Producing Complex Networks Using Coupled Oscillatory Circuits with Evolutionary Connections

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***Abstract*— In this study, we propose a method of generating complex networks by exploiting synchronization between coupled oscillatory circuits. To each node of a 2D fully connected network a van der Pol oscillator is assigned. We then study the topological evolution of the network in dependence on environmental conditions. These conditions are modeled by considering the distance between the oscillators and some small frequency errors that are added. By carrying out computer simulations, we confirm that different types of complex networks are obtained depending on different environmental conditions.**

I. Introduction

The synchronization phenomena observed from coupled oscillators are suitable models to analyze high-dimensional nonlinear phenomena [1], [2]. Many different coupled oscillatory networks have been proposed in the literature and a plethora of interesting synchronization phenomena have been discovered [3], [4]. Synchronization in dynamical complex networks has attracted a great deal of attention from various fields such as biology and neuroscience [5]-[8], engineering [9] and physics [10]-[13], economy [14] and social sciences [15]-[17]. In particular, understanding the relation between the topological structure and the functional behavior of a network is considered a significant topic for practical applications in many disciplines.

In a previous study, we have investigated synchronization phenomena in dynamical polygonal circuit networks with switching couplings [18]. We confirmed that new synchronization states can be produced when the network topology is changed by switching connections in the proposed system. Furthermore, we focused on brain networks as a particular example of dynamical complex networks. We applied three different types of coupling distributions: uniform, Gaussian and heavy tail. It was confirmed that the dynamical network with a heavy tail coupling distribution tended to avoid global synchronization [19]. Although these results are interesting, the proposed dynamical network was restricted to on/off couplings and the network topology did not grow up over time. In this contribution, we want to study evolutionary processes of networks depending on environmental conditions.

We propose a method of generating complex networks by using synchronization states of coupled oscillatory circuits. To this end, a van der Pol oscillator is assigned to each node of a given network. Initially, all nodes are fully connected with very weak coupling strengths. Additionally, small frequency errors are added to all oscillators. We then apply two rules for generating complex networks, inspired by existing real social networks. First, the connection between two nodes which are alike (that is, the frequencies of the two oscillators are very close), becomes stronger. Hence, the synchronization state between the two nodes is used to determine whether they are alike or not. Second, each node has an increasing influence on other nodes in a certain neighborhood. We then let the network evolve and investigate the final network structures, by changing the crucial parameters behind these two rules, namely frequency errors and neighbourhood distance. This two parameters define the environmental conditions.

By carrying out computer simulations, we confirm that different types of complex networks are obtained depending on the environmental conditions. We show that the generated networks vary substantially depending on these environmental conditions, even when the initial state of the network is the same. Hence, this model based on coupled oscillatory circuits offers an interesting model to study evolutionary processes in complex networks in general.

II. Coupled Circuits Network

A. Circuits Model

The initial arrangement of 100 van der Poll oscillators is shown in Fig. 1. All circuits are connected to each other by resistors. At the initial state, all nodes are fully connected with very weak coupling strengths. The van der Pol oscillator and the coupling method are shown in Fig. 2.

In the following, we introduce the circuit equations of the network model. The $v_k - i_{Rk}$ characteristics of the nonlinear resistor are approximated by the following third order polynomial equation,

$$i_k = -g_1 v_k + g_3 {v_k}^3 \quad (g_1, g_3 > 0), \qquad (k = 1, 2, ...13). \tag{1}$$

The normalized circuit equations governing the circuit are expressed as

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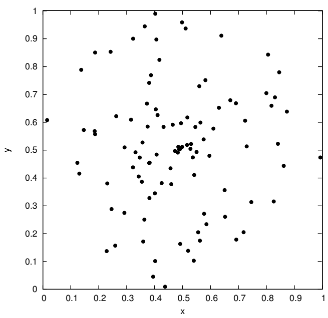
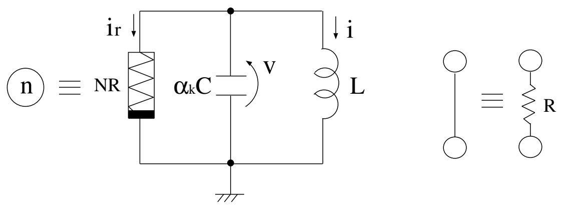


Fig. 1. Circuit arrangement in 2D space (N=100).



(a) van der Pol oscillator. (b) Coupling method.

Fig. 2. Circuit model.

[kth oscillator]

$$\begin{cases} \dfrac{dx_k}{d\tau} = \omega^2 \Big\{ \varepsilon \Big(1 - \dfrac{1}{3}{x_k}^2\Big) x_k - y_k \\ \qquad\qquad - (\gamma_0 + \Delta\gamma_s + \Delta\gamma_d) \displaystyle\sum_{k=1}^{n} y_k \Big\} \\ \dfrac{dy_k}{d\tau} = x_k \\ \qquad\qquad\qquad\qquad (k = 1, 2, ...N). \end{cases} \tag{2}$$

where

$$t = \sqrt{LC}\tau, \quad v_k = \sqrt{\frac{g_1}{3g_3}} x_k, \quad i_k = \sqrt{\frac{g_1}{3g_3}} \sqrt{\frac{C}{L}} y_k,$$

$$\alpha = \frac{1}{\omega^2}, \quad \varepsilon = g_1 \sqrt{\frac{L}{C}}, \quad \gamma = R\sqrt{\frac{C}{L}},$$

In these equations, γ_0 is the initial coupling strength. $\Delta\gamma_s$ and $\Delta\gamma_d$ denote the increasing coupling strengths influenced in the evolution process by the synchronization ratio and the neighborhood distance. ε denotes the nonlinearity of the oscillators and ω denotes the frequency error. The frequency error is set to 10%. For the computer simulations, we used a fourth-order Runge-Kutta method with step size $h = 0.01$. The parameter of nonlinearity of this circuit model is fixed to $\varepsilon = 0.1$. The number of van der Pol oscillators is set to $N = 100$.

B. Evolution Process of Network

We now explain the evolution process of the coupled circuits network. The network topology of the coupled circuits evolves according to the following steps.

[step-1] At the initial state, all nodes are fully connected with very weak coupling strengths (γ_0=0.0003).

[step-2] After a transient phase, we apply two rules for a sequence of generations. Each generation has length $\tau_g = 10,000$. The conceptual diagram of the computer simulation is shown in Fig. 3.

- (**Rule-1:** Influence of synchronization) In order to check whether two nodes are alike, we calculate the synchronization ratio for every pair of oscillators. If the synchronization ratio is larger than 90%, the corresponding coupling strength becomes stronger with $\Delta\gamma_s$.
  In order to analyze the synchronization ratio, we define a synchronization state as
  $$|x_k - x_n| < 0.1 \qquad (k \in S_n).$$
- (**Rule-2:** Influence of neighborhood distance) In order to apply the second rule (each node increasingly influences other nearby nodes), we calculate the distance between all pairs of nodes. If the distance is smaller than 0.1, the corresponding coupling strength $\Delta\gamma_d$ is increased.

[step-3] Step-2 is repeated until 100 generations are reached ($G = 100$).

[step-4] At the final state ($G = 100$), we check the synchronization ratio for every pair of oscillators. If the synchronization ratio is larger than 90%, the nodes are connected, otherwise the connection is cancelled. Finally, we obtain the network topology by affecting the frequency errors and distance.

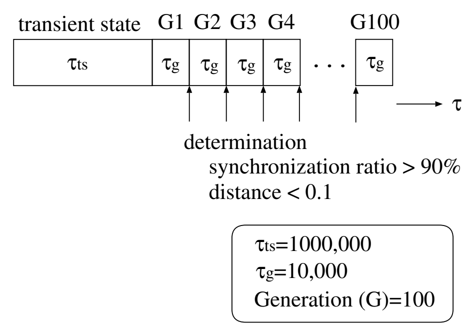
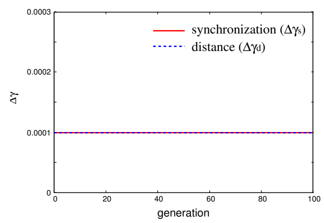
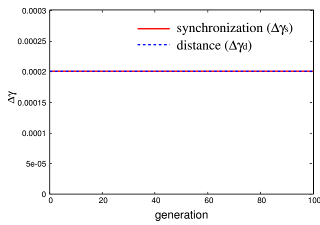


Fig. 3. Evolution process.

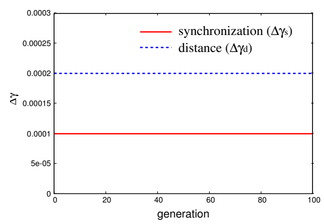
Next, we consider the methods of increasing the coupling strengths in each generation. Six types of methods are proposed as shown in Fig. 4. Figure 4 (a), (b) show the basic methods. The coupling strength change is the same for the influence of the synchronization and the distance condition. The coupling strength change shown in Fig. 4 (a) is $\Delta\gamma_s = \Delta\gamma_d = 0.0001$ and in Fig. 4 (b) is $\Delta\gamma_s = \Delta\gamma_d = 0.0002$. In the case of Fig. 4 (c), (d), the coupling strength change is different for the influence of the synchronization and the distance condition. The coupling strength change of the distance condition is twice as large as for the synchronization condition in Fig. 4 (c). While, in Fig. 4 (d), it is the other way around. In the last two methods, the coupling strength change for the distance and synchronization conditions varies with the generation sequence, as shown in Fig. 4 (e), (f). These six methods are named as Basic-1, 2, Constant-1, 2 and Varying-1, 2, respectively.



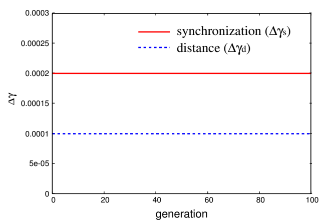
(a) Basic-1.
($\Delta\gamma_s = \Delta\gamma_d = 0.0001$)



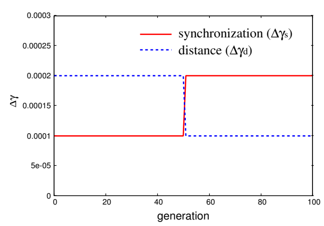
(b) Basic-2.
($\Delta\gamma_s = \Delta\gamma_d = 0.0002$)



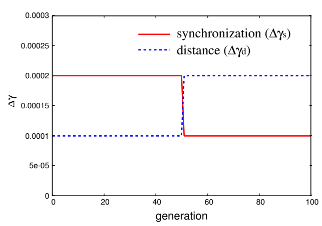
(c) Constant-1.
($\Delta\gamma_s = 0.0001, \Delta\gamma_d = 0.0002$)



(d) Constant-2.
($\Delta\gamma_s = 0.0002, \Delta\gamma_d = 0.0001$)



(e) Varing-1.



(f) Varing-2.

Fig. 4. Six methods of increasing the coupling strength depending on the influence of synchronization and distance (horizontal axis: generation, vertical axis: $\Delta\gamma$).

III. Simulation Results

For the computer simulations, we use 100 van der Pol oscillators as coupled circuits network and simulate for 20 different networks with frequency errors.

Table I summarizes the average network characteristics (average degree, average clustering coefficient and average path length). First, we compare the average degree of Basic-1 and Basic-2. When using a large coupling strength change, the average degree of Basic-2 is larger the average degree of Basic-1.

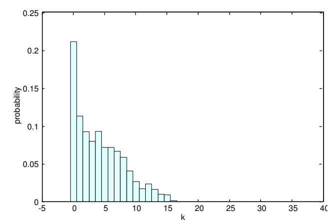
Next, we compare the average clustering coefficient and average path length of Constant-1 and Constant-2. Constant-2 has a large average clustering coefficient and a small average path length. Obviously, the network structure shows a small-world topology as the network has a high clustering coefficient and a low path length. From this result, we can see that the synchronization plays an important role for producing a small-world network.

A comparison of Varying-1 and Varying-2 reveals that they have a similar average degree and a similar average clustering coefficient. However, Varying-1 has a smaller average path length than Varying-2. Thus, Varying-1 also shows a small-world topology. To produce small-world topology, the network should be affected from synchronization at the end of the generations.

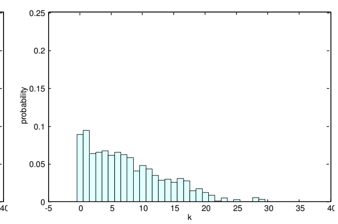
Figure 5 shows the average degree distributions of six methods for increasing the coupling strengths. From these figures, we confirm that the graphs do not follow a power-low scaling. The generated networks have no scale-free characteristics.

TABLE I
Network characteristics ($N = 100$)

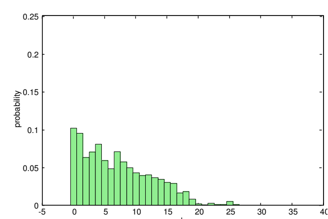
Network	Avg. Degree	Avg. Clustering Coefficient	Avg. Path Length
Basic-1	5.31	0.76	3.75
Basic-2	8.29	0.79	3.90
Constant-1	7.76	0.77	4.19
Constant-2	5.60	0.80	2.87
Varing-1	6.95	0.79	3.58
Varing-2	6.57	0.76	4.11



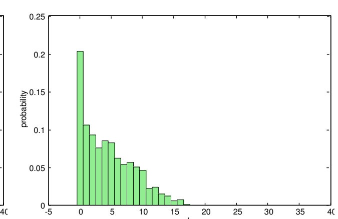
(a) Basic-1.



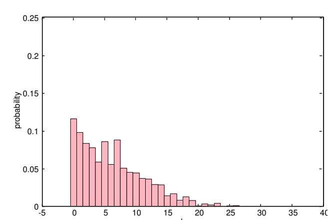
(b) Basic-2.



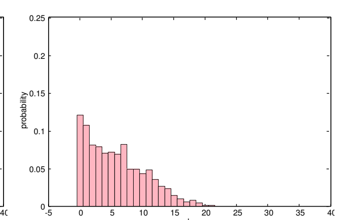
(c) Constant-1.



(d) Constant-2.



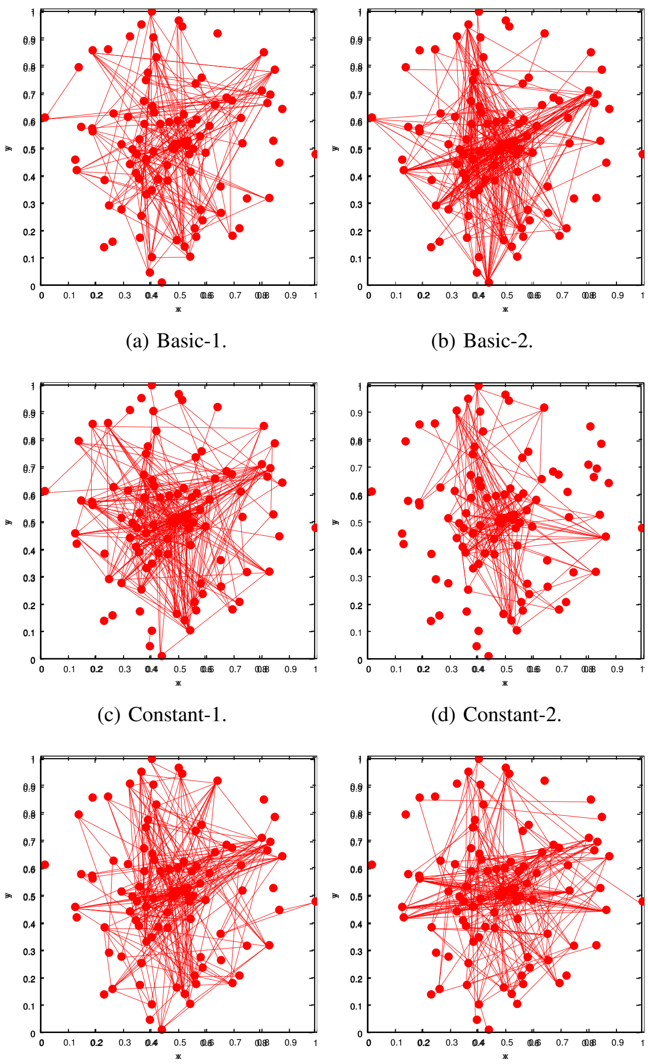
(e) Varing-1.



(f) Varing-2.

Fig. 5. Average degree distribution (horizontal axis: degree k, vertical axis: probability).

Figure 6 shows one example of the generated networks. Generally, we obtain different network topologies depending on the method for increasing the coupling strength. However, it is difficult to pick up the network characteristics. So, we use the layout algorithm called Fruchterman-Reingold Algorithm [20]. The idea of a force directed layout algorithm is to consider a force between any two nodes. In this algorithm, the nodes are represented by steel rings and the edges are springs between them. The generated network structures using Fruchterman-Reingold Algorithm are shown in Fig. 7. There are several clusters in each network. We can see that the nodes connect to nodes with similar degree. It is considered that the congenic selectivity of these networks is high.

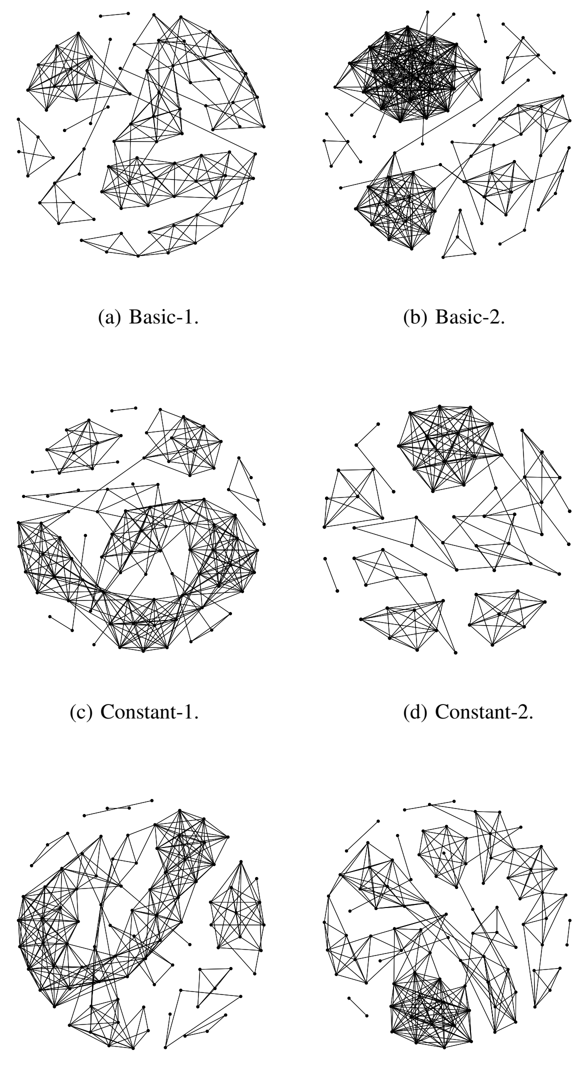


(a) Basic-1. (b) Basic-2.

(c) Constant-1. (d) Constant-2.

(e) Varing-1. (f) Varing-2.

Fig. 6. Produced network topologies.



(a) Basic-1. (b) Basic-2.

(c) Constant-1. (d) Constant-2.

(e) Varing-1. (f) Varing-2.

Fig. 7. Produced network topologies using Fruchterman-Reingold algorithm.

IV. Conclusions

In this study, we have proposed a method of generating complex networks by exploiting synchronization between coupled oscillatory circuits. To each node of a 2D fully connected network a van der Pol oscillator was assigned. We studied the topological evolution of the network in dependence on environmental conditions. These conditions were modeled by considering the neighborhood distance of the oscillators and some small frequency errors. By carrying out computer simulations, we confirmed that the network structure of Constant-2 showed a small-world topology depending on the two rules. It was also considered that the congenic selectivity of these produced networks was high.

In our future work, we would like to investigate the evolutional processes for creating complex networks using the proposed circuits system in detail. Furthermore, we would like to model a mechanism of opinion formation in complex networks by improving the proposed coupled nonlinear circuits system.

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