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Synchronization Phenomena in a Ring of van der Pol Oscillators Coupled by Time-Varying Resistor

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Abstract

In this study, we investigate the synchronization phenomena in a ring of van der Pol oscillators coupled by Time-Varying Resistor (TVR). We realize the TVR by switching a positive and a negative resistor periodically. When the frequency is high, positive resistor and negative resistor switch quickly. We use three TVRs, and each TVR has different frequencies. And, we observe the synchronization phenomena in the proposed circuit system by changing the frequency of TVRs and the duty ratio.

1. Introduction

The synchronization phenomena are the fundamental phenomena in nature and the synchronization phenomena are observed over the various fields; for example, biological fields, physical fields and electrical fields. The natural synchronization phenomena can be imitated with electric circuit. The reason why we use electric circuit is guaranteed high reproducibility of the experiment and short experimental time. Hence, many researchers are investigating the relationship among the synchronization phenomena coupled by oscillators and the natural synchronization phenomena.

This time, we use van der Pol oscillators. van der Pol oscillator has very simple structure, this oscillator consists of nonlinear resistor, conductor, and inductor. van der Pol oscillators are used to model natural phenomena.

In our previous research, we have investigated the synchronization phenomena in three van der Pol oscillators coupled by Time-Varying Resistor (TVR) [1]. TVR is a device which positive resistor and negative resister are periodically switched. We change the frequency of TVR, when the frequency is high, positive resistor and negative resistor switch quickly.

In this study, we propose coupled oscillatory systems such as a ring of van der Pol oscillators coupled by TVR. We use three TVRs, and each TVR has different frequencies. We investigate the synchronization phenomena observed in the proposed circuit system by changing the frequency of TVR[2].

2. System model

Figure 1 shows van der Pol oscillator and Fig. 2 shows a system model constituted van der Pol oscillators (VDP). We use a ring of van der Pol oscillators, three VDPs are connected by Time-Varying Resistor (TVR1, TVR2 and TVR3). We realize the TVR by switching a positive and a negative resistor periodically as shown in Fig. 3. We investigate the synchronization phenomena, how changing by the frequency of TVR1, TVR2 and TVR3 and the duty ratio. When three TVRs are connected by resistors, it stables in-phase synchronization phenomena. On the other hand, when three TVRs are connected by negative resistors, it stables three phase synchronization phenomena. When the duty ratio p is larger than 0, it stables three phase synchronization phenomena. When the duty ratio p is smaller than 0, it stables in-phase synchronization phenomena.

Firstly, the i_{gk}-v_k characteristics of the nonlinear resistor are defined as follows,

$$i_{gk} = -g_1 v_k + g_3 v_k^3. \qquad (1)$$

By changing variable and the parameters,

$$v_k = \sqrt{\frac{g_1}{g_3}} x_k,\ i_k = \sqrt{\frac{g_1}{g_3}} \sqrt{\frac{C}{L}} y_k,\ t = \sqrt{LC}\tau \qquad (2)$$

$$\varepsilon = g_1 \sqrt{\frac{L}{C}},\ \gamma = \frac{1}{r}\sqrt{\frac{L}{C}}, \qquad (3)$$

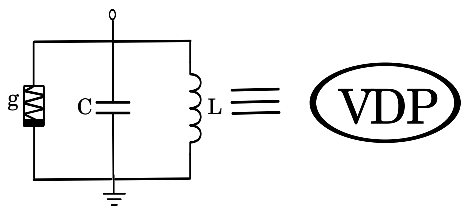


Figure 1: van der Pol oscillator.

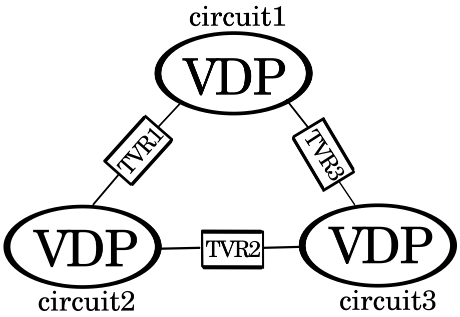


Figure 2: Circuit model.

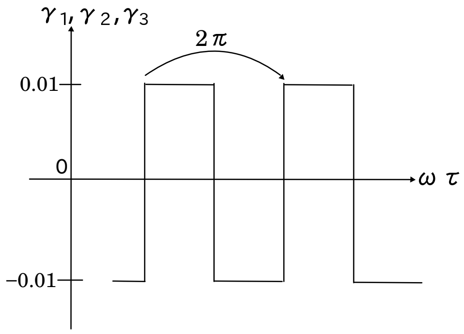


Figure 3: Characteristics of the TVR.

The normalized equations of VDP circuit are given as follows:

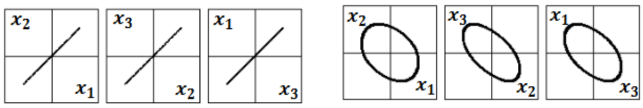
$$\begin{cases} \dot{x_1} = \varepsilon(x_1 - x_1^3) - y_1 + \gamma_1(x_2 - x_1) + \gamma_3(x_3 - x_1) \\ \dot{x_2} = \varepsilon(x_2 - x_2^3) - y_2 + \gamma_1(x_1 - x_2) + \gamma_2(x_3 - x_2) \\ \dot{x_3} = \varepsilon(x_3 - x_3^3) - y_3 + \gamma_2(x_1 - x_3) + \gamma_3(x_2 - x_3) \\ \dot{y_n} = x_n. \end{cases} \quad (4)$$

In these equations, n is the number of circuits and n = 1,2,3. ε denotes the nonlinearity of the oscillators, γ_1, γ_2, and γ_3 denote the coupling strengths of the TVR1, TVR2 and TVR3.

3. Simulation results

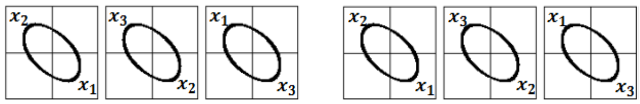
3.1 Same frequency

The simulation results of the system model are shown from Fig. 4 and Fig. 5. The value of the parameters is set to $\varepsilon = 0.1$, $\gamma_1 = \pm 0.01$, $\gamma_2 = \pm 0.01$ and $\gamma_3 = \pm 0.01$. The frequencies of TVR1, TVR2 and TVR3 sets with f_1, f_2 and f_3. Where, $f_n = 2\pi\omega_n$. The figure on the left shows the phase difference when the initial condition set with in-phase. The figure on the right shows phase difference when the initial condition set with 3-phase. Firstly, in case of $f_1 = f_2 = f_3$ = 0.09, and duty ratio p = 0, we can observe in-phase synchronization phenomena and 3-phase synchronization phenomena. We can observe both synchronization phenomena due to change by initial value. Secondly, in case of $f_1 = f_2 = f_3$ = 0.1, we observe only three phase synchronization phenomena, it is observed regardless of initial value.



(a) in-phase. (b) 3-phase.

Figure 4: Phase difference ($f_1 = f_2 = f_3 = 0.09$).



(a) in-phase. (b) 3-phase.

Figure 5: Phase difference ($f_1 = f_2 = f_3 = 0.1$).

3.2 Different frequency

Next, we investigate the influence of the duty ratio. The duty ratio sets with p. If p is larger than 0, synchronization stables three phase synchronization phenomena. If p is smaller than 0, synchronization stables in-phase synchronization phenomena.

Figure 6 shows the characteristics of the TVR in the case of the duty ratio p included.

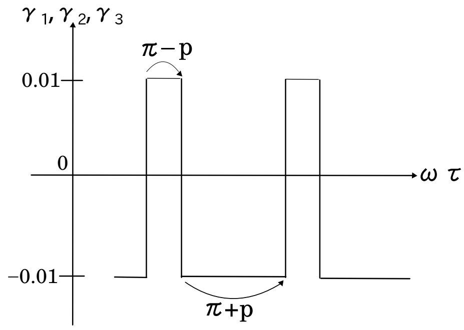


Figure 6: Characteristics of the TVR.

Figures 7 and 8 show the influence of the duty ratio p. In case of $f_1 = f_2 = f_3 = 0.09$, when p is between -0.01 and 0.005, we can observe both synchronization phenomena. In case of $f_1 = f_2 = f_3 = 0.1$, when p is between -0.01 and -0.001, we can observe both synchronization phenomena. Hence, we can handle the synchronization phenomena easily by changing the duty ratio.

Next, we investigate the phase difference of TVR when each TVR has different frequencies. At first, we show the phase difference between the oscillators. The value of the parameters is set to $\varepsilon = 0.1$, $\gamma_1 = \pm 0.01$, $\gamma_2 = \pm 0.01$, $\gamma_3 = \pm 0.01$, and $p = 0$.

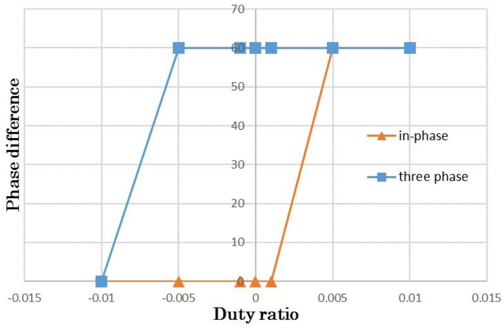


Figure 7: Phase difference ($f_1 = f_2 = f_3 = 0.09$).

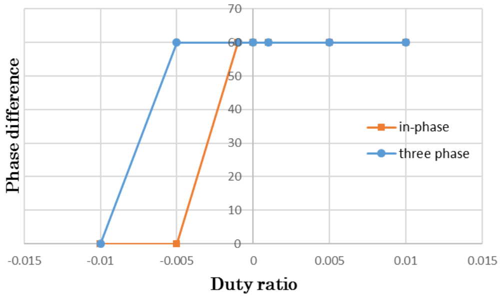
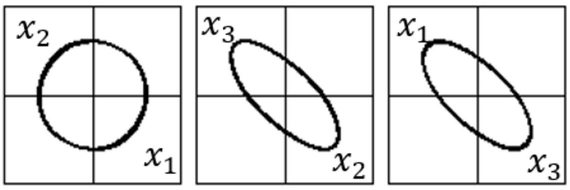
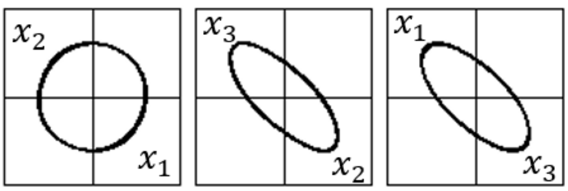


Figure 8: Phase difference ($f_1 = f_2 = f_3 = 0.1$).

In case of $f_1 = 0.1$, $f_2 = 0.2$, $f_3 = 0.15$, we can observe only three synchronization phenomena.
From Figs. 9 to 12 show the influence of the duty ratio,



(a) in-phase.
Figure 9: Phase difference ($f_1 = 0.1$, $f_2 = 0.2$, $f_3 = 0.15$).



(b) three-phase.
Figure 10: Phase difference ($f_1 = 0.1$, $f_2 = 0.2$, $f_3 = 0.15$).

Figures 11 and 12 show the influence of the duty ratio. In Fig. 11, initial condition set with in-phase. In Fig. 12, initial condition set with three phase.

When the phase difference is near at 0, this is in-phase synchronization phenomena. When the phase difference is near at 60, this is three phase synchronization phenomena.

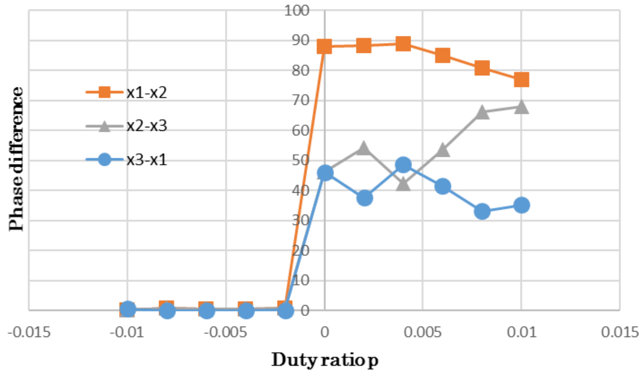


Figure 11: Phase difference (in-phase).

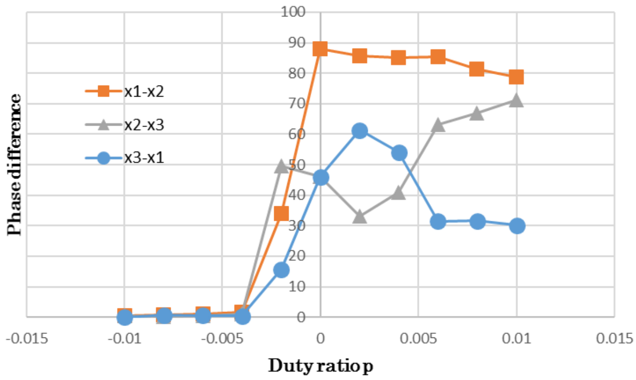


Figure 12: Phase difference (three phase).

In Fig. 11, when the duty ratio p is larger than 0, we can observe almost three phase synchronization phenomena. And when the duty ratio p is smaller than -0.002, we can observe in-phase synchronization phenomena.

In Fig. 12, when the duty ratio p is larger than -0.002, we can observe almost three phase synchronization phenomena. And when the duty ratio p is smaller than -0.004, we can observe in-phase synchronization phenomena.

4. Conclusions

We have proposed a system model using a ring of three van der Pol oscillators coupled by TVR. We can observe various synchronization phenomena by varying frequency of TVR and the duty ratio. When three frequencies of TVR (f_1, f_2, f_3) equal, we can observe in-phase synchronization phenomena, and 3-phase synchronization phenomena. However, when we changed the duty ratio, we can choose the kind of the synchronization phenomena. And we realize that we can handle the synchronization phenomena easily.

In the future, we would like to investigate the synchronization phenomena using a more complicated system by changing the frequency and the duty ratio.

References

[1] Y. Uwate, Y. Nishio, "Two van der Pol Oscillators Coupled by Chaotically Varying Resistor", Proceedings of International Workshop on Nonlinear Dynamics of Electric Systems (NDES'06), pp.189-192, Jul. 2006.

[2] D. Nariai, M. Tran, Y. Uwate, Y. Nishio, "Synchronization in Two Rings of Coupled Three van der Pol Oscillators", Proceedings of RISP International Workshop on Nonlinear Circuits, Communications and Signal Processing (NCSP'17), pp. 325-328, Feb. 2017.