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Synchronization of Two Memristor-Based Chaotic Circuits Coupled by Resistor

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Abstract

Memristor is the fourth fundamental circuit element after the resistor, the capacitor and the inductor. The memristor has attracted worldwide attention because it has a great potential. In this study, we investigate synchronization phenomena of two memristor-based chaotic circuits coupled by a resistor. We confirm synchronization phenomena by changing the coupling strength. Also, we investigate synchronization phenomena when changing the characteristic of the memristor.

1. Introduction

Synchronization phenomena can be found in our life. Especially, synchronization phenomena have a closely related to nature world and animate beings. For example, we can confirm flashing firefly lights, croaking of a frog, bird call and so on. Synchronization phenomena have been studied in various fields because it is widely applicable.

Chaos is an unpredictable phenomenon. Nishio-Inaba circuit is one of the simplest chaotic circuits [1]. We can confirm chaotic synchronization phenomena in cases where two chaotic circuits are coupled. Chaotic synchronization phenomena have been studied extensively [2].

On the other hand, a memristor is the fourth fundamental circuit element after the resistor, the capacitor and the inductor. Leon O. Chua proposed memristor in 1971 [3]. After that, the memristor was called "missing" element for many years. However, the element was implemented by Strukov et al. [4]. The "memristor" is short for memory resistor. The element has a relationship between charge and flux. $M_{(q)}$, which is the resistance value of the memristor is changing by passing charge. $M_{(q)}$ is called memristance. The memristor has attracted worldwide attention because it has a great potential. For example, the memristor is expected to apply to super-dense nonvolatile computer memory and neural computers. Also, memristor-based chaotic circuit is an interesting research topic because memristor is useful for designing nonlinear oscillators [5]. In [5], a nonlinear oscillator was derived from Chua's circuit by replacing Chua's diode with a memristor.

In this study, we investigate synchronization phenomena of two memristor-based chaotic circuits coupled by a resistor. We confirm synchronization phenomena by changing coupling strength. Also, we investigate synchronization phenomena when changing the characteristic of the memristor.

2. System model

Figure 1 shows a system model in this study. The system model is derived from two Nishio-Inaba circuits by replacing the diodes with a memristor. Each chaotic circuit consists of two inductors L_1 and L_2, one capacitor C, one linear negative resistor $-r$ and one memristor $M_{(q)}$. q is a charge that pass through the memristor. The chaotic circuits are coupled by a resistor.

We assume that the memeristor is characterized by the monotone-increasing and the piecewise-linear nonlinearity [5]. a and b are the slope of the piecewise-linear function where $a > 0$, $b > 0$. If $a \neq b$, $M_{(q)}$ is discontinuous.

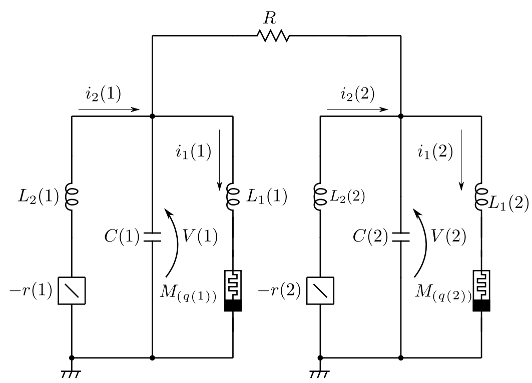


Figure 1: System model.

The circuit equations of this study are given as follows:

$$
\begin{cases}
L_1(1)\dfrac{di_1(1)}{dt} &= V(1) - M_{(q(1))}i_1(1), \\
C(1)\dfrac{dv(1)}{dt} &= i_2(1) - i_1(1) - \dfrac{1}{R}(V(1) - V(2)), \\
L_2(1)\dfrac{di_2(1)}{dt} &= -V(1) + r(1)i_2(1), \\
\dfrac{dq(1)}{dt} &= i_1(1), \\
L_1(2)\dfrac{di_1(2)}{dt} &= V(2) - M_{(q(2))}i_1(2), \\
C(2)\dfrac{dv(2)}{dt} &= i_2(2) - i_1(2) + \dfrac{1}{R}(V(1) - V(2)), \\
L_2(2)\dfrac{di_2(2)}{dt} &= -V(2) + r(2)i_2(2), \\
\dfrac{dq(2)}{dt} &= i_1(2).
\end{cases}
\tag{1}
$$

By changing the parameters:

$$
\begin{cases}
x(n) = i_1(n), \;\; y(n) = V(n), \;\; z(n) = i_2(n), \\
w(n) = q(n), \\
\alpha(n) = \dfrac{1}{L_1(n)}, \;\; \beta(n) = \dfrac{1}{L_2(n)}, \;\; \gamma(n) = \dfrac{r(n)}{L_2(n)}, \\
\delta = \dfrac{1}{R}, \;\; C(n) = 1.0, \qquad\qquad (n = 1, 2)
\end{cases}
\tag{2}
$$

the normalized circuit equations are given as follows:

$$
\begin{cases}
\dfrac{dx(1)}{dt} = \alpha(1)(y(1) + M_{(w(1))})x(1), \\
\dfrac{dy(1)}{dt} = z(1) - x(1) - \delta(y(1) - y(2)), \\
\dfrac{dz(1)}{dt} = -\beta(1)y(1) + \gamma(1)z(1), \\
\dfrac{dw(1)}{dt} = x(1), \\
\dfrac{dx(2)}{dt} = \alpha(2)(y(2) + M_{(w(2))})x(2), \\
\dfrac{dy(2)}{dt} = z(2) - x(2) + \delta(y(1) - y(2)), \\
\dfrac{dz(2)}{dt} = -\beta(2)y(2) + \gamma(2)z(2), \\
\dfrac{dw(2)}{dt} = x(2).
\end{cases}
\tag{3}
$$

The parameter δ is the coupling strength between the circuits. The memristance $M_{(w)}$ is given as follows:

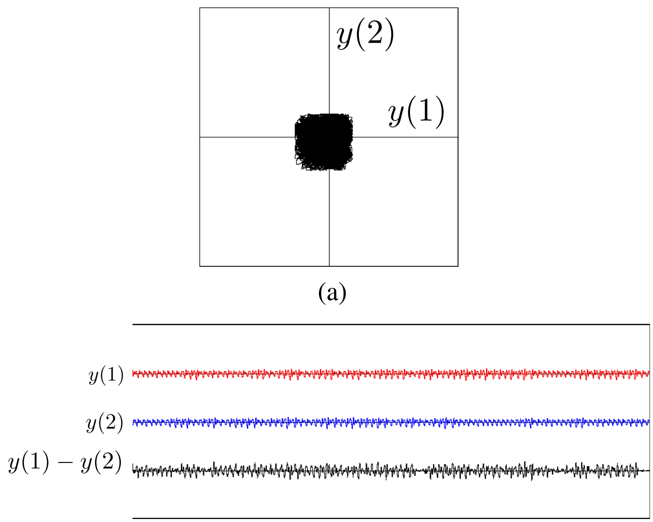
$$
M_{(w(n))} = \frac{d\varphi_{(w(n))}}{dw(n)}, \quad (n = 1, 2)
\tag{4}
$$

where $M_{(w)}$ is a when $|w| < 1.0$, and b when $|w| > 1.0$.

3. Simulation results

We set the parameters of the circuit as $\alpha(n) = 4.0$, $\beta(n) = 1.0$, $\gamma(n) = 0.525$, $a = 0.2$ and $b = 10.0$. Also, we set the time step value of the Runge-Kutta method h is 0.0001. We investigate synchronization phenomena by increasing the coupling strength δ between the circuits.

Figure 2 shows the simulation result $\delta = 0.0000001$. Figure 2 (a) shows a Lissajous figure. In the Lissajous figure, vertical axis and horizontal axis are $V(2)$ and $V(1)$. We can determine synchronization or asynchronous when the Lissajous figure is diagonal or square. Figure 2 (b) shows a voltage difference. In the waveform $y(1)$, $y(2)$ shows a voltage $V(1)$ and $V(2)$. We can determine synchronization or asynchronous when the amplitude of the waveform is high or low. In case of $\delta = 0.0000001$, two chaotic circuits are asynchronized.



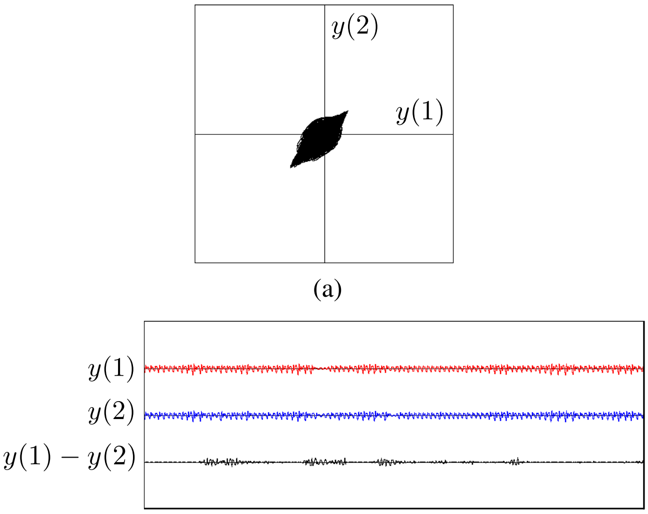
(a)

(b)

Figure 2: Simulation result. $\delta = 0.0000001$, $a = 0.2$, $b = 10.0$. (a)Lissajous figure. (b)Waveform.

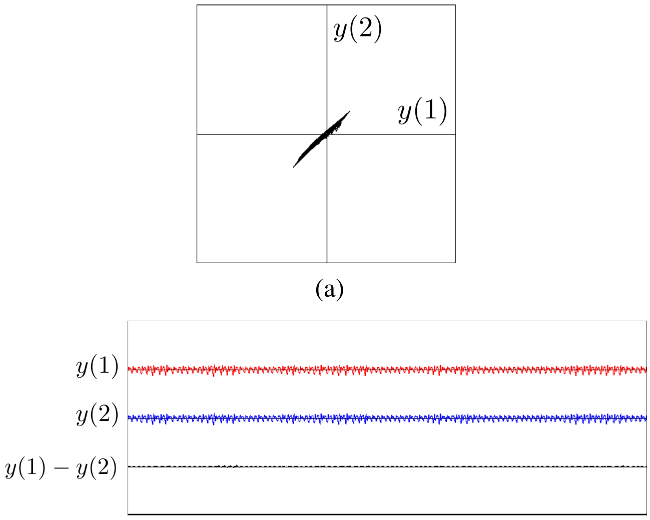
Figure 3 shows the simulation result $\delta = 0.2$, In case of $\delta = 0.2$, two chaotic circuits are synchronized than $\delta = 0.0000001$.

Figure 4 shows the simulation result $\delta = 0.6$. In case of $\delta = 0.6$, two chaotic circuits are synchronized.



(a) (b)

Figure 3: Simulation result. $\delta = 0.2$, $a = 0.2$, $b = 10.0$. (a)Lissajous figure. (b)Waveform.



(a) (b)

Figure 4: Simulation result. $\delta = 0.6$, $a = 0.2$, $b = 10.0$. (a)Lissajous figure. (b)Waveform.

We investigate synchronization phenomena by changing characteristic of memristor b. Figure 5 shows the simulation result $\delta = 0.2$, $a = 0.2$, $b = 5.0$. In case of $b = 5.0$, two chaotic circuits are synchronized than $b = 10.0$.

Figure 6 shows the simulation result $\delta = 0.2$, $a = 0.2$, $b = 100.0$. In case of $b = 100.0$, two chaotic circuits are asynchronized than $b = 10.0$.

We confirm that two chaotic circuits are syncrhonized by changing coupling strength. Also, we confirm that the synchronization phenomena are changed when changing the characteristic of the memristor b.

4. Conclusions

In this study, we have investigated the system model using two memristor-based circuits that are coupled by a resistor. As a result, two chaotic circuits are synchronized when increasing coupling strength. Also, we confirm that the chaotic circuits are synchronized when decreasing the characteristic of the memristor b and asynchronized when increasing b.

In the future works, we will investigate the effect of the memristor on the chaotic circuit by comparing with two coupled original Nishio-Inaba circuits. Also, we investigate the effect of when changing the characteristic of the memristor a.

Acknowledgment

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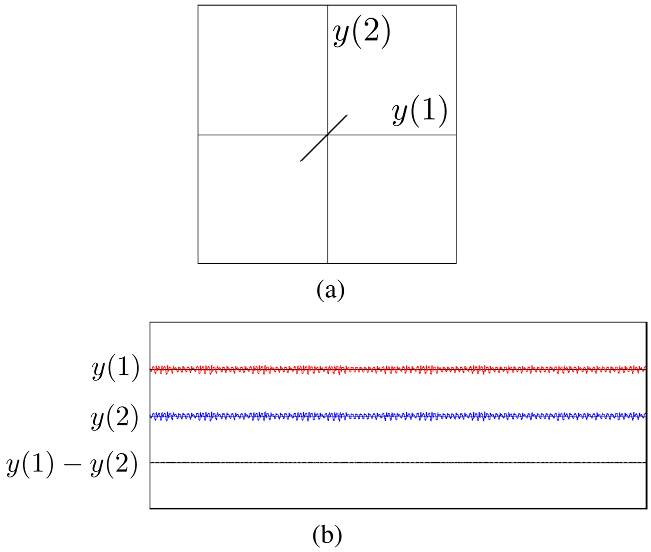

Figure 5: Simulation result. $\delta = 0.2$, $a = 0.2$, $b = 5.0$. (a)Lissajous figure. (b)Waveform.

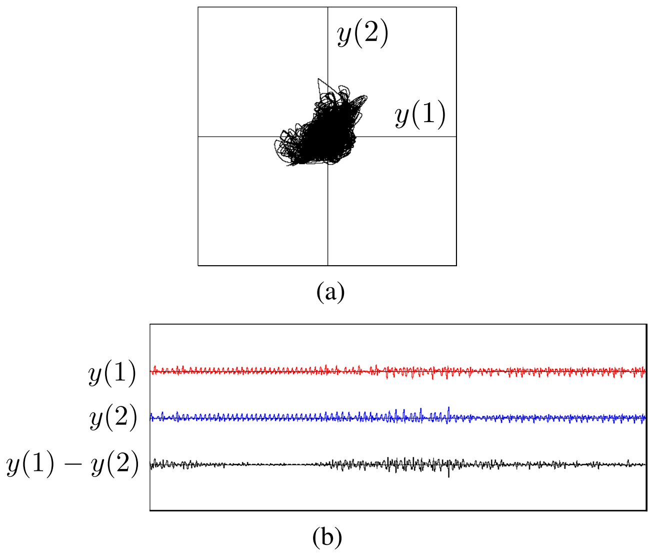

Figure 6: Simulation result. $\delta = 0.2$, $a = 0.2$, $b = 100.0$. (a)Lissajous figure. (b)Waveform.