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Influence of Coupling Strength on Synchronization in Two Rings of Coupled van der Pol Oscillators

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1. Introduction

In this study, we propose to the coupled oscillatory system. By computer simulations and circuit experiments, we investigate synchronization phenomena observed in the proposed circuit system by changing one of the coupling strength.

2. System model

Figure 1 shows two circuits and a model of the system. The circuit of a van der Pol oscillator is called circuit VDP. The circuit with a slightly modified VDP structure is called NC. We use six van der Pol oscillators in this study. We use two ring circuits of van der Pol oscillators. The three VDP of the first ring are connected by resistors. The three NC of the second ring are connected by inductors and resistors. When the two rings are not connected, the oscillators in the first ring exhibit in-phase synchronization and oscillators of the second ring exhibit three-phase synchronization. The first and second rings are connected by resistors (R_1, R_2, R_3). We observe the synchronization phenomena of adjacent oscillators. We investigate how the synchronization phenomena change upon changing the value of R_2.

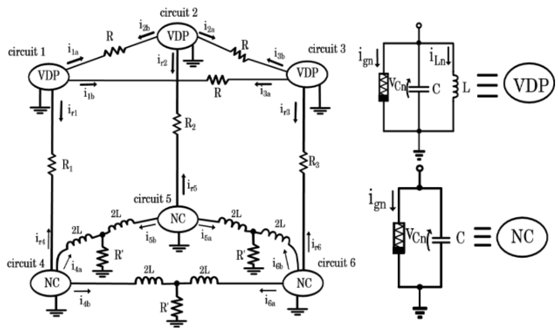

Fig1 Circuit model.

A nonlinear resistor defined as follows:

$$i_{gn} = -g_1 v_n + g_3 v_n^3. \tag{1}$$

the normalized equations of the first ring are given as:

$$\begin{cases} \dot{x}_n = \varepsilon(1 - x_n^2)x_n - y_n + \alpha(-2x_n + x_i + x_j) - \gamma_n(x_n - x_{n+3}) \\ \dot{y}_n = x_n \qquad\qquad (n = 1,2,3) \end{cases} \tag{2}$$

and the normalized equations of the secondo ring are given as:

$$\begin{cases} \dot{x}_n = \varepsilon(1 - x_n^2)x_n - y_{an} - y_{bn} - \gamma_n(x_n - x_{n-3}) \\ \dot{y}_{an} = (x_n - \beta(y_{an} + y_{bi})) \\ \dot{y}_{bn} = (x_n - \beta(y_{bn} + y_{aj})) \qquad\qquad (n = 4,5,6) \end{cases} \tag{3}$$

where n, i and j denote the number of the circuit (n = 1,2,3,4,5,6, i = 2,3,1,5,6,4 and j = 3,1,2,6,4,5). The parameters ε is non-linear strength. The parameters α, β and γ denote the coupling strengths of the resistor R, resistor R' and resistor R_n, respectively.

3. Results

The simulation results of the system model are shown from Fig. 2 to Fig. 4. The value of the parameters is set to ε = 0.05, α = 0.05, β = 0.05. Figures 2 - 4 show the graph of the relationship between coupling strength and the phase difference. We change the value of γ_2 from 0.000 to 0.030 at intervals of 0.001. In Fig. 2, we fix the value of other coupling strengths and initial value. The values of other coupling strengths are set to $\gamma_1 = \gamma_3$ = 0.015. In Fig.3, The values of other coupling strengths are set to $\gamma_1 = \gamma_3$ = 0.020. In Fig. 4, the values of other coupling strengths are set to $\gamma_1 = \gamma_3$ = 0.025.

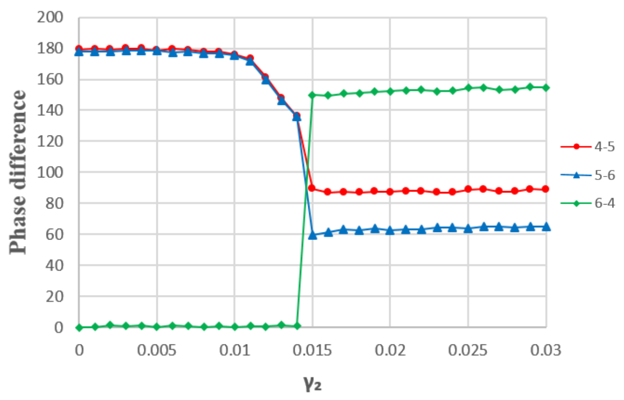

Fig2 The graph of phase difference ($\gamma_1 = \gamma_3 = 0.015$).

In Fig. 2, when γ_2 is even smaller than 0.015, circuit 4 - circuit 6 exhibit in-phase synchronization. When γ_2 is from 0.021 to 0.03, the phase difference of circuit 4 - circuit 5 is 85°. The phase difference of circuit 5 - circuit 6 is 65°. The phase difference of circuit 6 - circuit 4 is 155°. This result shows the phase difference changes depending on whether the value of γ_2 is larger or smaller than 0.015.

In Fig. 3, when γ_2 is even smaller than 0.020, circuit 4 circuit 6 exhibit in-phase synchronization. When γ_2 is from 0.020 to 0.03, the phase difference of circuit 4 - circuit 5 is 60°. The phase difference of circuit 5 - circuit 6 is 55°. The phase difference of circuit 6 - circuit 4 is 120°. This result shows the phase difference changes depending on whether the value of γ_2 is larger or smaller than 0.020.

In Fig. 4, when γ_2 is even smaller than 0.025, circuit 4 circuit 6 exhibit in-phase synchronization. When γ_2 is from 0.025 to 0.03, the phase difference of circuit 4 - circuit 5 is 30°. The phase difference of circuit 5 - circuit 6 is 25°. The phase difference of circuit 6 - circuit 4 is 55°. This result shows the phase difference changes depending on whether the value of γ_2 is larger or smaller than 0.025.

Figure 5 shows the time waveform of oscillators of the first ring and the second ring. The values of coupling strengths are set to $\gamma_1 = \gamma_3 = 0.020$ and $\gamma_2 = 0.030$. The oscillators of the first ring exhibit in-phase synchronization and the oscillators of the second ring exhibit three-phase synchronization gravitate toward in-phase synchronization.

From these results, we consider the synchronization phenomena change depending on whether the value of γ_2 is larger or smaller than the values of γ_1 and γ_3. When the value of γ_2 is smaller than the values of γ_1 and γ_3, circuits exhibit in-phase synchronization. When the value of γ_2 is larger than the values of γ_1 and γ_3, the oscillators of the first ring exhibit in-phase synchronization and the oscillators of the second ring exhibit three-phase synchronization gravitate toward in-phase synchronization. When the value of γ_2 is larger than the values of γ_1 and γ_3, the phase difference of the oscillators of the second ring becomes smaller by increasing γ_1 and γ_3. We can obtain new synchronization which is neither in-phase synchronization nor three-phase synchronization.

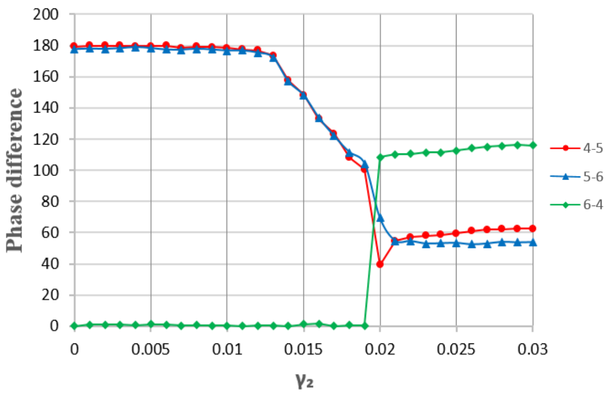

Fig3 The graph of phase difference ($\gamma_1 = \gamma_3 = 0.020$).

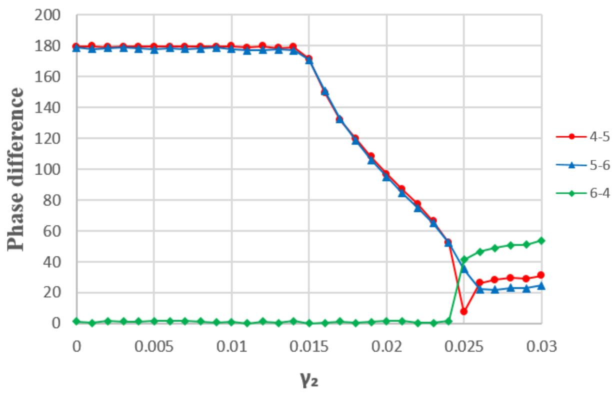

Fig4 The graph of phase difference ($\gamma_1 = \gamma_3 = 0.025$).

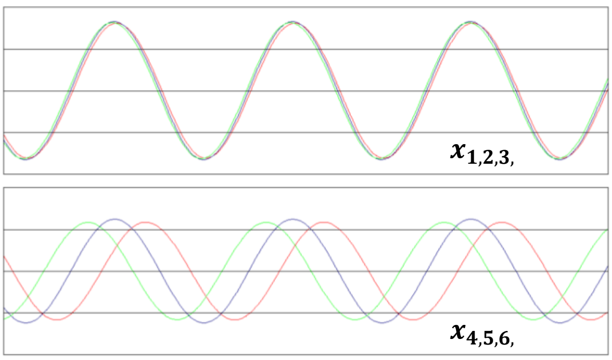

Fig5 The time waveform of oscillators.

4. Conclusion

We have proposed a system model using two rings of coupled three van der Pol oscillators coupled by resistors or inductors. We investigate how the synchronization changes upon changing the value of γ_2. We observed various synchronization phenomena by varying the coupling strengths. We found two factors that determine the phase difference of the second ring. The first factor is the synchronization phenomena change depending on whether the value of γ_2 is larger or smaller than the values of γ_1 and γ_3. The second factor is the values of the γ_1 and γ_3 decide the phase difference of the oscillators of the second ring. When the value of γ_2 is larger than the values of γ_1 and γ_3, the oscillators of the first ring exhibit in-phase synchronization and the oscillators of the second ring exhibit three-phase synchronization gravitate toward in-phase synchronization

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Acknowledgment

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