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Relationship between a Synchronization Rate and a Dopant Mobility of a Memristor on Chaotic Circuits Coupled by Two Memristors

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Abstract

In our previous study, a relationship between observed phenomena and the coupling strength in coupled chaotic circuits including memristors was investigated. However, there was no influence from the memristor. Basically, a memristor developed by HP reacts from a few Helz to a few hundred Helz. A main frequency band of chaotic circuits in our previous study is kilo Helz. Therefore, a behavior of the memristor become a resistor. In this study, the relationship between a synchronization rate and a dopant mobility of a memristor on chaotic circuits coupled by two memristors are investigated.

1. Introduction

Recently, a memristor, which is called as the fourth fundamental passive element of the electric circuit, has been remarked by many researchers. The memristor was proposed by L. O. Chua in 1971 [1]. In 2008, D. B. Strukov et al. implemented the memristor as a semiconductor element [2]. The memristor has a simple structure which sandwiches the TiO_2 between two electrodes of Pt. Additionally, the memristor can keep a value as a resistance. These features have been attracted many researchers.

On the other hand, chaos is studied in various fields such as mathematics, physics, engineering, economics, chemistry and so on. In the field of electrical engineering, chaotic phenomena observed in electric circuits are investigated. Shinriki-Mori circuit [3], which is one of the chaos circuits, has been studied by many researchers. Some of them reported about coupled these circuits. In these coupled systems, various kinds of interesting phenomena are observed [4]-[6].

Gambuzza et al. applied memristors to coupled chaotic circuits [7]. Two memristors are coupled in antiparallel as the coupling element. The system coupling multiple chaotic circuits have been investigated. However, a relationship between coupling strength and observed phenomena was not investigated.

In our previous study, the relationship between observed phenomena and the coupling strength in coup lied chaotic circuits including memristors was investigated. However, there was no influence from the memristor. Basically, a memristor developed by HP reacts from a few Helz to a few hundred Helz. A main frequency band of chaotic circuits in our previous study is kilo Helz. Therefore, a behavior of the memristor become a resistor.

In this study, the relationship between a synchronization rate and a dopant mobility of a memristor on chaotic circuits coupled by two memristors are investigated. In Sect. 2, the system model is shown. Computer simulation results are shown in Sect. 3. Finally, concluding remarks are mentioned in Sect. 4.

2. System Model

2.1 Memristor Model

Figure 1 shows the memristor model proposed by Strukov of Hewlett-Packard et al. [2]. The voltage applied to the memristor is v, and the current flowing is i. The equation of relationship between the voltage v and the current i is described by the following equation [7].

$$v(t) = (R_{ON}\omega(t) + R_{OFF}(1 - \omega(t)))i(t), \quad (1)$$

where R_{ON} has a low resistance value and R_{OFF} has a high resistance value. The variable ω is described by the following equation.

$$\omega = \frac{w}{D}. \quad (2)$$

The w is the width of the doped region in the memristor. The variable ω is normalized by the maximum width D of the memristor. The variable ω means the proportion occupied by R_{ON}. Therefore, ω is limited to a value between 0 and 1.

The memristor model is described by the following equation.

$$\frac{d\omega(t)}{dt} = \eta \frac{\mu_v R_{ON}}{D^2} F_{(\omega(t),i(t))} i(t), \quad (3)$$

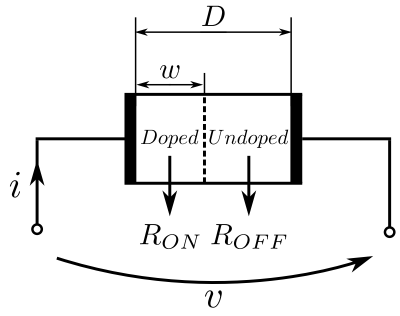

Figure 1: Memristor model.

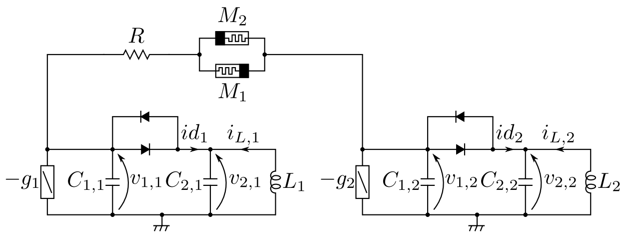

Figure 2: System model.

where η ($\eta = 1$ or $\eta = -1$) is the memristor polarity, μ_v is the dopant mobility and D is thickness of the memristor made by TiO_2. The current $i(t)$ is given by Eq. (1) as follows:

$$i(t) = \frac{v(t)}{R_{ON}M}, \tag{4}$$

where $M = \omega(t) + \beta(1 - \omega(t))$ with $\beta = R_{OFF}/R_{ON}$. $F_{(\omega(t),i(t))}$ is the Biolek window function [8]. The Biolek window function is given by:

$$F_{(\omega(t),i(t))} = 1 - (\omega(t) - \mathrm{stp}(-i(t)))^2, \tag{5}$$

where $\mathrm{stp}(i) = 1$ when $i \geq 0$, and $\mathrm{stp}(i) = 0$ when $i < 0$.

In this study, two memristors are coupled in antiparallel as a coupling element.

2.2 System Model

Figure 2 shows a system model in this study. It is a model combining the nodes of the negative resistance of two Shinriki-Mori circuits with a resistor and two memristors coupled in antiparallel. g_1 and g_2 are negative resistors.

Figure 3 shows a diode model in the system model. Figure 3 (a) shows the circuit diagram, and Fig. 3 (b) shows the voltage-current characteristic. This model is a piecewise linear model.

The circuit equation derived from these model is shown as

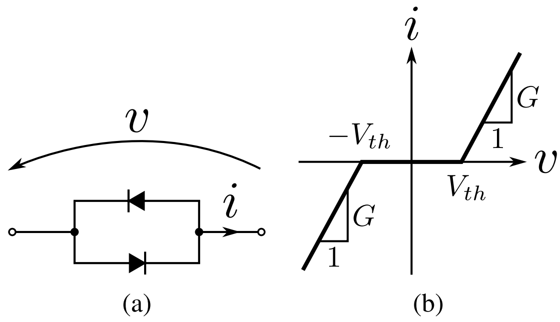

Figure 3: Bidirectionally coupled diode model. (a)Circuit diagram. (b)The voltage-current characteristic.

follows:

$$\left\{\begin{array}{rcl}
C_{1,1}\dfrac{dv_{1,1}}{dt} & = & -id_1 + g_1 v_{1,1} \\
& & +\dfrac{(M_1+M_2)(v_{1,2}-v_{1,1})}{R(M_1+M_2)+R_{ON}M_1M_2}, \\
C_{2,1}\dfrac{dv_{2,1}}{dt} & = & id_1 + i_{L,1}, \\
L_1\dfrac{di_{L,1}}{dt} & = & -v_{2,1}, \\
C_{1,2}\dfrac{dv_{1,2}}{dt} & = & -id_2 + g_2 v_{1,2} \\
& & +\dfrac{(M_1+M_2)(v_{1,1}-v_{1,2})}{R(M_1+M_2)+R_{ON}M_1M_2}, \\
C_{2,2}\dfrac{dv_{2,2}}{dt} & = & id_2 + i_{L,2}, \\
L_2\dfrac{di_{L,2}}{dt} & = & -v_{2,2}, \\
\dfrac{d\omega_1}{dt} & = & \eta_1 F_{(\omega_1)}\dfrac{\mu_v R_{ON}}{D^2}\dfrac{v_{1,2}-v_{1,1}+v_R}{R_{ON}M_1}, \\
\dfrac{d\omega_2}{dt} & = & \eta_2 F_{(\omega_2)}\dfrac{\mu_v R_{ON}}{D^2}\dfrac{v_{1,2}-v_{1,1}+v_R}{R_{ON}M_2},
\end{array}\right. \tag{6}$$

where M_1, M_2 are described as follows:

$$\left\{\begin{array}{l}
M_1 = \omega_1 + \beta(1-\omega_1), \\
M_2 = \omega_2 + \beta(1-\omega_2).
\end{array}\right. \tag{7}$$

The voltage of the resistor v_R is described as follows:

$$v_R = \frac{\frac{R}{R_{on}}(M_1+M_2)(v_{1,1}-v_{1,2})}{\frac{R}{R_{on}}(M_1+M_2)+M_1M_2}. \tag{8}$$

By using the following variables and parameters, the normalized circuit equation is derived.

$$
\begin{aligned}
&x_1 = \frac{v_{1,1}}{v_0},\ \ x_2 = \frac{v_{2,1}}{v_0},\ \ x_3 = \sqrt{\frac{L_1}{C_{2,1}}}\frac{i_{L,1}}{v_0},\\
&x_4 = \frac{v_{1,2}}{v_0},\ \ x_5 = \frac{v_{2,2}}{v_0},\ \ x_6 = \sqrt{\frac{L_1}{C_{2,1}}}\frac{i_{L,2}}{v_0},\\
&x_7 = \omega_1,\ \ x_8 = \omega_2,\ \ \frac{d}{dt} = \text{"}\,.\,\text{"},\\
&\tau = \frac{1}{\sqrt{C_{2,1}L_1}}t,\ \ \alpha = G\sqrt{\frac{L_1}{C_{2,1}}},\\
&\beta = \frac{R_{OFF}}{R_{ON}},\ \ \gamma = \frac{L_1}{L_2},\\
&\delta = \frac{C_{2,1}}{C_{1,1}},\ \ \varepsilon = \frac{C_{2,1}}{C_{1,2}},\ \ \zeta = \frac{C_{2,1}}{C_{2,2}},\\
&\eta_1 = 1,\ \ \eta_2 = -1,\ \ \theta_1 = g_1\sqrt{\frac{L_1}{C_{2,1}}},\\
&\theta_2 = g_2\sqrt{\frac{L_1}{C_{2,1}}},\ \ \iota = \sqrt{C_{2,1}L_1}\frac{\mu_v}{D^2}v_0,\\
&k = \frac{1}{R_{ON}}\sqrt{\frac{L_1}{C_{2,1}}},\ \ \lambda = \frac{R}{R_{ON}}\ \ \mathit{and}\ \ \xi = \frac{V_{th}}{v_0}.
\end{aligned} \tag{9}
$$

The normalized circuit equation is described as follows:

$$
\left\{
\begin{aligned}
\dot{x}_1 =\ & \delta\left[-\alpha\left\{(x_1 - x_2) + \frac{1}{2}\left(|x_1 - x_2 - \xi|\right.\right.\right.\\
& \left.\left.-|x_1 - x_2 + \xi|\right)\right\} + \theta_1 x_1\\
& \left.+k\frac{(m_1 + m_2)}{\lambda(m_1+m_2)+m_1 m_2}(x_4 - x_1)\right],\\
\dot{x}_2 =\ & \alpha\left\{(x_1 - x_2) + \frac{1}{2}\left(|x_1 - x_2 - \xi|\right.\right.\\
& \left.\left.-|x_1 - x_2 + \xi|\right)\right\} + x_3,\\
\dot{x}_3 =\ & -x_2,\\
\dot{x}_4 =\ & \varepsilon\left[-\alpha\left\{(x_4 - x_5) + \frac{1}{2}\left(|x_4 - x_5 - \xi|\right.\right.\right.\\
& \left.\left.-|x_4 - x_5 + \xi|\right)\right\} + \theta_2 x_4\\
& \left.+k\frac{(m_1 + m_2)}{\lambda(m_1+m_2)+m_1 m_2}(x_1 - x_4)\right],\\
\dot{x}_5 =\ & \zeta\left[\alpha\left\{(x_4 - x_5) + \frac{1}{2}\left(|x_4 - x_5 - \xi|\right.\right.\right.\\
& \left.\left.\left.-|x_4 - x_5 + \xi|\right)\right\} + x_6\right],\\
\dot{x}_6 =\ & -\gamma x_5,\\
\dot{x}_7 =\ & \eta_1\iota\left[1 - \left\{x_7 - \mathrm{stp}\left(-i_{M_1}\right)\right\}^2\right] i_{M_1},\\
\dot{x}_8 =\ & \eta_2\iota\left(1 - \left[x_8 - \mathrm{stp}\left\{-\left(-i_{M_2}\right)\right\}\right]^2\right) i_{M_2},
\end{aligned}
\right. \tag{10}
$$

where m_1, m_2 are described as follows:

$$
\left\{
\begin{aligned}
m_1 &= x_7 + \beta(1 - x_7),\\
m_2 &= x_8 + \beta(1 - x_8),
\end{aligned}
\right. \tag{11}
$$

and i_{M_1}, i_{M_2} are described as follows:

$$
\left\{
\begin{aligned}
i_{M_1} &= \frac{(x_4 - x_1)m_1 m_2}{m_1\{\lambda(m_1+m_2)+m_1 m_2\}},\\
i_{M_2} &= \frac{(x_4 - x_1)m_1 m_2}{m_2\{\lambda(m_1+m_2)+m_1 m_2\}}.
\end{aligned}
\right. \tag{12}
$$

In this study, simulations are carried out using these normalized circuit equations.

3. Simulation Results

Figures 4 and 5 show simulation results in this study. Figures 4 and 5 are time series waveforms. From the top, vertical axes show voltages of two circuits, the voltage difference between two circuits, variables w_1 and w_2 of memristors, and the difference between variables w_1 and w_2. Horizontal axes show time.

Initial values and parameters are set as follows: $x_1 = 0.001$, $x_2 = 0.001$, $x_3 = 0.001$, $x_4 = 0.001$, $x_5 = 0.001$, $x_6 = 0.0011$, $x_7 = 0.0$, $x_8 = 0.0$, $\alpha = 20.0$, $\beta = 100.0$, $\gamma = 1.0001$, $\delta = 3.3$, $\varepsilon = 3.3$, $\zeta = 1.0$, $\eta_1 = 1.0$, $\eta_2 = -1.0$, $\theta_1 = 0.19$, $\theta_2 = 0.19$, $k = 5.423$, $\lambda = 310.0$, $\xi = 0.6$, where γ is added as the parameter mismatch 0.01%. Parameter λ and ι show the resistor R as a coupling strength and the dopant mobility μ_v of the memristor, respectively.

In Figure 4, parameter ι is set as $\iota = 1.0$. Movements of variables x_7 and x_8 are very slow comparing with oscillations of two circuits. In this case, the system behaves two chaotic circuit coupled by a resistor.

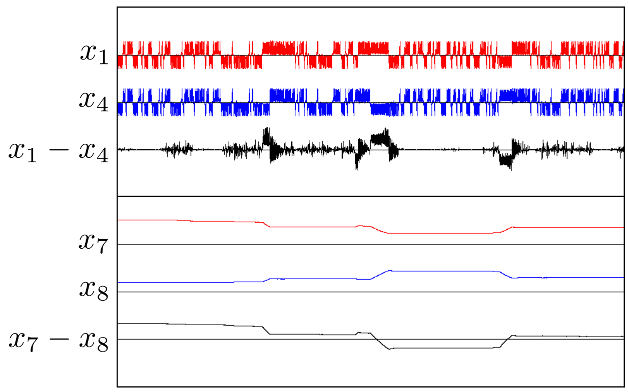

Figure 4: Simulation result. The parameter mismatch is 0.01%, $\iota = 1.0$, $\lambda = 310.0$. Time series waveform.

By increasing parameter ι, the memristor behaves as a memristor. In Figure 5, parameter ι is set as $\iota = 70.0$. In this case, the resistance of the memristor is changed by current flow of the memristor. Increasing parameter ι makes increasing the current of the memristor. In case of $\iota > 70.0$, the memristor current value does not have a reality.

Interesting relationship between oscillations of two circuit and memristor variables is observed in Fig. 5. When two memristor variables become saturated states, oscillation biases of two circuits become the opposite side. When two memristor variables are stables, two circuits are synchronizing. When two memristor variables move intensely, oscillation biases of two circuits become the same side and two circuits are not synchronizing. These three states can be observed randomly. By some computer simulations, it was considered that there are some relationship between these three states and parameter ι.

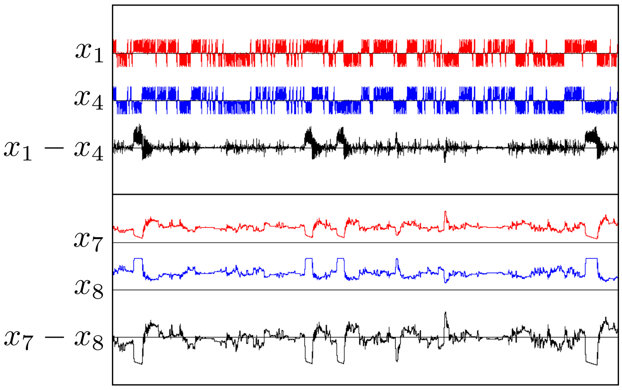

Figure 5: Simulation result. The parameter mismatch is 0.01%, $\iota = 70.0$, $\lambda = 310.0$. Time series waveform.

In order to investigate the relationship between these states and ι in detail, three states are defined as follows. Synchronization: the extreme value of the difference of $x_1 - x_4$ and the previous extreme value are in the range of ± 0.05. Asynchronization: $x_1 \cdot x_4$ is minus. Synchronization of switching phenomena: without synchronization and asynchronization. Using this definition, the relationship between three states and ι are investigated.

Figure 6 shows relationship between three state and parameter ι. The horizontal axis shows ι and the vertical axis shows the rate of each state.

When $\iota = 16$, synchronization state becomes the lowest value and synchronization of switching state and asynchronization state become the highest value. Therefore, $\iota = 16$ is the most effective value for synchronization phenomena of the coupled system.

4. Conclusions

In this study, the relationship between a synchronization rate and a dopant mobility of a memristor on chaotic circuits

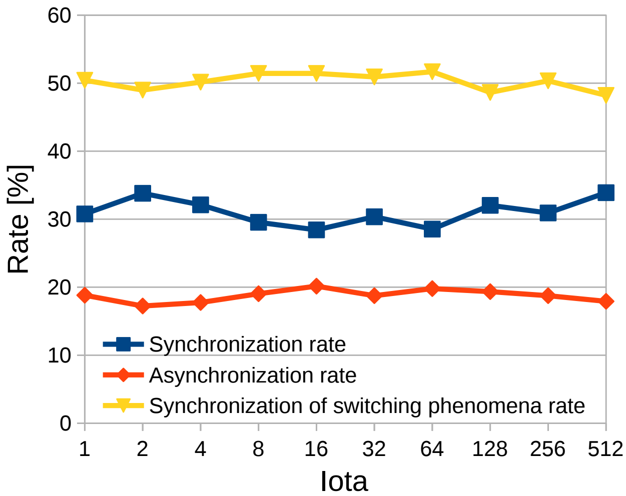

Figure 6: Relationship between three state and parameter ι. The parameter mismatch is 0.01%, $\lambda = 310.0$.

coupled by two memristors has been investigated. As a result, the most effective value of a dopant mobility of a memristor for synchronization phenomena of the coupled system. As a future work, investigating the influence of memristors using the most effective value to the coupled chaotic circuits is considered.

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