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Amplitude Death in Large-Scale Polygonal Oscillatory Networks

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Abstract—The study of AD is important for understanding the control mechanisms and efficient regulators of a system's dynamics. In this study, we investigate the occurrence of AD in two coupled large-scale oscillatory networks.

1. Introduction

Synchronization phenomena in coupled oscillators are suitable models for analyzing a number of natural occurrences [1],[2]. Therefore, many researchers have proposed different coupled oscillatory networks, and some interesting synchronization phenomena have been discovered [3]-[6].

Oscillation quenching (oscillation and amplitude death), another fundamental emergent phenomenon in coupled nonlinear systems, can be caused by several factors [7],[8]. Amplitude death (AD) occurs in strongly coupled nonlinear oscillators when their interaction causes a pair of fixed points to become stable and attracting. Setou et al. reported AD in ring coupled oscillators when the frequencies of the coupled units differ [9].

We have investigated synchronization phenomena in coupled polygonal oscillatory networks that share branches [10], [11]. In this system, van der Pol oscillators are connected to every corner of each polygonal network. The first and the second oscillators, which are connected to both polygonal networks, are called "shared oscillators," and each polygonal network has an odd number of oscillators. We then observe N-phase synchronization. Through computer simulations and theoretical analysis, we confirmed that the coupled oscillators tended to synchronize to minimize the power consumption of the whole system. The phase difference of the shared oscillators was determined by finding the minimum value of the power consumption function. Additionally, we proposed a new polygonal circuit system that includes actual inductor models (with loss) at all ground parts [12]. Synchronization phenomena in coupled polygonal oscillatory networks with strong frustration are investigated. Strong frustration is realized using conflicting coupling organization in the network and by increasing the coupling strength.

We confirmed that the amplitude of the oscillators decreases as the value of the coupling strength increases, and that AD occurs in the polygonal oscillatory networks. If one of the polygonal networks is triangular, we observe global AD. However, for other types of networks, AD appears in a complicated way. First, AD occurs at the oscillators located farthest from the shared oscillators. Next, AD occurs simultaneously in all other oscillators as the coupling strength increases. We explained the mechanism by which AD occurs using a theoretical approach.

In this study, we investigate the occurrence of AD in more large-scale networks. For considering the large-scale networks, the number of oscillators connected to the one side of polygonal network is increased until around M = 101. By using computer simulations, we observe similar AD phenomena in the large-scale polygonal network with previous studies.

2. Two Coupled Small-Scale Oscillatory Networks [12]

In our previous study, we investigated AD in two coupled small-scale oscillatory networks. Two polygonal oscillatory networks are coupled by sharing a branch, as shown in Fig. 1. In this circuit model, we consider the coupling method in which two adjacent oscillators tend to synchronize in the anti-phase state. The number of oscillators coupled to left and the right polygonal networks is denoted by N and M, respectively. The values of N and M are set to odd numbers to produce N or M phase synchronizations. The first and second oscillators, which are connected to both sides of the polygonal networks, are called "shared oscillators."

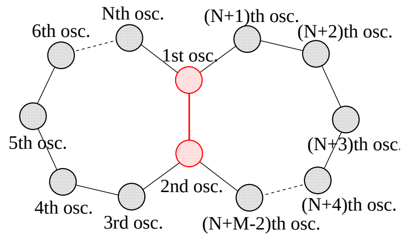
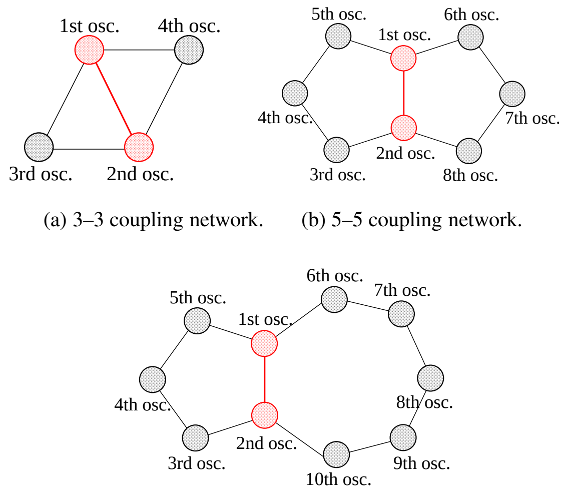


Figure 1: Two coupled oscillatory networks (N–M coupling network).

Examples of the network models used in this study are shown in Fig. 2. Here, the 3–3 and 5–5 coupling networks are symmetric models and the 5–7 coupling network is an asymmetric model.



(a) 3–3 coupling network. (b) 5–5 coupling network.

(c) 5–7 coupling network (N=5, M=7).

Figure 2: Examples of two coupled small-scale oscillatory networks.

Figure 3 shows the circuit model of the 3–3 coupling network. The earth resistances are inserted to the 3rd and 4th oscillators to model actual inductors and realize symmetry in the circuit network model. Tiny resistors (r_m) are inserted to avoid an L-loop in the computer simulations.

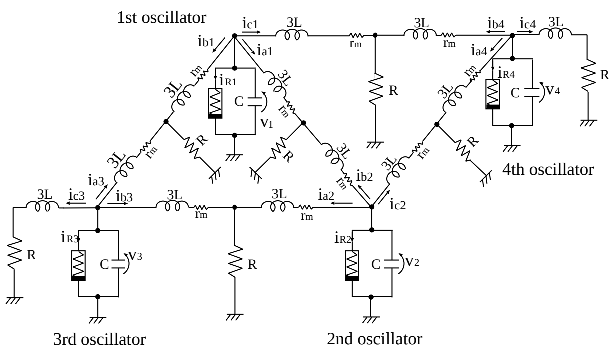


Figure 3: Coupling model (3–3 coupling network).

Next, we develop an expression for the circuit equations of the N–M coupling oscillatory network in Fig. 3. The $v_k - i_{Rk}$ characteristics of the nonlinear resistor are approximated by the following third-order polynomial equation:

$$i_{Rk} = -g_1 v_k + g_3 {v_k}^3 \quad (g_1, g_3 > 0), \qquad (1)$$
$$(k = 1, 2, 3, ..., N + M - 2).$$

Using the variables and parameters

$$t = \sqrt{LC}\tau, \quad v_k = \sqrt{\frac{g_1}{3g_3}} x_k,$$

$$i_{ak} = \sqrt{\frac{g_1}{3g_3}} \sqrt{\frac{C}{L}} y_{ak}, \quad i_{bk} = \sqrt{\frac{g_1}{3g_3}} \sqrt{\frac{C}{L}} y_{bk},$$

$$i_{ck} = \sqrt{\frac{g_1}{3g_3}} \sqrt{\frac{C}{L}} y_{ck}, \quad i_n = \sqrt{\frac{g_1}{3g_3}} \sqrt{\frac{C}{L}} y_n,$$

$$\varepsilon = g_1 \sqrt{\frac{L}{C}}, \quad \gamma = R\sqrt{\frac{C}{L}}, \quad \eta = r_m \sqrt{\frac{C}{L}},$$
$$(k = 1, 2, 3, ..., N + M - 2),$$

the normalized circuit equations governing the circuit are expressed as:

$$\begin{cases} \dfrac{dx_k}{d\tau} = \varepsilon\left(1 - \dfrac{1}{3}{x_k}^2\right)x_k - (y_{ak} + y_{bk} + y_{ck}) \\ \dfrac{dy_{ak}}{d\tau} = \dfrac{1}{3}\left\{x_k - \eta y_{ak} - \gamma(y_{ak} + y_n)\right\} \\ \dfrac{dy_{bk}}{d\tau} = \dfrac{1}{3}\left\{x_k - \eta y_{bk} - \gamma(y_{bk} + y_n)\right\} \\ \dfrac{dy_{ck}}{d\tau} = \dfrac{1}{3}\left\{x_k - \eta y_{ck} - \gamma(y_{ck} + y_n)\right\} \\ \qquad (k = 1, 2, 3, ..., N + M - 2). \end{cases} \qquad (2)$$

In this equation, γ is the coupling strength, ε denotes the nonlinearity of the oscillators, and y denotes the current of the inductor of the connected oscillator with the kth oscillator. For the computer simulations, we calculate Eq. (2) using the fourth-order Runge–Kutta method with step size $h = 0.005$. We set the parameters of this circuit model to $\varepsilon = 0.1$ and $\eta = 0.0001$. The coupling strength γ between the oscillators changes from a small to a large value.

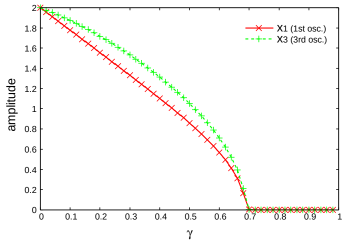
Figure 4 shows the change in amplitude according to the size of the network. In the case of the 3–3 coupling network, global AD occurs at the same time, whereas in the 5–5 and 5–7 coupling networks, AD first occurs in the oscillators located farthest from the shared oscillators, and then the other oscillators stop oscillating at the same time.

3. Two Coupled Large-Scale Oscillatory Networks

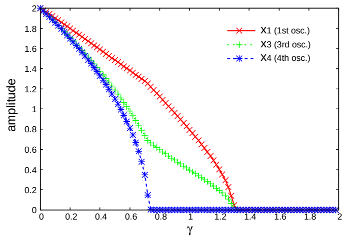
In this section, we investigate the occurrence of AD in more large-scale networks. Figure 5(a) shows the circuit model which is that triangular network and odd number polygonal network are coupled with one branch. Where M denotes the number of coupled oscillators of odd polygonal network. Similarly, Fig. 5(b) and (c) show the circuit model which is that pentagonal and heptagonal network are used for left side of polygonal network.

Figure 6 shows the change in amplitude when the number of oscillators M is set to 101. In the case of the triangle network, global AD occurs at the same time, whereas in the pentagonal and heptagonal networks, AD first occurs in the oscillators located farthest from the shared oscillators, and then the other oscillators stop oscillating at the same time. These results are similar with the small-scale network model. Namely, we can say that these AD phenomena do not depend on the number of oscillators in polygonal networks.

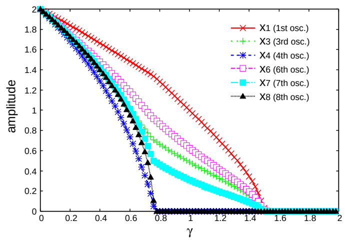
Finally, we investigate the coupling strength when the first and second AD occurs. The simulation results are



(a) 3–3 coupling network.

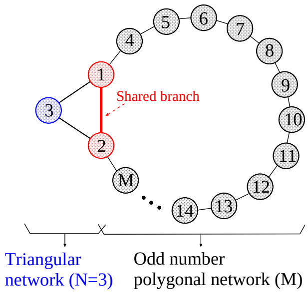


(b) 5–5 coupling network.

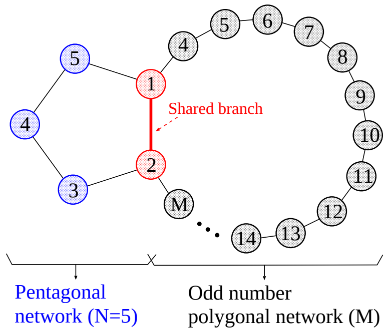


(c) 5–7 coupling network.

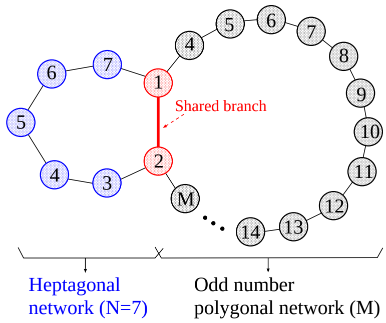
Figure 4: Amplitude of two coupled small-scale oscillatory networks.



(a) 3–3 coupling network (N=3, M=3, 5, 7, ... ,101).



(b) 5–5 coupling network (N=5, M=5, 7, 9, ... ,101).



(c) 5–7 coupling network (N=5, M=7, 9, 11, ... ,101).

Figure 5: Examples of two coupled large-scale oscillatory networks.

shown in Fig. 7. In the case of triangular network (Fig. 7 (a)), the coupling strength of global AD increases with the number of oscillators in right side polygonal network. If M is larger than 20 the coupling strength of AD converges to 1.0. While, in the cases of pentagonal and heptagonal networks, the coupling strength of first AD increases with the number of oscillators in right side polygonal network. However, the coupling strength of second AD is almost same even if the number of oscillators in right side polygonal network is increased. The coupling strength of second AD of heptagonal network has larger value than the pentagonal network.

4. Conclusions

In this study, we investigated the occurrence of AD in more large-scale networks. For considering the large-scale networks, the number of oscillators connected to the one side of polygonal network is increased until around M = 101. By using computer simulations, we observed similar AD phenomena in the large-scale polygonal network with previous studies.

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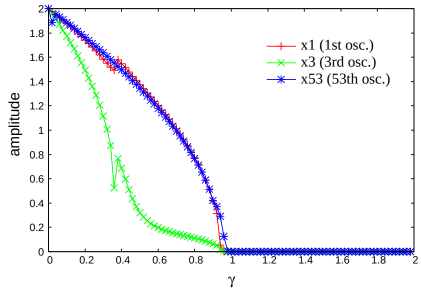
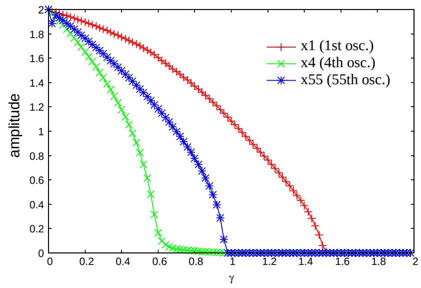
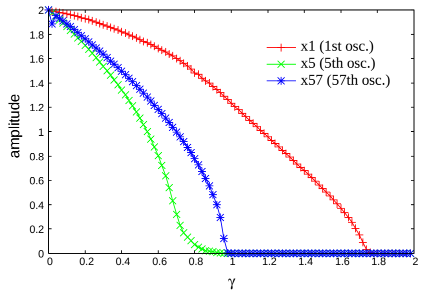


(a) 3–101 coupling network.

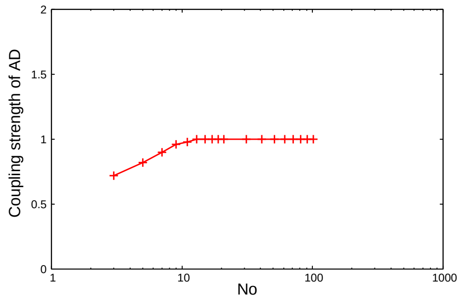


(b) 5–101 coupling network.

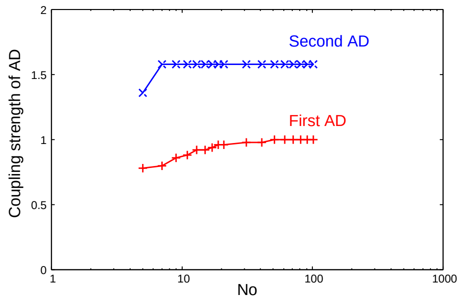


(c) 5–101 coupling network.

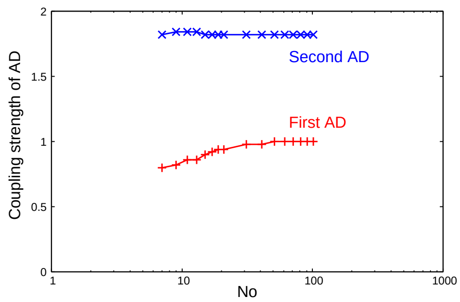
Figure 6: Amplitude of two coupled large-scale oscillatory networks.



(a) Triangle network (global AD).



(b) Pentagonal network (partial AD).



(c) Heptagonal network (partial AD).

Figure 7: Coupling strength at AD.

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