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Synchronization of Two Different Hierarchical Networks using Coupled Chaotic Circuits

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Abstract

We present results obtained for a new paradigm of coupled oscillatory networks, called competitive interaction networks. In the proposed circuit system, there are two hierarchical networks including multi-directed connections. We investigate what kind of strategy in network is effective for wining in the battle. We focus on dependencies on relationship of coupling strength and network structure.

1. Introduction

Coupled oscillatory circuits provide simple models for describing high-dimensional nonlinear phenomena occurring in our everyday world. Synchronization, in particular, is one of the most important features that can be described and explored with the help of oscillators, because, upon their coupling, strongly correlated rhythms among the oscillators emerge, called synchronized states. Therefore, many researchers have proposed different coupled oscillatory networks and have discovered many interesting synchronization phenomena [1].

Recently, synchronization in complex networks with different types of interactions has been extensively investigated for understanding important role played by the interactions. This is because interactions in networks leads to the emergence of key synchronization phenomena, especially competitive coupling can be observed in real world networks. In our research group, we have investigated synchronization phenomena in complex networks by using analog electrical oscillators [2]. Giron et al. investigated synchronization phenomena in plant communities where usually both facilitation and competition coexist, playing a key role in the structure and organization of these communities. They showed that synchronization provides an efficient way to unveil how species sharing facilitative interactions group of the plant community [3].

However, a node in a complex network is expressed by a mathematical model in most studies of synchronization of complex networks with competitive interactions. Although, it is very important to use mathematical model for complex networks in order to understand the synchronization states by approaching theoretical methods, we also need to consider physical models for future engineering applications.

In this study, we focus on synchronization state observed in two networks of chaotic circuits which are coupled in multi-direction hierarchically. We analyze the role of synchronization by changing the competitive coupling strategies.

2. Network Model

Figure 1 shows the hierarchical network model of this study. The chaotic circuits located at every layers correspond to chess piece.

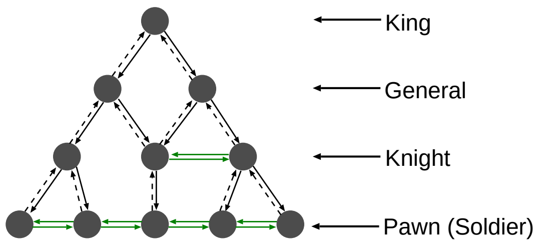


Figure 1: Hierarchical Network model.

Figure 2 shows the proposed network model called competitive interaction network. There are two hierarchical networks (Group A and Group B) including multi-direction connections. Each network consists of 11 chaotic circuits and the adjacent chaotic circuits are coupled by resistors. The coupling strength in the group and between the groups are summarized in Table 1.

We define competitive coupling strategy between two hierarchical networks. The lowest five chaotic circuits (A7 - A11) in Group-A and (B7 - B11) in Group-B attack to in front of the chaotic circuits. We call this competitive coupling "distributed attack".

In the proposed network, each node is expressed by the chaotic circuit. Figure 3 shows the chaotic circuit which is

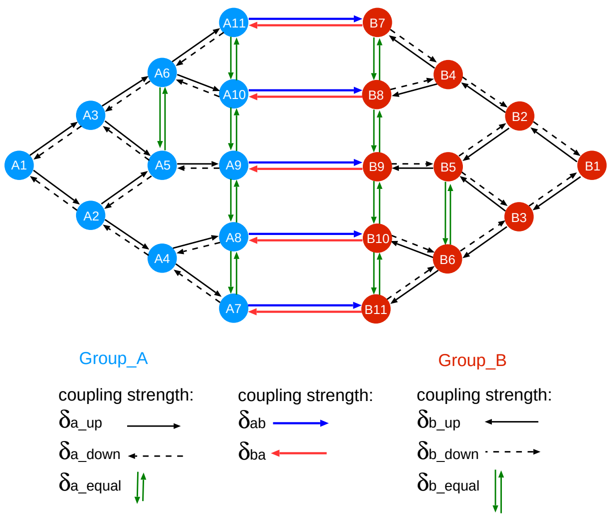


Figure 2: Competitive interaction network.

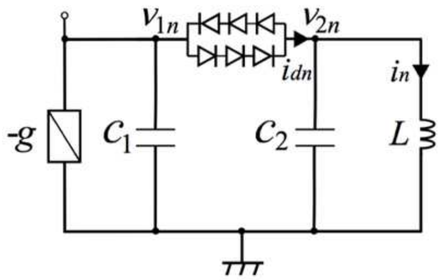
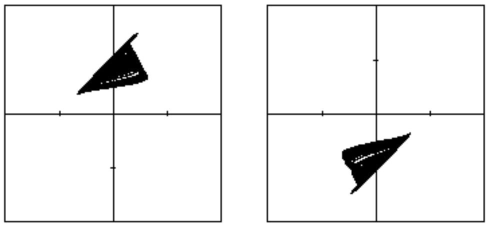


Figure 3: Chaotic circuit.



(a) Group-A. (b) Group-B.

Figure 4: Attractors ($\alpha = 0.4$, $\beta = 20.0$, $\gamma = 0.5$).

Table 1: Definition of coupling strength.

Group	Place	Name
In Group-A	From higher to lower layer	δ_{a-up}
	From lower to higher layer	δ_{a-down}
	Same layer	$\delta_{a-equal}$
In Group-B	From higher to lower layer	δ_{b-up}
	From lower to higher layer	δ_{b-down}
	Same layer	$\delta_{b-equal}$
Between Group-A and B	From Group-A to Group-B	δ_{ab}
	From Group-B to Group-A	δ_{ba}

three-dimensional autonomous circuit proposed by Shinriki et al. in Ref. [4]. This circuit is composed by an inductor, a negative resistance, two condensers and dual-directional diodes. The circuit generates asymmetric attractor as shown in Fig. 4. In this simulation, the attractor of Fig. 4 (a) is used to Group-A, and the attractor of Fig. 4 (b) is used to Group-B.

Next, we develop the expression for the circuit equations of the circuit model as shown in Fig. 3. The $i-v$ characteristics of the nonlinear resistor are approximated by the following three-segment piecewise-linear function,

$$i_{dn} = \begin{cases} G_d(v_{1n} - v_{2n} - V) & (v_{1n} - v_{2n} > V) \\ 0 & (|v_{1n} - v_{2n}| \leq V) \\ G_d(v_{1n} - v_{2n} + V) & (v_{1n} - v_{2n} < -V) \end{cases} \quad (1)$$

The normalized circuit equations governing the circuit are expressed as follows.

$$\begin{cases} \dfrac{dx_n}{d\tau} = z_n \\ \dfrac{dy_n}{d\tau} = \alpha\gamma y_n - \alpha f(y_n - z_n) - \alpha\delta \displaystyle\sum_{k\in S_n} (y_n - y_k) \\ \dfrac{dz_n}{d\tau} = f(y_n - z_n) - x_n. \end{cases} \quad (2)$$

where

$$t = \sqrt{LC_2}\tau, \quad i_n = \sqrt{\frac{C_2}{L}} V x_n,$$
$$v_{1n} = V y_n, \quad v_{2n} = V z_n,$$
$$\alpha = \frac{C_2}{C_1}, \quad \beta = \sqrt{\frac{L}{C_2}} G_d,$$
$$\gamma = \sqrt{\frac{L}{C_2}} g, \quad \delta = \frac{1}{R}\sqrt{\frac{L}{C_2}}.$$

where $n = 1, 2, 3, ..., 22$ and S_n is set to nodes which are connected to chaotic circuits. The nonlinear function $f()$ corresponds to the $i-v$ characteristics of the nonlinear resistors consisting of the diodes and are described as follows:

$$f(y_n - z_n) = \begin{cases} \beta(y_n - z_n - 1) & (y_n - z_n > 1) \\ 0 & (|y_n - z_n| \leq 1) \\ \beta(y_n - z_n + 1) & (y_n - z_{nn} < -1) \end{cases} \quad (3)$$

For the computer simulations, we calculate Eq. (2) using the fourth-order Runge-Kutta method with the step size $h = 0.005$.

3. Synchronization Phenomena

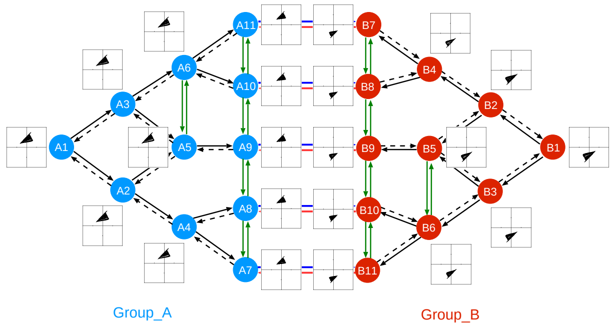
3.1 Dependency on relationship of coupling strength

As first simulation, we focus on the relationship of multi-directed coupling strength by using same network structure. Table 2 summarizes the setting of coupling strength values. In Group-A, the coupling strength from higher layer to lower layer has larger value than other direction coupling strength. While in Group-B, the both direction couplings have same value. We investigate synchronization state by changing the coupling strength between two groups.

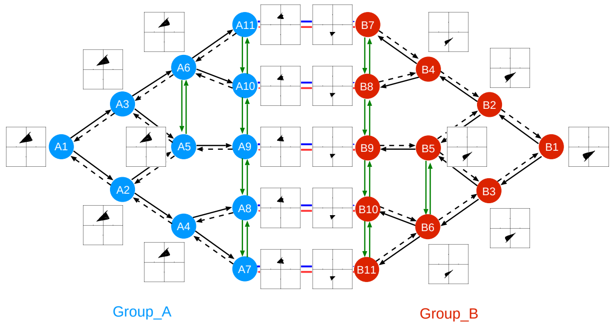
Table 2: Setting of coupling strength.

Group	Coupling strength	Value
In Group-A	δ_{a-up}	0.5
	δ_{a-down}	0.1
	$\delta_{a-equal}$	0.3
In Group-B	δ_{b-up}	0.3
	δ_{b-down}	0.3
	$\delta_{b-equal}$	0.3
Between Group-A and B	δ_{ab}	variability
	δ_{ba}	variability

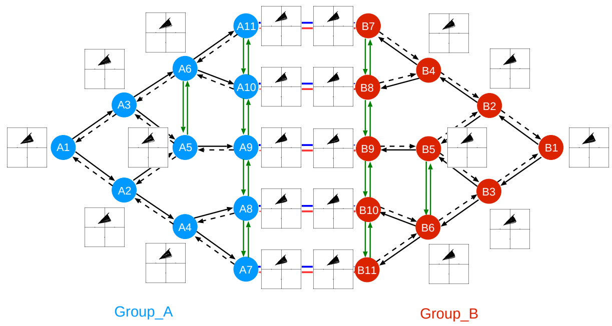
Figure 5 shows the obtained attractors from the computer simulations. When the coupling strength $\delta_{ab} = \delta_{ba}$ is set to 0.1 and 0.2, the amplitude of chaotic attractor located at lowest layer becomes small than the original attractor. In the case of $\delta_{ab} = \delta_{ba}$=0.3, all chaotic circuits oscillate at upper side as shown in Fig. 5 (c). Namely, Group-A wins against Group-B. We confirm that if the network structure of two groups is same, the coupling strength from higher to lower layers should have larger value than other direction coupling for wining at the battle.



(a) $\delta_{ab} = \delta_{ba}$=0.1.



(b) $\delta_{ab} = \delta_{ba}$=0.2.



(c) $\delta_{ab} = \delta_{ba}$=0.3.

Figure 5: Attractors.

3.2 Dependency on network structure

Next, we consider dependency on network structure. In this simulation, the relationship of the coupling strength is same in two groups. The network structure of two group has different connection pattern. One connection is rewired in Group-B. There are two types of rewiring connection.

Figure 6 shows the competitive interaction network which Group-B has the connection between top layer to third layer (1 layer skip connection). Figure 7 shows the competitive interaction network which Group-B has the connection between top layer to bottom layer (2 layers skip connection). For the simulations, the coupling strength in each group and between groups are changed. We focus on wining or losing of the networks.

The summarized simulation results of competitive interaction network Fig. 6 is shown in Fig. 8. When the coupling strength from higher layer to lower layer has larger value than the other direction, the both networks can not win. In the case of δ_{a-up}=0.34 and 0.32, wining group depends on the coupling strength of between two groups. When the coupling strength δ_{ab} and δ_{ba} is small, we obtain draw. By increasing δ_{ab} and δ_{ba}, we confirm Group-A wins at certain range. After that Group-B wins with large value of δ_{ab} and δ_{ba}.

The summarized simulation results of competitive interaction network Fig. 7 is shown in Fig. 9. In this case, we obtain the only two results; ”draw” and ”Group-B wind”. We can not observe the situation that Group-A wins. From these results, we confirm that the network structure especially layer skipping connection has important role for wining at the battle.

delta_ab=delta_ba		0.25	0.26	0.27	0.28	0.29	0.3	0.31	0.32	0.33	0.34	0.35	0.36	0.37	0.38	0.39	0.4	0.41	0.42	0.43	0.44	0.45	0.46	0.47	0.48	0.49	0.5
delta_a_up delta_b_up	delta_a_down delta_b_down																										
0.5	0.1	draw																									
0.4	0.2																										
0.38	0.22																										
0.36	0.24																										
0.34	0.26	draw								Group A													Group B				
0.32	0.28	draw					Group A		Group B																		
0.3	0.3	draw				Group B																					

Figure 8: Wining or losing results (Group-B: 1 layer skip connection).

delta_ab=delta_ba		0.25	0.26	0.27	0.28	0.29	0.3	0.31	0.32	0.33	0.34	0.35	0.36	0.37	0.38	0.39	0.4	0.41	0.42	0.43	0.44	0.45	0.46	0.47	0.48	0.49	0.5
delta_a_up delta_b_up	delta_a_down delta_b_down																										
0.5	0.1	draw																									
0.4	0.2																										
0.38	0.22																										
0.36	0.24																										
0.34	0.26	draw									Group B																
0.32	0.28	draw						Group B																			
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Figure 9: Wining or losing results (Group-B: 2 layers skip connection).

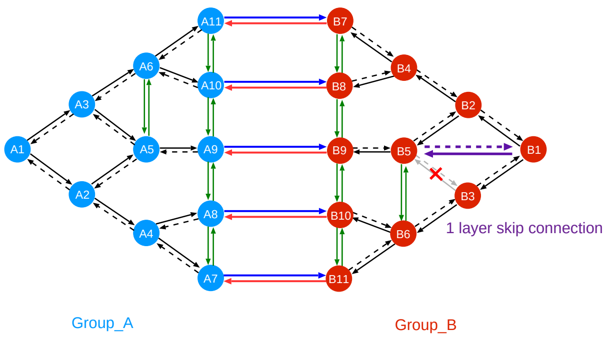

Figure 6: Competitive interaction network (Group-B: 1 layer skip connection).

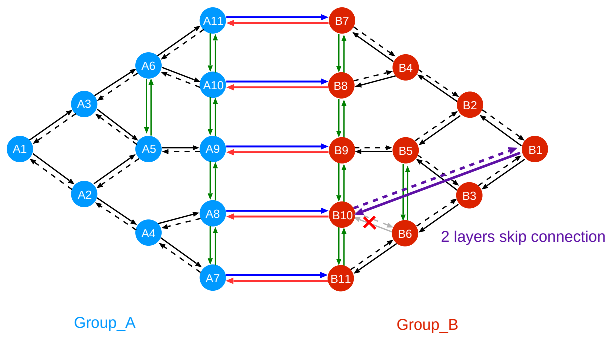

Figure 7: Competitive interaction network (Group-B: 2 layers skip connection).

4. Conclusions

In this study, we have investigated wining or losing observed in two networks of chaotic circuits which are coupled in multi-direction hierarchically. We confirmed that the network structure especially layer skipping connection has important role for wining at the battle.

In our future works, we would like to investigate effect of competitive strategies and apply this model to more complex networks such as smart grid network and social network.

Acknowledgment

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