Synchronization Phenomena of Chaotic Circuit Networks with Distributed Hub Including Positive and Negative Coupling

Shuhei Hashimoto, Yoko Uwate, and Yoshifumi Nishio
Dept. of Electrical and Electronic Eng.,
Tokushima University
2-1 Minami-Josanjima,
Tokushima 770-8506, Japan
Email:{s-hashimoto, uwate, nishio}@ee.tokushima-u.ac.jp

***Abstract*—In this study, we focus on network structure with a hub of Scale-free network. We consider the synchronization of coupled chaotic circuit network whose hub nodes are split two nodes and we investigate two network models. In one of them, we replace a hub node with two distributed nodes connected by a positive resistor. In another, we replace a hub node with two distributed nodes connected by a negative resistor. From the simulation results, we can confirm that connection between hubs plays an important role in network structure.**

I. Introduction

Synchronization phenomena can be observed everywhere in our life. For example, we can confirm metronome, flashing firefly lights, beating rhythm of the heart and so on. Especially, synchronization phenomena of oscillatory network are interesting. In addition, complex networks attract attention from various fields. The feature of networks is characterized by the degree distribution, the path length and the clustering coefficient.

Numerous researches on complex networks have been made so far [1]-[7]. Especially, there are many studies related with hub nodes and these are very interesting. Because, in the real world, large-scale complex networks often include huge nodes with a large number of edges, and such "hub" nodes play key roles in the networks.

In this study, we pay our attention on the behavior of the networks whose hub nodes are split into several nodes. Namely, when such a hub node is split into several nodes, how other nodes connected to the hub node change their behavior? Furthermore, when such split hub nodes agree with or oppose to each other, how the whole network behaves to cooperate or to compete? We construct the complex networks by using coupled chaotic circuits and evaluate the behavior by using synchronization rate between a pair of the chaotic circuits. In the first model, we replace a hub node with two distributed hub nodes connected by a positive resistor. In the second model, we replace a hub node with two distributed hub nodes connected by a negative resistor. We evaluate these models by synchronization rate and compare their behaviors. In distributed hubs connected by a positive resistor, we compare the synchronization phenomena of the network whose hub node replaced with two distributed nodes and the network before splitting a hub. Further, we investigate the effect of the network structure around the hub node on the behavior of the whole network.

Next, as the third model, we replace a hub node with three distributed hub nodes. We connect three distributed hub nodes mutually and we make that connection ring hub. In this study, we investigate a network of three distributed nodes connected by positive/positive/negative resistors and positive/negative/negative resistors. We investigate the effect of the connection type on the behavior of the whole network.

II. Circuit model

The chaotic circuit model which proposed by Shinriki et $al.$ [8][9] is shown in Fig. 1. This circuit consists of a negative resistor, an inductor, two capacitors and dual-directional diodes. In this study, we use a network in which each node is replaced by a chaotic circuit. The circuit equation is shown in Eq. (1).

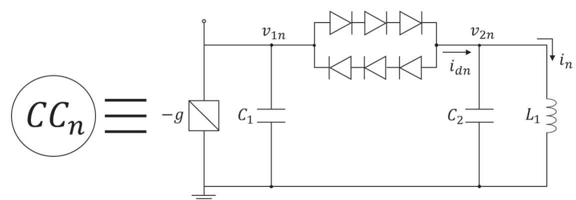


Fig. 1. Circuit model.

$$\begin{cases} L\dfrac{di_n}{dt} = v_{2n} \\ C_1\dfrac{dv_{1n}}{dt} = gv_{1n} - i_{dn} - \dfrac{1}{R}\displaystyle\sum_{k\in S_n}(v_{1n} - v_{1k}) \\ C_2\dfrac{dv_{2n}}{dt} = -i_n - i_{dn}, \end{cases} \quad (1)$$

978-1-5386-4881-0/18/$31.00 ©2018 IEEE

where $n = 1, 2, 3, \cdots, 10$ and S_n is the set of nodes which are directly connected to the node n. The characteristic of nonlinear resistor which consists of dual diodes is following Eq. (2).

$$i_{dn} = \begin{cases} G_d(n_{1n} - v_{2n} - V) & (v_{1n} - v_{2n} > V) \\ 0 & (|n_{1n} - v_{2n}| \le V) \\ G_d(n_{1n} - v_{2n} + V) & (v_{1n} - v_{2n} < -V). \end{cases} \quad (2)$$

By changing the variables and parameters,

$$\begin{cases} i_n = \sqrt{\frac{C_2}{L}} V x_n, \quad v_{1n} = V y_n, \quad v_{2n} = V z_n \\ \alpha = \frac{C_2}{C_1}, \quad \beta = \sqrt{\frac{L}{C_2}} G_d, \quad \gamma = \sqrt{\frac{L}{C_2}} g, \\ \delta = \frac{1}{R}\sqrt{\frac{L}{C_2}}, \quad t = \sqrt{LC_2}\tau, \quad ``\cdot" = \frac{d}{dt}, \end{cases} \quad (3)$$

the normalized equations of chaos circuits are given as follows:

$$\begin{cases} \dot{x} = z_n \\ \dot{y} = \alpha\gamma y_n - \alpha\beta f(y_n - z_n) - \alpha\delta \sum_{k \in S_n} (y_n - y_k) \\ \dot{z} = \beta f(y_n - z_n) - x_n. \end{cases} \quad (4)$$

The parameter δ corresponds the coupling strength between the circuits. The parameter γ is the bifurcation parameter of chaotic circuit. The nonlinear function $f(y_n - z_n)$ corresponds to the characteristics of the nonlinear resistor consisting of the diodes and described as follows:

$$f(y_n - z_n) = \begin{cases} y_n - z_n - 1 & (y_n - z_n > 1) \\ 0 & (|y_n - z_n| \le 1) \\ y_n - z_n + 1 & (y_n - z_n < -1). \end{cases} \quad (5)$$

This circuit generates asymmetric chaotic attractor in the parameter $\gamma = 0.5$ and symmetric chaotic attractor in the parameter $\gamma = 0.65$ as shown in Fig. 2.

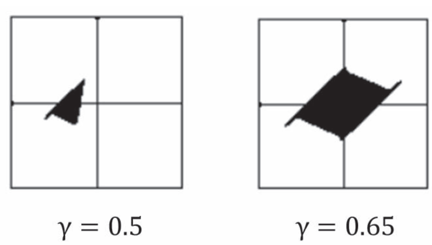
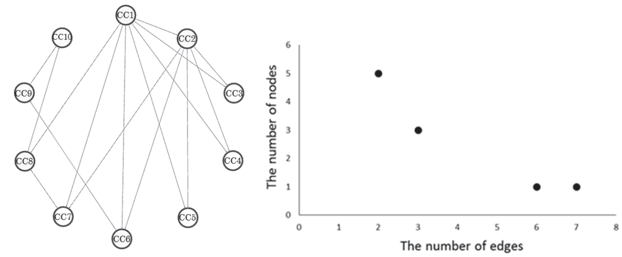


Fig. 2. Attractors with the different parameter.

III. Network model

We made the basic network which is 10 coupled chaotic circuits and each circuit connected by a resistor. This network model and the degree distribution of this network are shown in Fig. 3.



(a) Network model (b) The degree distribution

Fig. 3. Basic network model.

In this study, we propose network model which a hub node replaced with two distributed nodes. The example of splitting hub is shown in Fig. 4. Right network model in Fig. 5 is the network that applied method shown in Fig. 4 to CC1 as a hub of the basic network. Furthermore, we investigated synchronization of network in two different ways of connecting. In the first model, distributed hubs are connected by a positive resister and other nodes are connected by a positive resistor each other. In the second model, distributed hubs are connected by a negative resistor and other nodes are same at the first model.

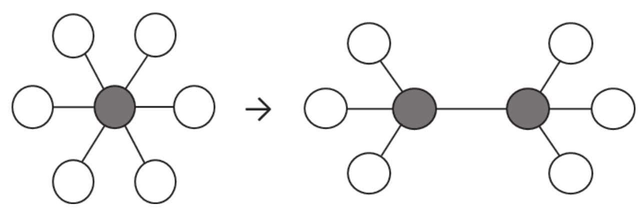


Fig. 4. Splitting a node.

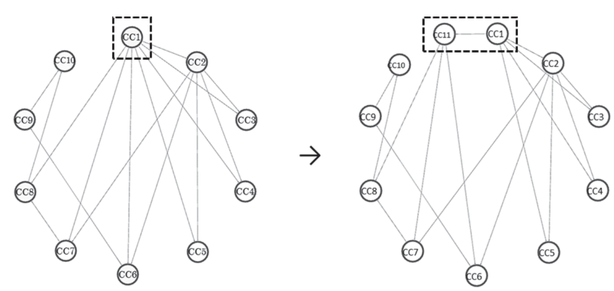


Fig. 5. Neteork model with distributed hub.

IV. SIMULATION RESULT

Definition of synchronization in this study is determined a voltage difference waveform. We define synchronization as the following Eq. (6).

$$|z_j - z_i| < 0.15 \qquad (i, j = 1, 2, \cdots, 10) \qquad (6)$$

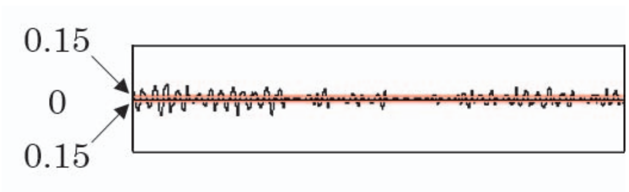


Fig. 6. A difference waveform.

Figure 6 is a different waveform which was observed in this simulation. The two lines in Fig. 6 correspond threshold which is given Eq. (6). It is determined that wave within two lines which is the threshold is synchronization. Therefore, we propose and investigate the synchronization probability denoted the synchronization rate during a certain time interval. In this research, we fix a certain time interval as (τ=1,000,000 and step=0.02τ) and investigate the synchronization rate in the entire network of 10 coupled chaotic circuits.

A. Comparing network with distributed hub and basic network

In this simulation, we use two bifurcation parameters of chaotic circuit γ. Attractors of these parameters are shown in Fig. 2. We compared synchronization rate of basic network and network with distributed hub nodes which connected by positive resistor. Where the coupling strength other than between distributed hub nodes is defined as δ and the coupling strength between distributed nodes is defined as δ'. In basic network, we measured synchronization rate by changing the coupling strength $\delta = 0.01$ to $\delta = 0.1$. In network with distributed hub, we measured synchronization rate by changing the coupling strength $\delta = 0.01$ to $\delta = 0.01$ and the coupling strength $\delta' = 0.02$ to $\delta' = 0.2$. Then we calculated difference of synchronization rate of basic network and network with distributed hub nodes in each coupling strength δ. Figure 7 shows average of synchronization rate of the coupling strength δ' in two parameters $\gamma = 0.5$ and $\gamma = 0.65$. In Fig. 7, we can observed that synchronization rate is almost unchanged, even if a hub is replaced with two distributed hub nodes.

B. Synchronization phenomena of network with distributed hub which connected by negative resistor

In this section, we observe synchronization phenomena of network with distributed hub nodes which connected by negative resistor. We measure synchronization rate of the network by changing the coupling strength $\delta' = -0.02$ to $\delta' = -0.2$ in the coupling strength $\delta = 0.1$. Then, we grouped edges that are similar in connection and we averaged synchronization

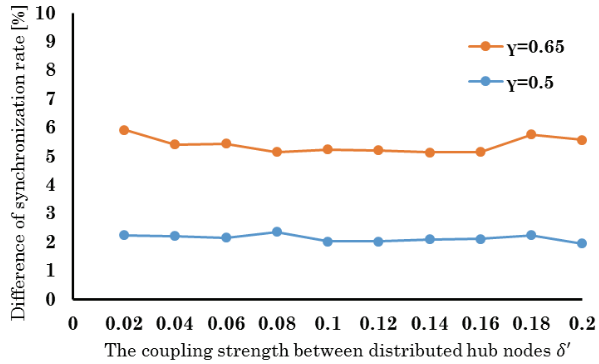


Fig. 7. Average of difference in synchronization rate.

rate for each group. The first group is edges connected to CC1 which is distributed hub. The second group is edges connected CC11 which is distributed hub. The third group is edges between the nodes connected to CC1 of distributed hub. The fourth group is edges between the nodes connected to CC11 of distributed hub. The fifth group is edges between node connected to CC1 and node connected to CC11, that is, connection between nodes which are coupled with nodes that are in asynchronous relationship.

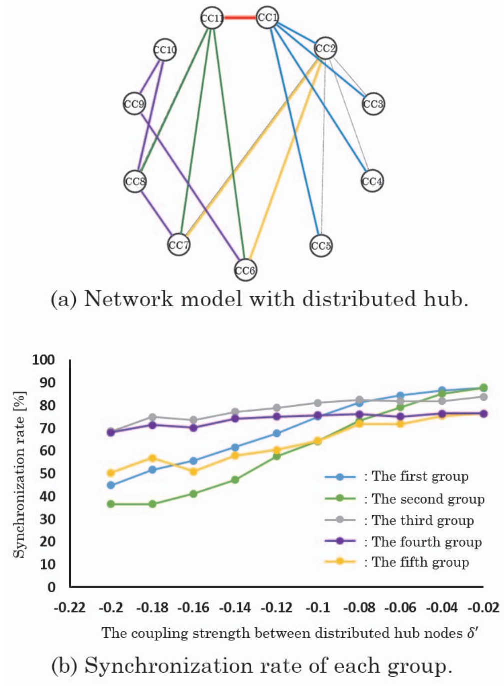
Figure 8 shows averages of synchronization rate of each group in the coupling strength $\delta' = -0.02$ to $\delta' = -0.2$. In Fig. 8, we can observed that synchronization rates of the first group and the second group are similar and that of the third group and the fourth group are similar. Synchronization rates of the first group and the second group are lower as the negative coupling is stronger. Synchronization rates of the third group and the fourth group are almost unchanged even if the negative coupling changed. Synchronization rates of the fifth group are lower as the negative coupling is stronger, although how to coupling of fifth group is similar to the third group and the fourth group. This results are considered that fifth group is connections between nodes which are coupled with nodes that are in asynchronous relationship.

C. Synchronization phenomena of extended network with ring hub

In this section, we investigated synchronization phenomena of network which have three distributed hub nodes in extended network ($N = 29$). Three distributed hub nodes are conneted each other and they are connected by positive/positive/negative resistors, positive/negative/negative resistors.

Figure 9 shows network model which used in this section. We grouped edges that are similar in connection with the previous section and we measure average synchronization rate for each group. The first group is edges connected to CC1 which is distributed hub. The second group is edges connected CC28 which is distributed hub. The Third group is edges connected to CC29 which is distributed hub. The fourth group is the edges between the nodes connected to CC1 of the distributed hub node. The fifth group is the edges between

978-1-5386-4881-0/18/$31.00 ©2018 IEEE



(a) Network model with distributed hub.

(b) Synchronization rate of each group.

Fig. 8. Synchronization rate of each group.

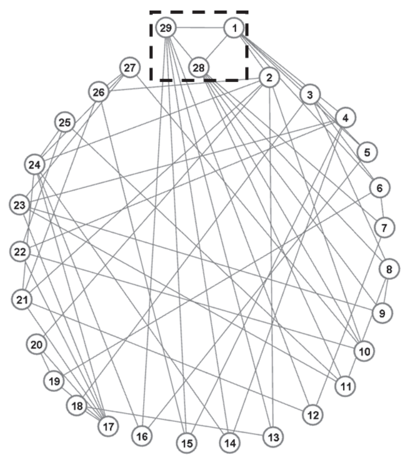


Fig. 9. Extended network model ($N = 29$).

the nodes connected to CC of the distributed hub node. The sixth group is edges between node connected to CC1 and node connected to CC28. The seventh group is edges between node connected to CC1 and node connected to CC29. The eighth group is edges between node connected to CC28 and node connected to CC29.

Figure 10 and 11 show average of synchronization rate of each group. In Fig. 10, CC1 and CC28, CC1 and CC29 are connected by positive resistor, CC28 and CC29 are connected by negative resistor. In Fig. 11, CC28 and CC29 are connected by positive resistor, CC1 and CC28, CC1 and CC29 are connected by negative resistor. From these results, groups of edge which connected to distributed hub with negative resistors obtained low synchronization rate like ten coupled chaotic circuits network.

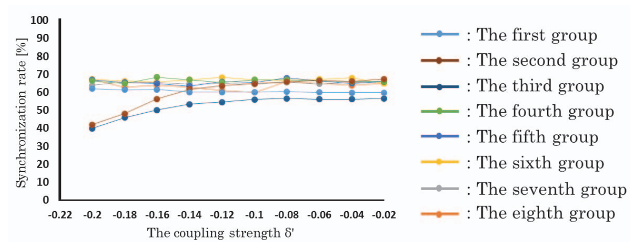


Fig. 10. Synchronization rate of each group in extended network (positive/positive/negative).

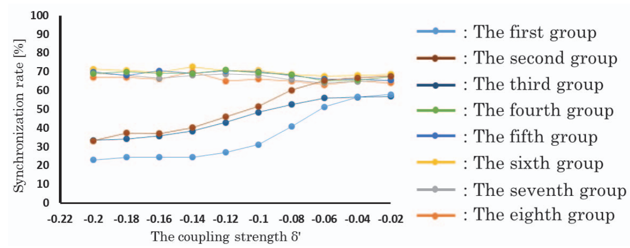


Fig. 11. Synchronization rate of each group in extended network (positive/negative/negative).

V. Conclusions

In this study, we have investigated synchronization phenomena of network whose hub node is replaced with two distributed hub nodes. Furthermore, we connected distributed hub nodes in two ways. In the first model, distributed hub nodes connected by a positive resistor. In the second model, distributed hub nodes connected by a negative resistor.

In the result of the first model, when we compared synchronization rate of undelying network and network with distributed hub, we can observed that synchronization rate is almost unchanged, even if a hub is replaced with two distributed hub nodes. In the simulation of the second model, we compare edges grouping similar connection. From the simulation result, synchronization rates of group which is connection between nodes which are coupled with nodes that are in asynchronous relationship are lower as the negative coupling is stronger. Furthermore, in the extended model, we can confirmed results similar to ten chaotic circuits network. However, we need further investigation because these results may change depending on the initial value.

As our future work, we investigate network with distributed hub nodes coupled by negative resistor in detail. And we need to investigate synchronization phenomena of network with distributed hub nodes in larger networks.

Acknowledgment

This work was partly supported by JSPS Grant-in-Aid for Scientific Research 16K06357.

References

[1] D. J. Watts and S. H. Strogatz, "Collective Dynamics of Small-world", Nature, vol.393, pp.440-442, 1998.
[2] A. L. Barabasi and R. Albert, "Emergence of Scaling in Random Networks", Science, vol.286, pp.509-512, 1999.
[3] J. Aguirre, R. Sevilla-Escoboza, R. Gutirrez, D. Papo, and J.M. Buld, "Synchronization of Interconnected Networks: The Rolw of Connector Nodes", Phys. Rev. Lett. 112, 248701 Published 16 June 2014.
[4] K. Ago, Y. Uwate, Y. Nishio, "Influence of Bridge on Coupled Chaotic Circuit Network", Proc. of RISP International Workshop on Nonlinear Circuits, Communications and Signal Processing (NCSP'14), pp. 417-420, Mar. 2014.
[5] K. Ago, Y. Uwate, Y. Nishio, "Investigation of Partical Synchronization in Coupled Chaotic Circuit Network with Local Bridge", Proc. of IEEE Workshop on Nonlinear Cirsuit Networks (NCN'14), pp.63-66, Dec. 2014.
[6] T. Chikazawa, Y. Uwate, Y. Nishio, "Chaos Propagation in Coupled Chaotic Circuits with Multi-Ring Combination", Proc. of IEEE Asia Pacific Conference on Circuits and Systems (APCCAS'16), pp. 65-68, Oct. 2016.
[7] S. Hashimoto, T. Chikazawa, Y. Uwate, Y. Nishio, "Synchronization Phenomena in Complex Networks of Coupled Chaotic Circuits with Different Degree Distribution", Proc. of RISP International Workshop on Nonlinear Circuits, Communications and Signal Processing (NCSP'17), pp. 333-336, Feb. 2017.
[8] M. Shinriki, M. Yamamoto and A. Mori, "Multimode Oscillations in a Modified van der Pol Oscillator Conraining a Positive Nonlinear Conductance", Proc. IEEE, vol. 69, pp. 394-395, 1981.
[9] N. Inaba, T. Saito and S. Mori, "Chaotic Phenomena in a Curcuit with a Negative Resistance and an Ideal Switch of Diodes", Trans. of IEICE, vol.E70, no. 8, pp. 744-754, 1987.