Synchronization of Coupled Chaotic Circuit Network with Positive and Negative Coupling

Shuhei Hashimoto , Yoko Uwate and Yoshifumi Nishio
Tokushima University, Tokushima 770-8501, Japan
* *E-mail: {s-hashimoto, uwate, nishio}@tokushima-u.ac.jp*

1. Introduction

Synchronization phenomena can be observed everywhere in our life. For example, we can confirm metronome, flashing firefly lights, beating rhythm of the heart and so on. Especially, synchronization phenomena of oscillatory network are interesting. In addition, complex networks attract attention from various fields. Numerous researches on complex networks have been made so far [1]-[7]. Especially, there are many studies related with hub nodes and these are very interesting. Because, in the real world, large-scale complex networks often include huge nodes with a large number of edges, and such "hub" nodes play key roles in the networks.

In this study, we focus on network structure with a hub which is consisted of Scale-free network. We consider the synchronization of coupled chaotic circuit network whose hub nodes are split two nodes and we investigate two network models. In one of them, we replace a hub node with two distributed nodes connected by a positive resistor. In another, we replace a hub node with two distributed nodes connected by a negative resistor. From the simulation results, we can confirm that connection between hubs plays an important role in network structure.

2. System model

The chaotic circuit model which proposed by Shinriki et al. [8][9] is shown in Fig. 1. This circuit consists of a negative resistor, an inductor, two capacitors and dual-directional diodes. In this study, we use a network in which each node is replaced by a chaotic circuit. The normalized equation of this chaotic circuits is shown in Eq. (1).

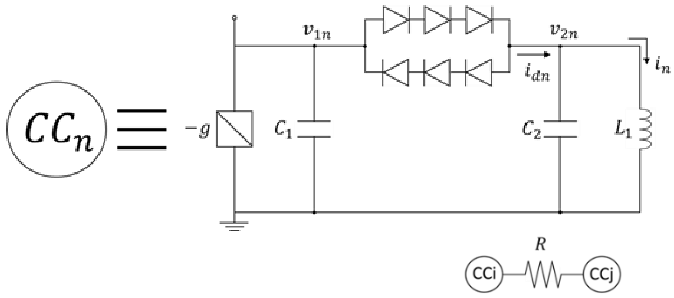
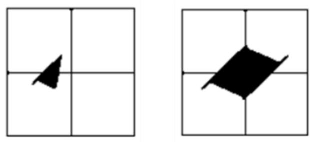


Fig. 1. Circuit model.



γ = 0.5 γ = 0.65

Fig. 2. Attractors with the different parameter.

$$\begin{cases} \dot{x}_n = z_n \\ \dot{y}_n = \alpha\gamma y_n - \alpha\beta f(y_n - z_n) - \alpha\delta \sum\limits_{k\in S_n}(y_n - y_k) \\ \dot{z}_n = \beta f(y_n - Z_n) - x_n \end{cases} \quad (1)$$

$$\begin{cases} i_n = \sqrt{\frac{C_2}{L}} V x_n \,, v_{1n} = V y_n \,, v_{2n} = V z_n \\ \alpha = \frac{C_2}{C_1} \,, \beta = \sqrt{\frac{L}{C_2}} G_d \,, \gamma = \sqrt{\frac{L}{C_2}} g \\ \delta = \frac{1}{R}\sqrt{\frac{L}{C_2}} \,, t = \sqrt{LC_2}\tau \,, \text{"}\cdot\text{"} = \frac{d}{d\tau} \end{cases} \quad (2)$$

Equation (2) shows parameters of this circuit and Fig. 2 shows chaotic attractor obtained from this circuit. The parameter δ corresponds the coupling strength between the circuits. The parameter α is the strength of negative resistor.

In this study, we propose network model which a hub node replaced with two distributed nodes. The example of splitting hub is shown in Fig. 4. Right network model in Fig. 5 is the network that applied method shown in Fig. 4 to CC1 as a hub of the basic network. Furthermore, we investigated synchronization of network in two different ways of connecting. In the first model, distributed hubs are connected by a positive resister and other nodes are connected by a positive resistor each other. In the second model, distributed hubs are connected by a negative resistor and other nodes are same at the first model.

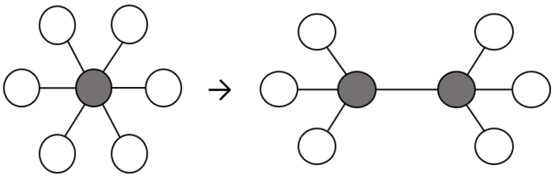


Fig. 4. Splitting a node.

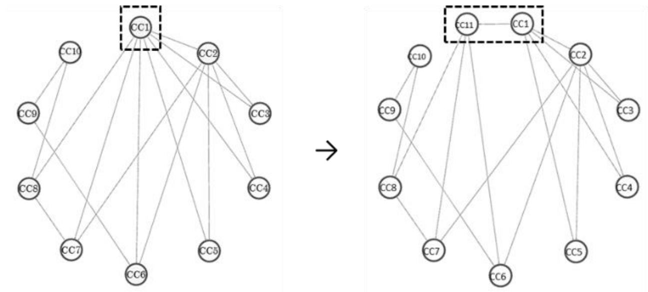
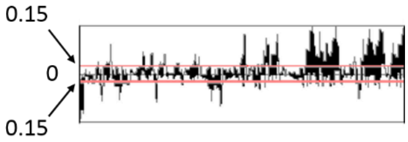


Fig. 5. Network model with distributed hub.

3. Simulation result

Definition of synchronization in this study is determined a voltage difference waveform. We define synchronization as the following Eq. (3).



$$|z_j - z_i| < 0.15 \qquad (i, j = 1,2,3, \dots, N) \qquad (3)$$

Fig. 6. A difference waveform

Figure 6 is a different waveform which was observed in this simulation. The two lines in Fig. 6 correspond threshold which is given Eq. (3). It is determined that wave within two lines which is the threshold is synchronization. Therefore, we propose and investigate the synchronization probability denoted the synchronization rate during a certain time interval. In this research, we fix a certain time interval as (τ=1,000,000 and step=0.02τ) and investigate the synchronization rate in the entire network of 10 coupled chaotic circuits.

3-1. Comparing network with distributed hub and basic network

In this simulation, we use two bifurcation parameters of chaotic circuit γ. Attractors of these parameters are shown in Fig. 2. We compared synchronization rate of basic network and network with distributed hub nodes which connected by positive resistor. Where the coupling strength other than between distributed hub nodes is defined as δ and the coupling strength between distributed nodes is defined as δ′. In basic network, we measured synchronization rate by changing the coupling strength δ = 0.01 to δ = 0.1. In network with distributed hub, we measured synchronization rate by changing the coupling strength δ = 0.01 to δ = 0.01 and the coupling strength δ′ = 0.02 to δ′ = 0.2. Then we calculated difference of synchronization rate of basic network and network with distributed hub nodes in each coupling strength δ. Figure 7 shows average of synchronization rate of the coupling strength δ′ in two parameters γ = 0.5 and γ = 0.65. In Fig. 7, we can observed that synchronization rate is almost unchanged, even if a hub is replaced with two distributed hub nodes.

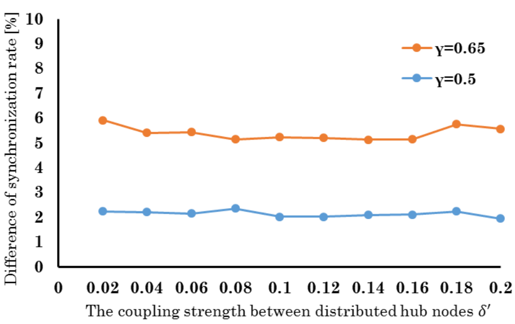


Fig. 7. Average of difference in synchronization rate.

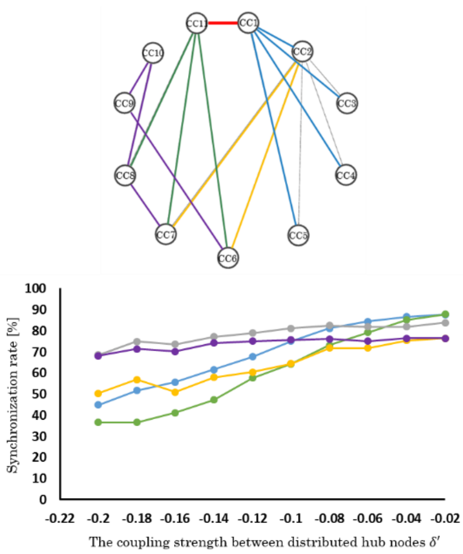


Fig. 8. Synchronization rate of each group.

3-2. Synchronization Phenomena of network with distributed hub which connected by negative resistor

In this section, we observe synchronization phenomena of network with distributed hub nodes which connected by negative resistor. We measure synchronization rate the network by changing the coupling strength $\delta' = -0.02$ to $\delta' = -0.2$ in the coupling strength $\delta = 0.1$. Then, we grouped edges that are similar in connection and we averaged synchronization rate for each group. The first group is edges connected to CC1 which is distributed hub. The second group is edges connected CC11 which is distributed hub. The third group is edges between the nodes connected to CC1 of distributed hub. The fourth group is edges between the nodes connected to CC11 of distributed hub. The fifth group is edges between node connected to CC1 and node connected to CC11, that is, connection between nodes which are coupled with nodes that are in asynchronous relationship. Figure 8 shows averages of synchronization rate of each group in the coupling strength $\delta' = -0.02$ to $\delta' = -0.2$. In Fig. 8, we can observed that synchronization rates of the first group and the second group are similar and that of the third group and the fourth group are similar. Synchronization rates of the first group and the second group are lower as the negative coupling is stronger. Synchronization rates of the third group and the fourth group are almost unchanged even if the negative coupling changed. Synchronization rates of the fifth group are lower as the negative coupling is stronger, although how to coupling of fifth group is similar to the third group and the fourth group. This results are considered that fifth group is connections between nodes which are coupled with nodes that are in asynchronous relationship.

4. Conclusion

In this study, we have investigated synchronization phenomena of network whose hub node is replaced with two distributed hub nodes. Furthermore, we connected distributed hub nodes in two ways. In the first model, distributed hub nodes connected by a positive resistor. In the second model, distributed hub nodes connected by a negative resistor. In the result of the first model, when we compared synchronization rate of undelying network and network with distributed hub, we can observed that synchronization rate is almost unchanged, even if a hub is replaced with two distributed hub nodes. In the simulation of the second model, we compare edges grouping similar connection. From the simulation result, synchronization rates of group which is connection between nodes which are coupled with nodes that are in asynchronous relationship are lower as the negative coupling is stronger.

Acknowlegement

This work was partly supported by JSPS Grant-in-Aid for Scientific Research 16K06357.

References

[1] D. J. Watts and S. H. Strogatz, “Collective Dynamics of Small-world”, Nature, vol.393, pp.440-442, 1998.
[2] A. L. Barabasi and R. Albert, “Emergence of Scaling in Random Networks”, Science, vol.286, pp.509-512, 1999.
[3] J. Aguirre, R. Sevilla-Escoboza, R. Gutirrez, D. Papo, and J.M. Buld, “Synchronization of Interconnected Networks: The Rolw of Connector Nodes”, Phys. Rev. Lett. 112, 248701 Published 16 June 2014. [4] K. Ago, Y. Uwate, Y. Nishio, “Influence of Bridge on Coupled Chaotic Circuit Network”, Proc. of RISP International Workshop on Nonlinear Circuits, Communications and Signal Processing (NCSP’14), pp. 417420, Mar. 2014.
[5] K. Ago, Y. Uwate, Y. Nishio, “Investigation of Partical Synchronization in Coupled Chaotic Circuit Network with Local Bridge”, Proc. of IEEE Workshop on Nonlinear Cirsuit Networks (NCN’14), pp.63-66, Dec. 2014.
[6] T. Chikazawa, Y. Uwate, Y. Nishio, “Chaos Propagation in Coupled Chaotic Circuits with Multi-Ring Combination”, Proc. of IEEE Asia Pacific Conference on Circuits and Systems (APCCAS’16), pp. 65-68, Oct. 2016.
[7] S. Hashimoto, T. Chikazawa, Y. Uwate, Y. Nishio, “Synchronization Phenomena in Complex Networks of Coupled Chaotic Circuits with Different Degree Distribution”, Proc. of RISP International Workshop on Nonlinear Circuits, Communications and Signal Processing (NCSP’17), pp. 333-336, Feb. 2017.
[8] M. Shinriki, M. Yamamoto and A. Mori, “Multimode Oscillations in a Modified van der Pol Oscillator Conraining a Positive Nonlinear Conductance”, Proc. IEEE, vol. 69, pp. 394-395, 1981.
[9] N. Inaba, T. Saito and S. Mori, “Chaotic Phenomena in a Curcuit with a Negative Resistance and an Ideal Switch of Diodes”, Trans. of IEICE, vol.E70, no. 8, pp. 744-754, 1987.