2018 RISP International Workshop on Nonlinear Circuits,
Communications and Signal Processing (NCSP2018)
Honolulu, Hawaii, USA, March 4-7, 2018

Propagation of Chaotic and Periodic Solutions in Ladder Coupled Chaotic Circuits with Different Parameters

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Abstract

In this study, we investigate the propagation of chaotic solutions and periodic solutions in ladder coupled chaotic circuits. The state of propagation is different for each type of circuit networks. We change the parameters of chaotic circuits by regularity in the ladder structure. We use chaotic circuits which are coupled by resistors.

1. Introduction

In reality network, various things are connected such as cities, traffic and human beings. These connects are important factor in the modern. Incidentally, these things have little different characters in each even if they are the same type. In this situation, such a differentiation of characters is sometimes propagated to other things. There are some cases that this propagation affects a great deal in the network. For example, the traffic jam of the transportation network, the pandemic outbreak of viral infection, deterioration of human relations and so on are the bad influences. Therefore, the investigation of the propagation of these characters is important to block the effects. Then, in the circuit network, we investigate the state of similar phenomenon by propagation of chaotic solution and periodic solution. The behavior of the network affects various phenomena under some different situations.

In this study, we investigate the propagation of chaotic and periodic solution in coupled chaotic circuits. These chaotic circuits have little difference of each other by the regularly changed of parameter. In this model, five circuits generate three periodic solutions and other five circuits generate chaotic solutions. Ten chaotic circuits are coupled by resistor, and we change the value of resistors, in other word, changing the coupling strength. We observe how chaotic and periodic solution propagate in this ladder structure by changing the coupling strength. Moreover, we change the number of chaotic circuits, and we confirm the state of propagation.

2. System model

The chaotic circuit is shown in Fig. 1. This circuit consists of a negative resistor, two inductors, a capacitor and dual-directional diodes. This chaotic circuit is called Nishio-Inaba circuit.

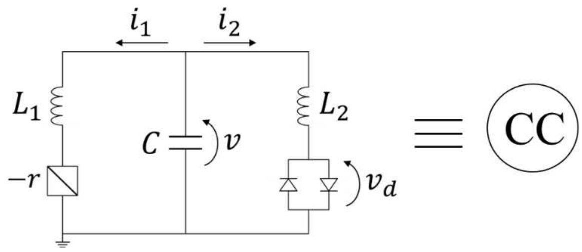

Figure 1: Chaotic circuit

The circuit equations of this circuit are described as follows:

$$\begin{cases} L_1 \dfrac{di}{dt} = v + ri \\ L_2 \dfrac{di}{dt} = v - v_d \\ C \dfrac{dv}{dt} = -i_1 - i_2. \end{cases} \tag{1}$$

The characteristic of nonlinear resistance is described as follows:

$$v_d = \frac{r_d}{2} \left(\left| i_2 + \frac{V}{r_d} \right| - \left| i_2 - \frac{V}{r_d} \right| \right). \tag{2}$$

By changing the variables and parameters:

$$\begin{cases} i_1 = \sqrt{\dfrac{C}{L_1}} V x_n,\ \ i_2 = \dfrac{\sqrt{L_1 C}}{L_2} V y_n,\ \ v = V z_n \\ \alpha = r\sqrt{\dfrac{C}{L_1}},\ \ \beta = \dfrac{L_1}{L_2},\ \ \delta = r_d \dfrac{\sqrt{L_1 C}}{L_2}, \\ t = \sqrt{L_1 C_2}\tau. \end{cases} \tag{3}$$

The normalized circuit equations are given as follows:

$$\begin{cases} \dfrac{dx_1}{d\tau} = \alpha x_1 + z_1 \\ \dfrac{dy_1}{d\tau} = z_1 - f(y_1) \\ \dfrac{dz_1}{d\tau} = -x_1 - \beta y_1. \end{cases} \tag{4}$$

$f(y_i)$ is described as follows:

$$f(y_i) = \frac{1}{2}\left(\left| y_i + \frac{1}{\delta} \right| - \left| y_i - \frac{1}{\delta} \right| \right). \tag{5}$$

In our system, each chaotic circuit is coupled by one resistor R. Figure 2 shows the system model. We couple ten chaotic circuits in ladder structure.

By changing the variables and parameters:

$$\begin{cases} i_1 = \sqrt{\dfrac{C}{L_1}} V x_n,\ \ i_2 = \dfrac{\sqrt{L_1 C}}{L_2} V y_n,\ \ v = V z_n \\ \alpha = r\sqrt{\dfrac{C}{L_1}},\ \ \beta = \dfrac{L_1}{L_2},\ \ \delta = r_d \dfrac{\sqrt{L_1 C}}{L_2}, \\ \gamma = \dfrac{1}{R},\ \ t = \sqrt{L_1 C_2}\tau. \end{cases} \tag{6}$$

In Eq. (6), γ is the coupling strength. R is the resistor that couples each chaotic circuit.

The normalized circuit equations are given as follows:

$$\text{CC}_1 \begin{cases} \dfrac{dx_1}{d\tau} = \alpha x_1 + z_1 \\ \dfrac{dy_1}{d\tau} = z_1 - f(y_1) \\ \dfrac{dz_1}{d\tau} = -x_1 - \beta y_1 - \gamma(z_1 - z_2). \end{cases} \tag{7}$$

$$\text{CC}_n \begin{cases} \dfrac{dx_n}{d\tau} = \alpha x_n + z_n \\ \dfrac{dy_n}{d\tau} = z_n - f(y_n) \\ \dfrac{dz_n}{d\tau} = -x_n - \beta y_n \\ \qquad - \gamma(z_n - z_{n-1}) - \gamma(z_n - z_{n+1}) \\ \qquad\qquad (n = 2, 3, \cdots, 9). \end{cases} \tag{8}$$

$$\text{CC}_{10} \begin{cases} \dfrac{dx_{10}}{d\tau} = \alpha x_{10} + z_{10} \\ \dfrac{dy_{10}}{d\tau} = z_{10} - f(y_{10}) \\ \dfrac{dz_{10}}{d\tau} = -x_{10} - \beta y_{10} - \gamma(z_9 - z_{10}). \end{cases} \tag{9}$$

In this study, we set the parameters of the system as $\beta = 3.0$ and $\delta = 470.0$. The parameters α are set from 0.411 to 0.420 with step size 0.001 from CC1 to CC10. Figure 3 shows the attractors from CC1 to CC5. We define these attractors as three periodic solutions. Figure 4 shows the attractor from CC6 to CC10. We define these attractors as chaotic solution.

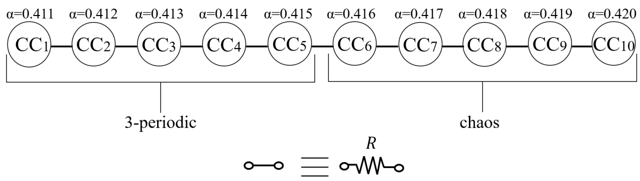

Figure 2: System model

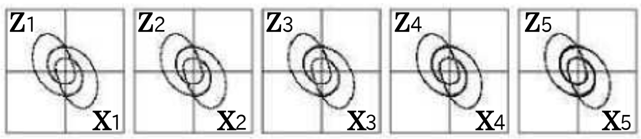

Figure 3: Three periodic attractots

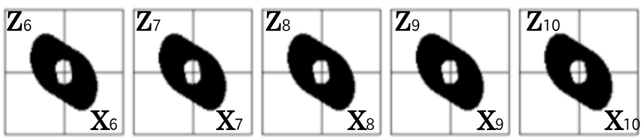

Figure 4: Chaotic attractors

3. Simulation results

We investigate the propagation of chaotic solutions and three periodic solutions in ladder coupled chaotic circuits by changing the coupling strength. All coupling strength is changed evenly. We define that when chaotic attractors change three periodic attractors, three periodic attractors are propagated. When three periodic attractors change chaotic attractors, chaotic attractors are propagated. First, we couple the chaotic circuits from CC1 to CC10. The result is shown in Fig. 5. We set the coupling strength as $\gamma = 0.001$. In this case, because coupling strength is weak, propagation has not occured.

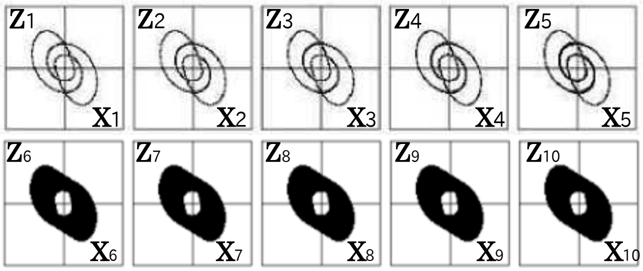

Figure 5: Attractors ($\gamma = 0.001$)

When we set the coupling strength as $\gamma = 0.0055$, three periodic attractor is propagated CC6 as shown in Fig. 6. Furthermore, when we increase the coupling strength, three periodic attractors are propagated the chaotic attractors gradually one by one. Figure 7 shows that three periodic attractors are propagated to CC7. Figure 8 shows that three periocid attractors are propagated to CC8. Figure 9 shows that three periodic attractors are propagated to CC9. Finally, Fig. 10 shows three periodic attractors are propagated to all coupled chaotic circuits.

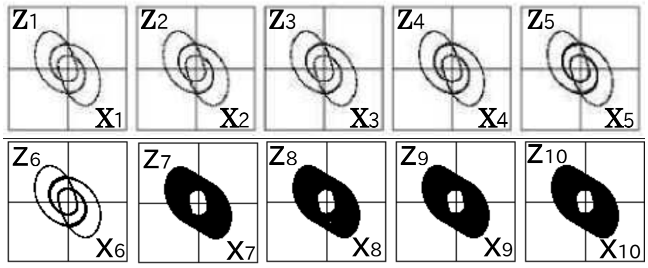

Figure 6: Attractors ($\gamma = 0.0055$)

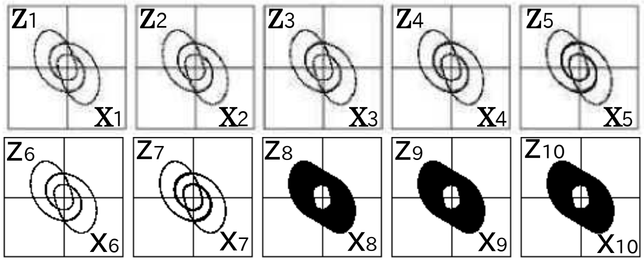

Figure 7: Attractors ($\gamma = 0.0064$)

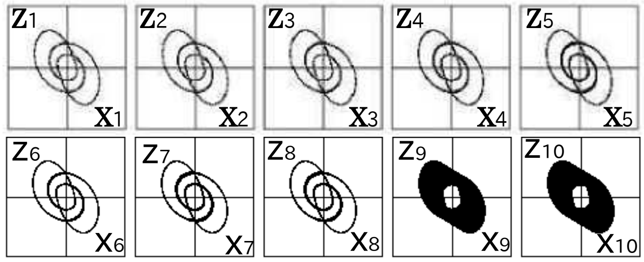

Figure 8: Attractors ($\gamma = 0.0068$)

However, when we set the coupling strength as $\gamma = 0.011$, chaotic attractors are propagated all three periodic attractors. Differently from the propagation of three periodic attractors, chaotic attractors propagate all three periodic attractors at once. Then, even though we increase the coupling strength moreover, these attractors do not change. Therefore, we obtain the result that when we set the coupling strength as more than $\gamma = 0.011$, chaotic attractors are propagated at all times.

Finally, we change the number of coupled chaotic circuits from $N = 7$ to $N = 10$. The parameters α are set with step size 0.001 from CC1 to CC7, from CC1 to CC8, from CC1 to CC9 and from CC1 to CC8. Likewise, the parameter α of CC1 is 0.411. We focus on the number of three periodic attractors. We investigate how the relationship between propagation of three periodic attractors and coupling strength is changed depending on the number of coupled chaotic circuits.

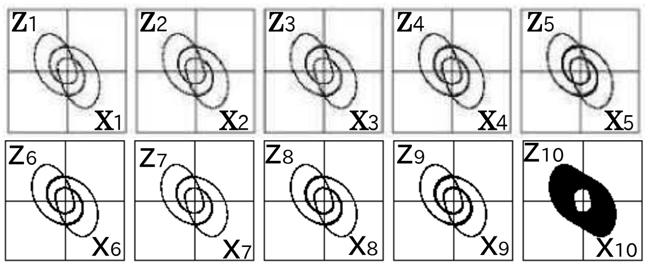

Figure 9: Attractors ($\gamma = 0.007$)

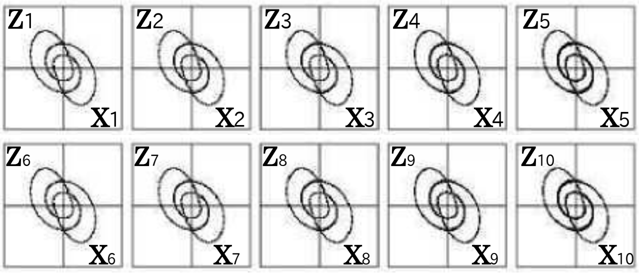

Figure 10: Attractors ($\gamma = 0.01$)

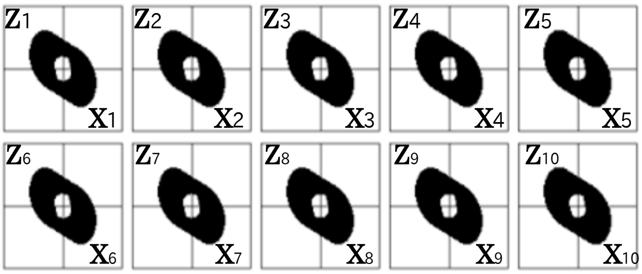

Figure 11: Attractors ($\gamma = 0.011$)

Figure 12 shows the graph of this result. The vertical line shows the number of three periodic attractors. The horizontal line shows the coupling strength. We ascertain that when we reduce the number of coupled chaotic circuits, three periodic attractors propagate by weaker coupling strength. Similarly, chaotic attractors propagate all at once by weaker coupling strength.

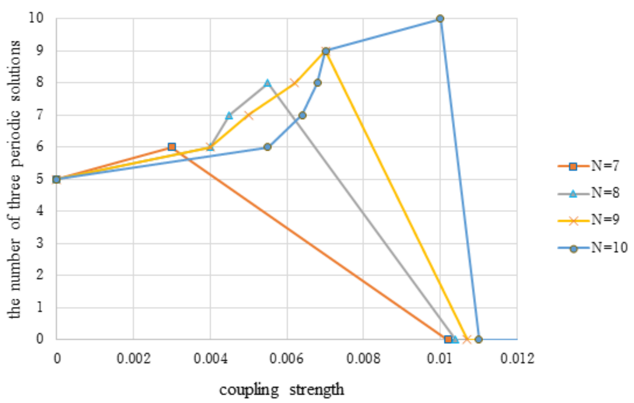

Figure 12: Relationship between the number of three periodic attractors and the coupling strength by the value of N.

4. Conclusions

In this study, we have proposed the system model that ten chaotic circuits with different parameters are coupled by resisters in ladder structure. The parameters α are set from 0.411 to 0.420 with step size 0.001 from CC1 to CC10. We have investigated the propagation of chaotic solutions and periodic solutions by changing the coupling strength γ. By the computer simulations, we confirm that the state of propagation of these attractors is altered when we change the coupling strength. As a result, in ladder ten coupled chaotic circuits, when we increase the coupling strength from $\gamma = 0.0055$ to 0.01, three periodic attractors are propagated chaotic attractors gradually one by one. Also, when we set the coupling strength as $\gamma = 0.011$, chaotic attractors are propagated all three periodic attractors at once. Next, we change the number of coupled chaotic circuits, and we investigate the propagation likewise. When we decrease the number of coupled chaotic circuits, propagation of chaotic attractors and three periodic attractors is occurred by weaker coupling strength.

In the future works, we will develop this model to couple some ladders and investigate the propagation of chaotic attractors and three periodic attractors.

References

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