negative resistor and other nodes are connected by a positive resistor each other.

4. Simulation Results

In this section, we show the synchronization phenomena of coupled chaotic circuits, which is a previous study, and that of coupled van der Pol oscillators of this study. Further, the coupling strength other than between distributed nodes is defined as δ and the coupling strength between distributed nodes is defined as δ'.



(a) Network model



(b) The degree distribution

Figure 2: Basic network model.

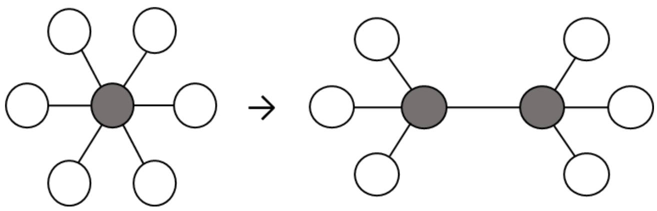


Figure 3: Splitting a node.

4.1 Synchronization phenomena of coupled chaotic circuits in previous study [8]

In previous study, we used chaotic circuit which proposed by Shinriki *et al.* [9][10]. The chaotic circuit is shown in Fig. 5.

And Fig. 6 shows synchronization phenomena of previous study. We evaluate synchronization by phase difference due to compare previous study and this study. In Fig. 6, the coupling strength δ is 0.1 and the coupling strength δ' is -0.2. We can confirm that the couplings which connected to distributed hub nodes are asynchronous and the coupling between distributed hub nodes is asynchronous. Furthermore, the couplings between nodes which do not connect to distributed hub node seems asynchronous. However, from previous study, the couplings between nodes which do not connect to distributed hub node are rather synchronous by using syn-

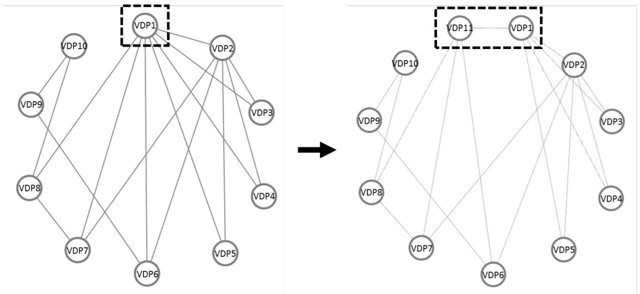


Figure 4: Network model with distributed hub nodes.

chronization rate. Figure 7 shows voltage difference of previous study. Figure 7(a) corresponds the couplings between nodes which do not connect to distributed hub node.

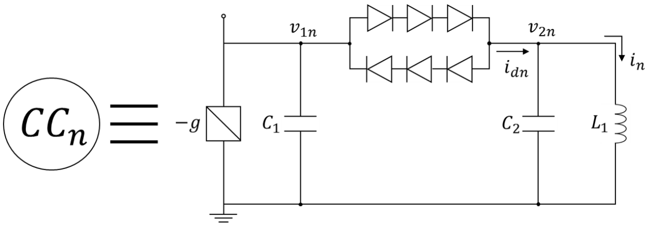


Figure 5: Mori-Shinriki circuit.

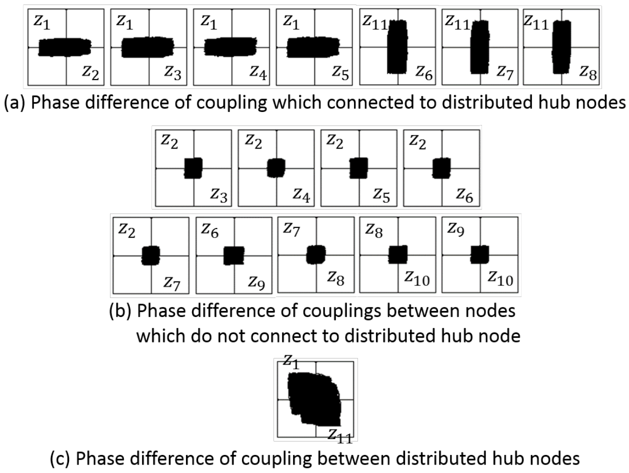


Figure 6: Phase difference in previous study network.

4.2 Synchronization phenomena of coupled van der Pol oscillators

In this section, we observed synchronization phenomena of coupled van der Pol oscillators. As mentioned above, we evaluate synchronization phenomena by phase difference.

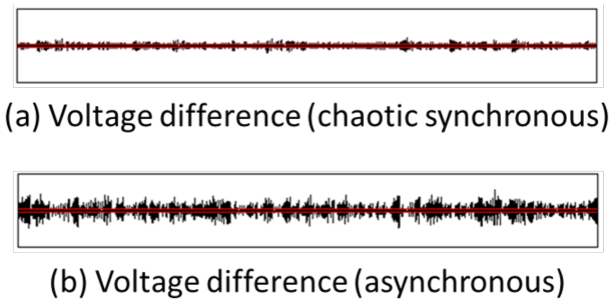


Figure 7: Voltage difference of network coupled chaotic circuits in previous study.

Figure 8 shows the result of this simulation. In Fig. 8, the coupling strength δ is 0.0002 and the coupling strength δ' is -0.02. We can confirm that couplings which connected to distributed hub nodes are asynchronous. Furthermore, the couplings between nodes which do not connect to distributed hub node are almost synchronous and the coupling between distributed hub nodes is anti-phase synchronization. This result is similar to the result of section 4.1.

From the result of section 4.1 and 4.2, we summarize the rekationship between synchronization phenomena and network model. Figure 9 shows network model visualizing synchronization phenomena. We can confirm that network is classified three groups which are synchronous, anti-phase synchronous and asynchronous.

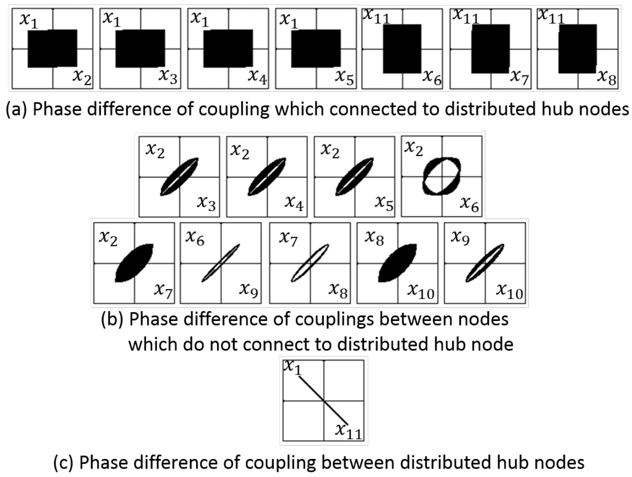


Figure 8: Phase difference of coupled van der Pol oscillators.

5. Conclusion

In this study, we have investigated synchronization phe-

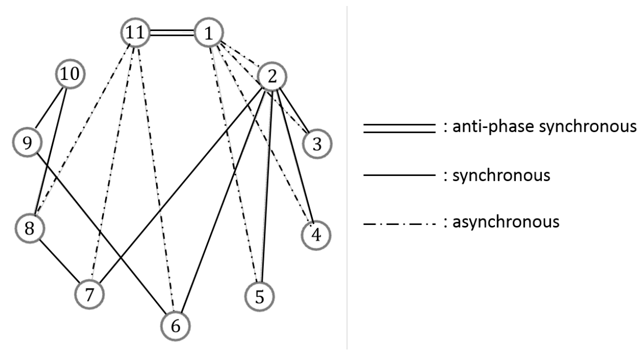


Figure 9: Network model visualizing synchronization phenomena.

nomena of coupled van der Pol oscillators. We used network whose hub node is replaced with two distributed hub nodes and distributed hub nodes are connected negative resistor. Furthermore, we compared synchronization phenomena of coupled van der Pol oscillators and that of coupled chaotic circuits which is already researched in previous study.

In the result of synchronization phenomana of the coupled van der Pol oscillators, we can observe synchronization phenomena changes due to differences in the structure of the coupling. At first, the coupling between distributed hub nodes which connected by negative resistor is anti-phase synchronization. Then the couplings which connected to distributed hub nodes are asynchronous and the couplings between nodes which do not connect to distributed hub node are almost synchronous. This result is similar to the result of coupled chaotic circuits of our previous study. However, it is necessary to investigate more detailed conditions of the coupling strength and initial value.

Ackowledgment

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