

# Influence of Coupling Strengths on Synchronization Phenomena in Two Rings of Coupled van der Pol Oscillators

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**Abstract**—In this study, we investigate the synchronization phenomena in two rings of van der Pol oscillators coupled by resistors. We propose a novel coupled oscillatory system comprising two rings of van der Pol oscillators with different coupling schema. We focus on the coupling strengths of the coupled van der Pol oscillators. By computer simulation, we investigate how the synchronization phenomena change by changing the coupling strengths. In these results, we observe various synchronization phenomena.

## 1. Introduction

The synchronization phenomena of coupled oscillators are familiar. Synchronization phenomena have been studied in various fields for many years, such as in electrical systems, mechanical systems and biological systems. Among them, the synchronization phenomena of van der Pol oscillators are similar to natural phenomena when the frequency is changed. A coupled system of van der Pol oscillators is simple and easy to handle. Many researchers have proposed various coupled oscillatory networks of van der Pol oscillators [1]-[3]. However, there are many portions not yet been clarified with respect to the phenomenon that the mechanism and attendant. Therefore, we investigate synchronization phenomena between two rings of van der Pol oscillators where different synchronization phenomena occur.

The van der Pol oscillator is a simple circuit. It consists of a resistor, inductor, capacitor and nonlinear resistor. It was invented by the electrical engineer Balthasar van der Pol. The equation of a van der Pol oscillator is a second-order differential equation.

In this study, we propose coupled oscillatory system including two rings of van der Pol oscillators. The ring consists of van der Pol oscillators connected by inductors and resistors. The phase difference of the ring depends on number of oscillators inside ring. When the number of oscillators is an odd number, the oscillators of the ring exhibit some phase synchronization. For example, five oscillators of the ring exhibit five-phase synchronization. We coupled two rings of different size by the resistor. Because we consider changes in synchronization phenomena due to difference in circuit structure, we consider two kinds of circuit

structures. We consider  $(2k + 1) - (2k + 3)$  coupled van der Pol oscillators ( $k = 1, 2$ ). By computer simulation, we investigate the synchronization phenomena observed in the proposed circuit system by changing the coupling strengths between two rings.

## 2. System Model

Figure 1 shows the circuit used in this study. We call this circuit NC. Figure 2 shows model of the system. We use two rings of van der Pol oscillators. The NC of the first and the second ring are connected by inductors and resistors. The first ring consist of  $2k + 1$  NC and the second ring consists of  $2k + 3$  NC ( $k = 1, 2$ ). We coupled two rings of different size by the resistors  $R$ . We observe the synchronization phenomena of adjacent oscillators. We investigate how the synchronization phenomena change upon changing the value of  $R$ .

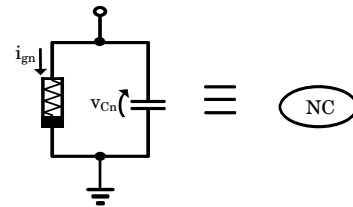


Figure 1: The circuit used in this study.

The circuit equations of the first ring are given as follows:

$$\begin{cases} C \frac{dv_{1n}}{dt} &= -i_{g1n} - i_{1an} \\ &-i_{1bn} - \frac{1}{R}(v_{1n} - v_{2n}) \\ 2L \frac{di_{1an}}{dt} &= v_{1n} - (i_{1an} + i_{1b(n+1)})R' \\ 2L \frac{di_{1bn}}{dt} &= v_{1n} - (i_{1bn} + i_{1a(n-1)})R'. \end{cases} \quad (1)$$

The circuit equations of the second ring are given as follows:

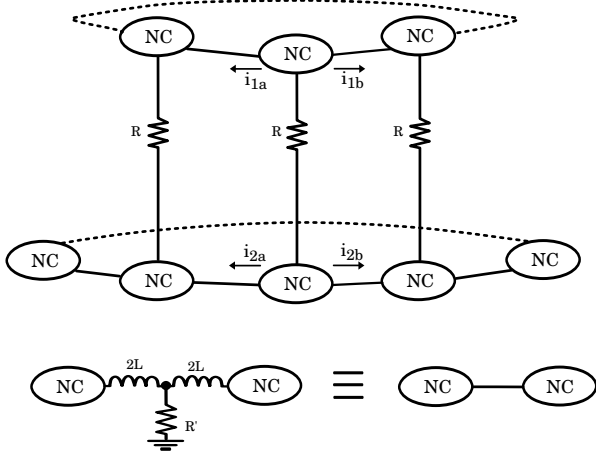


Figure 2: System model.

$$\left\{ \begin{array}{l} C \frac{dv_{2n}}{dt} = -i_{g2n} - i_{2an} - i_{2bn} \\ \quad -\frac{1}{R}(v_{2n} - v_{1n}) \\ \quad (n = 1 \sim 2n + 1) \\ C \frac{dv_{2n}}{dt} = -i_{g2n} - i_{2an} - i_{2bn} \\ \quad (n = 2n + 2, 2n + 3) \\ 2L \frac{di_{2an}}{dt} = v_{2n} - (i_{2an} + i_{2b(n+1)})R' \\ 2L \frac{di_{2bn}}{dt} = v_{2n} - (i_{2bn} + i_{2a(n-1)})R' \end{array} \right. \quad (2)$$

where  $1n$  and  $2n$  denote the number of the circuit of the first ring and the second ring ( $1n = 1, 2, \dots, 2k + 1$ ,  $2n = 1, 2, \dots, 2k + 3$ .)

A nonlinear resistor defined as follows:

$$i_{gn} = -g_1 v_n + g_3 v_n^3. \quad (3)$$

By changing the variables and parameters as follows.

$$\begin{aligned} t &= \sqrt{LC}\tau, v_n = \sqrt{\frac{g_1}{3g_3}} x_n, i_{an} = \sqrt{\frac{g_1 C}{3g_3 L}} y_{an}, \\ i_{bn} &= \sqrt{\frac{g_1}{3g_3 L}} y_{bn}, \varepsilon = g_1 \sqrt{\frac{L}{C}}, \\ \gamma &= \frac{1}{R} \sqrt{\frac{L}{C}}, \alpha = R' \sqrt{\frac{C}{L}} \end{aligned} \quad (4)$$

The normalized equations of the first ring are given as:

$$\left\{ \begin{array}{l} \dot{x}_{1n} = \varepsilon(x_{1n} - x_{1n}^3) - y_{1an} \\ \quad -y_{1bn} + \gamma(x_{1n} - x_{2n}) \\ \dot{y}_{an} = x_{1n} - \beta(y_{1an} + y_{1b(n+1)}) \\ \dot{y}_{bn} = x_{1n} - \beta(y_{1bn} + y_{1a(n-1)}) \end{array} \right. \quad (5)$$

and the normalized equations of the second ring are given as:

$$\left\{ \begin{array}{l} \dot{x}_{2n} = \varepsilon(x_{2n} - x_{2n}^3) - y_{2an} \\ \quad -y_{2bn} + \gamma(x_{2n} - x_{1n}) \\ \quad (n = 1 \sim 2n + 1) \\ \dot{x}_{2n} = \varepsilon(x_{2n} - x_{2n}^3) - y_{2an} - y_{2bn} \\ \quad (n = 2n + 2, 2n + 3) \\ \dot{y}_{2an} = x_{2n} - \beta(y_{2an} + y_{2b(n+1)}) \\ \dot{y}_{2bn} = x_{2n} - \beta(y_{2bn} + y_{2a(n-1)}) \end{array} \right. \quad (6)$$

where the parameters  $\varepsilon$  is non-linear strength. The parameters  $\alpha$  and  $\gamma$  denote the coupling strengths of the resistor  $R'$  and resistor  $R$ , respectively.

## 2.1. Simulation Results

First, we show the simulation results in  $k = 1$ . Figure 3 shows system model of  $k = 1$ . In this model, the first ring consists of 3 circuits and the second ring consists of 5 circuits.

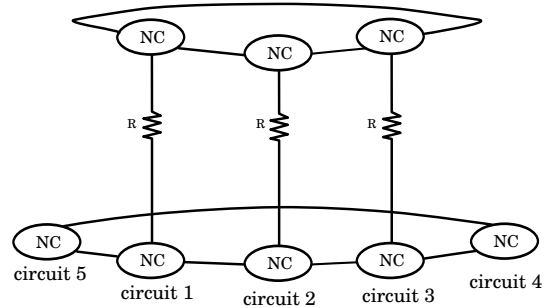


Figure 3: System model ( $k = 1$ ).

The simulation results of the system model are shown in Figs.4-6. The parameters are set to  $\varepsilon = 0.01$ ,  $\alpha = 0.01$ . We change the value of  $\gamma$  from 0.0000 to 0.0012 at intervals of 0.0002. We fix the value of initial value. Figure 4 shows the phase difference of the second ring. We focus on circuits connected to the first ring. When the coupling strength  $\gamma$  is increased, it can be observed that the phase difference of

circuit 1-circuit 2 and circuit 2-circuit 3 decrease. Figure 5 shows the time waveform of the first ring and the second ring, in the case of  $\gamma = 0.0000$ . The oscillators of first ring exhibit three-phase synchronization and the oscillators of second ring exhibit five-phase synchronization. Figure 6 shows the time waveform of the first ring and the second ring, in the case of  $\gamma = 0.005$ . The oscillators of first ring exhibit three-phase synchronization. Circuit 1, circuit 2 and circuit 3 of second ring exhibit the synchronization similar to three-phase synchronization when the coupling strength  $R$  increases. When  $k$  was large, we were able to observe the synchronization phenomena similar to the synchronization phenomenon of the first ring.

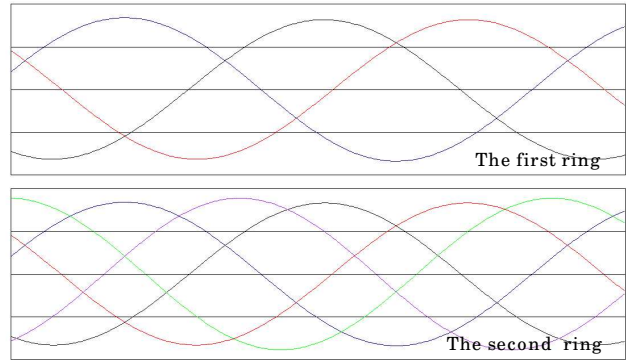


Figure 6: Time waveform ( $\gamma = 0.0012$ ).

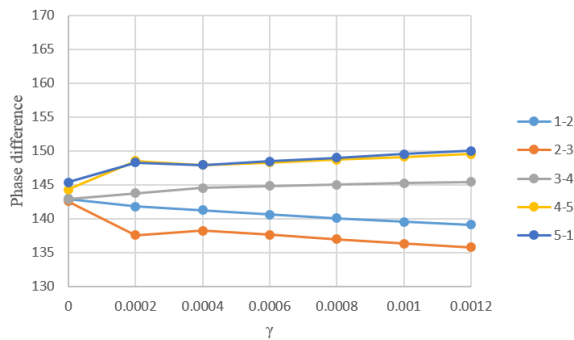


Figure 4: Phase difference of the second ring ( $k = 1$ ).

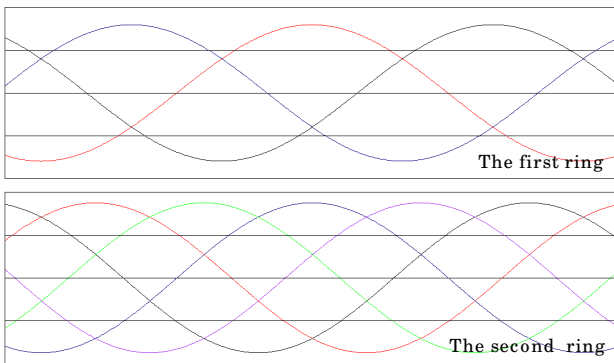


Figure 5: Time waveform ( $\gamma = 0.0000$ ).

Next, we show simulation result in  $k = 2$ . Figure 7 shows system model of  $k = 2$ . In this model, the first ring consists of 5 circuits and the second ring consists of 7 circuits.

Figure 8 shows the phase difference of the second ring. Circuits the second ring connected to the first ring exhibit the synchronization similar to five-phase synchronization when the coupling strength  $R$  increases. It is easier to observe this phenomena of  $k = 1$  than result of  $k = 2$ .

Circuits of the second ring connected to the first ring are affected by the first ring. When the coupling strength  $R$  increases, in the circuit of the second ring connected to the first ring, the synchronization phenomena similar to

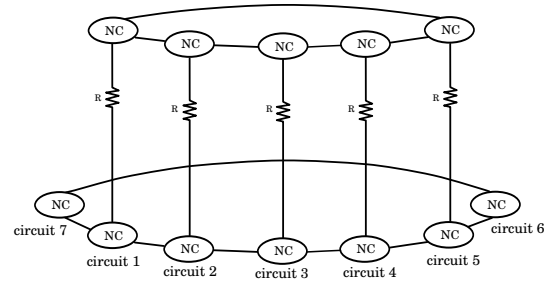


Figure 7: System model ( $k = 2$ ).

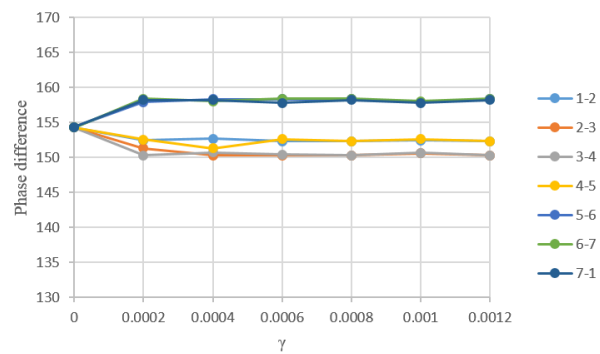


Figure 8: Phase difference of the second ring ( $k = 2$ ).

the synchronization phenomenon of the first ring exhibit. When  $k$  was large, we were able to observe the synchronization phenomena similar to the synchronization phenomenon of the first ring.

### 3. Conclusion

We have proposed a system model using two rings of coupled van der Pol oscillators. We observe time waveform and phase difference with using computer simulation. We observe various synchronization phenomena by changing the coupling strengths. Circuits of the second ring connected to the first ring are affected by the first ring. When the coupling strength  $R$  increases, in the circuit of the second ring connected to the first ring, the synchronization

phenomena similar to the synchronization phenomenon of the first ring exhibit. When  $k$  was large, we were able to observe this phenomena. In the future, we will investigate synchronization phenomena using other parameters and analyze the proposed circuit model.

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### **References**

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