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Synchronization Phenomena due to Changes in Network Structure of Coupled Chaotic Circuits in Complex Networks

Shuhei Hashimoto, Takahiro Chikazawa, Yoko Uwate, and Yoshifumi Nishio

Dept. of Electrical and Electronic Engineering, Tokushima University
2-1 Minami-Josanjima, Tokushima 770–8506, Japan
Email: {s-hashimoto, chikazawa, uwate, nishio}@ee.tokushima-u.ac.jp

Abstract

In this study, we consider the synchronization of coupled chaotic circuits which have different network structure. Especially, we focus on network structure with hub which is consisted of Scale-free network. So, we compare synchronization phenomena of some network models created by three types of degree distribution and evaluate them by synchronization rate. From the simulation results, we can confirm that connection between hubs is important network structure.

1. Introduction

Synchronization phenomena can be observed everywhere in our life. For example, we can confirm metronome, flashing firefly lights, beating rhythm of the heart and so on. Especially, synchronization phenomena of oscillatory network are interesting. In addition, complex networks attract attention from various fields. The feature of networks is characterized by the degree distribution, the path length and the clustering coefficient.

Complex networks of chaotic circuits have been studied [1]-[3]. However, many researchers have not been researched more about synchronization phenomena in complex networks of coupled chaotic circuits which compare degree distribution. In Previous study, we have investigated synchronization phenomena of coupled chaotic circuis network which made by three types of the degree distribution [4]. However, expected results were not obserbed.

In this study, we change the circuit which use in the research, and we compare synchronization phenomena of some network models created by three types of degree distribution. Furthermore, we focus on network structure like a hub. Complex networks of real world have characters like scale free property, cluster property, small world property [2][3]. So, we use degree distribution models based on the power law and the normal distribution. Scale free network follows the power law. Random network follows the normal distribution. Furthermore, we use the soaring distribution which is different from the others. We compare synchronization rate of some network models based on three types of degree distribution by changing the coupling strength. Also we investigate synchronization phenomena of each connection of network with hubs.

2. Circuit model

The chaotic circuit model which proposed by Shinriki $et\ al.$ [4][5] is shown in Fig. 1. This circuit consists of a negative resister, an inductor, two capacitors and dual-directional diodes. In this study, we use network model which is 10 coupled chaotic circuits. The circuit equation is shown in Eq. (1).

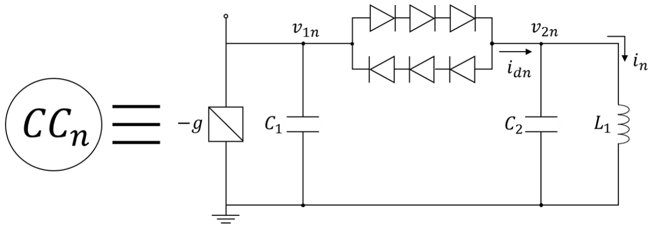

Figure 1: Circuit model.

$$\begin{cases} L\dfrac{di_n}{dt} = v_{2n} \\ C_1\dfrac{dv_{1n}}{dt} = gv_{1n} - i_{dn} - \dfrac{1}{R}\displaystyle\sum_{k\in S_n}(v_{1n} - v_{1k}) \\ C_2\dfrac{dv_{2n}}{dt} = -i_n - i_{dn}, \end{cases} \quad (1)$$

where $n = 1, 2, 3, \cdots, 10$ and S_n is the set of nodes which are directly connected to the node n. The characteristic of nonlinear resistor which consists of dual diodes is following Eq. (2).

$$i_{dn} = \begin{cases} G_d(n_{1n} - v_{2n} - V) & (v_{1n} - v_{2n} > V) \\ 0 & (|n_{1n} - v_{2n}| \le V) \\ G_d(n_{1n} - v_{2n} + V) & (v_{1n} - v_{2n} < -V). \end{cases} \quad (2)$$

By changing the variables and parameters,

$$
\begin{cases}
i_n = \sqrt{\dfrac{C_2}{L}} V x_n, \;\; v_{1n} = V y_n, \;\; v_{2n} = V z_n \\[2ex]
\alpha = \dfrac{C_2}{C_1}, \;\; \beta = \sqrt{\dfrac{L}{C_2}} G_d, \;\; \gamma = \sqrt{\dfrac{L}{C_2}} g, \\[2ex]
\delta = \dfrac{1}{R}\sqrt{\dfrac{L}{C_2}}, \;\; t = \sqrt{LC_2}\tau, \;\; \text{"}\cdot\text{"} = \dfrac{d}{dt},
\end{cases}
\tag{3}
$$

the normalized equations of this circuit are given as follows:

$$
\begin{cases}
\dot{x} = z_n \\[1ex]
\dot{y} = \alpha\gamma y_n - \alpha\beta f(y_n - z_n) - \alpha\delta \displaystyle\sum_{k \in S_n} (y_n - y_k) \\[1ex]
\dot{z} = \beta f(y_n - z_n) - x_n.
\end{cases}
\tag{4}
$$

The parameter δ corresponds the coupling strength between the circuits. The parameter α is the strength of negative resistor. The nonlinear function $f(y_n - z_n)$ corresponds to the characteristics of the nonlinear resistor consisting of the diodes and described as follows: where $f(y)$ is described as follows :

$$
f(y_n - z_n) = \begin{cases}
y_n - z_n - 1 & (y_n - z_n > 1) \\
0 & (|y_n - z_n| \leq 1) \\
y_n - z_n + 1 & (y_n - z_n < -1).
\end{cases}
\tag{5}
$$

This circuit generates asymmetric chaotic attractor in the parameter $\gamma = 0.5$ and symmetric chaotic attractor in the parameter $\gamma = 0.65$ as shown in Fig. 2.

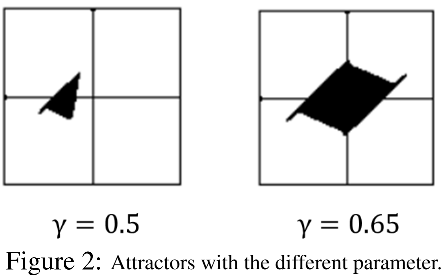

Figure 2: Attractors with the different parameter.

3. Network model

In this study, we create networks which are 10 coupled chaotic circuits and each circuit connected by a resistor. Each network is composed by each degree distribution. Network model which used in this research is shown in Fig. 3.

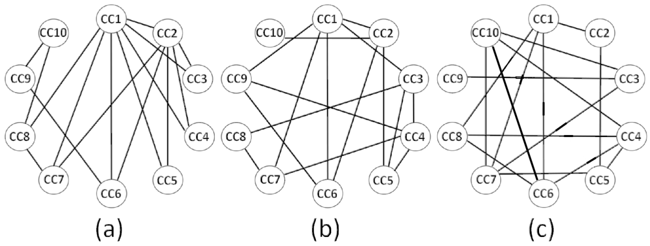

Figure 3: Network model.

Degree distribution A, B and C in Fig. 4 show the degree distribution which used for composing network in this study. Degree distribution A imitates the degree distribution of the power law. Degree distribution B imitates the degree distribution of a random network. Degree distribution C is the degree distribution of soaring. We fix that the number of circuits in each network model is 10 and the number of edges in each network model is 16.

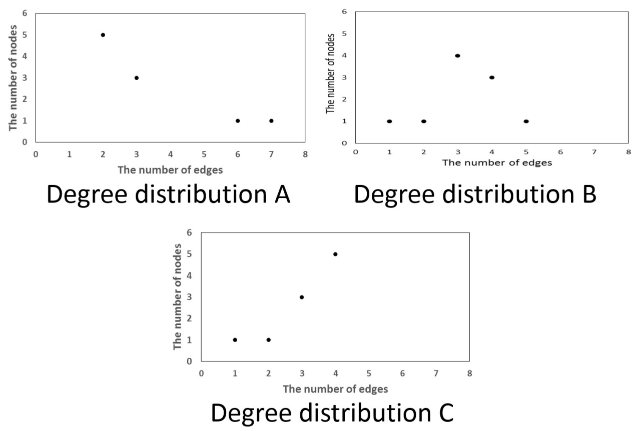

Figure 4: Degree distribution.

The path length of each network model is nearly the same value. In addition to these networks model, we make other model with the different path length. These network models in Fig. 5 also created based on each degree distribution. The path length of model (a) is longer than (a'). The path length of model (b) and (c) are shorter than model (b') and (c') each other. The path length of each network is shown in Table 1.

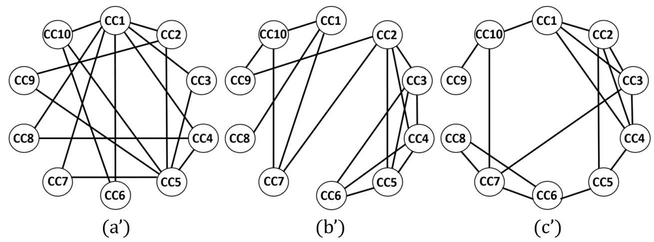

Figure 5: Network model with the different path length.

Table 1: Path length of each network model.

	Path length
(a)	1.822
(a')	1.689
(b)	1.844
(b')	2.156
(c)	1.889
(c')	2.0

4. Simulation Result

Definition of synchronization in this study is determined a voltage difference waveform. We define synchronization as the following Eq. (6).

$$|z_j - z_i| < 0.15 \qquad (i, j = 1, 2, \cdots, 10) \qquad (6)$$

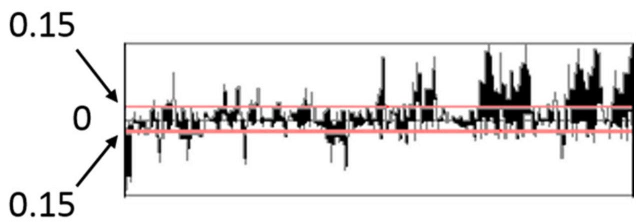

Figure 6: A difference waveform.

Figure 6 is a different waveform which was observed in this simulations. The two lines in Fig. 6 correspond threshold which is given Eq. (7). It is determined that wave within two lines which is the threshold is synchronization. Therefore, we propose and investigate the synchronization probability denoted the synchronization rate during a certain time interval. In this research, we fix a certain time interval as (τ=1,000,000 and step=0.02τ) and investigate the synchronization rate in the entire network of 10 coupled chaotic circuits.

4.1 Comparing synchronization rate of network model

In this simulation, we use two parameters of the strength of negative resistor. Attractors of these parameters are shown in Fig. 2. Synchronization rate which set the parameter $\gamma = 0.5$ is shown in Figs. 7 and the parameter $\gamma = 0.65$ is shown in Fig. 8.

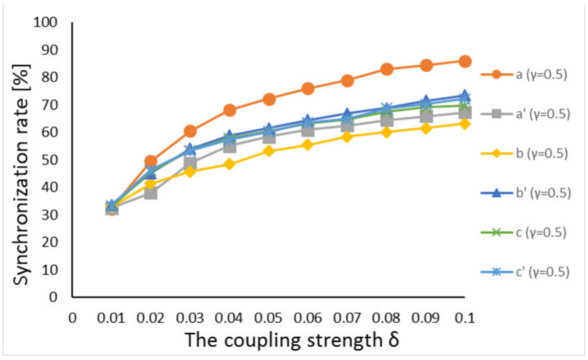

Figure 7: Shynchronization rate in the parameter $\gamma = 0.5$.

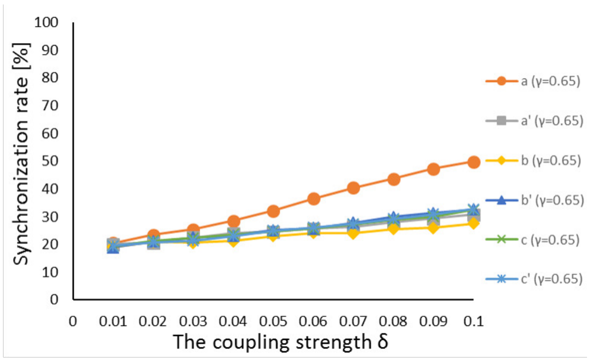

Figure 8: Shynchronization rate in the parameter $\gamma = 0.65$.

In the case of the parameter $\gamma = 0.5$, synchronization rate of the network model (a) is highest. Furthermore, when we compare synchronization rate with different path length, network model with long path length obtains high synchronization rate than short one model based on the same degree distribution. In the case of the parameter $\gamma = 0.65$, it is almost the same result above. However, synchronization rate of all network models is lower and less difference except for network model (a).

4.2 Synchronization rate of each connection in network model (a)

In the section 5.1, we observed synchronization rate of network model (a) is highest even if changing the parameter. So, we investigate synchronization rate of model (a) in detail. Figure 9 shows synchronization rate of each connection in network model (a). In Fig. 9, we can confirm that synchronization rate of one connection obtains high. This connection corresponds that of between hub. So, synchronization rate of network which have connection between node with many coupling is high.

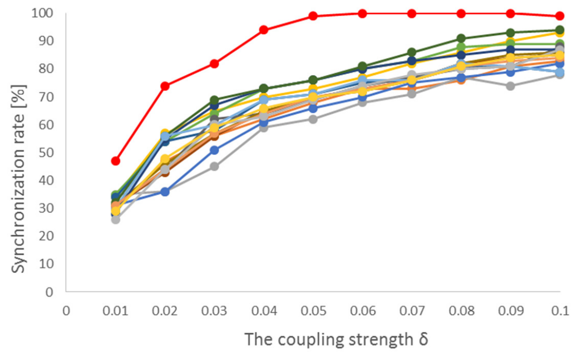

Figure 9: Shynchronization rate of each connection in network model (a).

5. Conclusion

In this study, we have investigated synchronization rate of several network topology structures by changing the coupling strength. Furthermore, we investigate synchronization phenomena with the different parameter. The network topology structures are following degree distribution of three types.

When we compare network model in the parameter $\gamma = 0.5$, we have confirmed that network model which have long path length get high synchronization rate than short one. When we compare network model in the parameter $\gamma = 0.65$, the difference of synchronization rate become small except network model (a). Then, we have investigated synchronization rate of each connection in network model (a). We have confirmed that connection between hub is important structure.

For the future works, we would like to develop the network model by adding a weight to the coupling. And we investigate synchronization phenomena of larger network.

Ackowledgment

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References

[1] K. Ago, Y. Uwate, Y. Nishio, "Influence of Bridge on Coupled Chaotic Circuit Network", Proc. of RISP International Workshop on Nonlinear Circuits, Communications and Signal Processing (NCSP'14), pp. 417-420, Mar. 2014.

[2] K. Ago, Y. Uwate, Y. Nishio, "Investigation of Partical Synchronization in Coupled Chaotic Circuit Network with Local Bridge", Proc. of IEEE Workshop on Nonlinear Cirsuit Networks (NCN'14), pp.63-66, Dec. 2014.

[3] T. Chikazawa, Y. Uwate, Y. Nishio, "Chaos Propagation in Coupled Chaotic Circuits with Multi-Ring Combination", Proc. of IEEE Asia Pacific Conference on Circuits and Systems (APCCAS'16), pp. 65-68, Oct. 2016.

[4] S. Hashimoto, T. Chikazawa, Y. Uwate, Y. Nishio, "Synchronization Phenomena in Complex Networks of Coupled Chaotic Circuits with Different Degree Distribution", Proc. of RISP International Workshop on Nonlinear Circuits, Communications and Signal Processing (NCSP'17), pp. 333-336, Feb. 2017.

[5] D. J. Watts and S. H. Strogatz, "Collective Dynamics of Small-world", Nature, vol.393, pp.440-442, 1998.

[6] A. L. Barabasi and R. Albert, "Emergence of Scaling in Random Networks", Science, vol.286, pp.509-512, 1999.

[7] M. Shinriki, M. Yamamoto and A. Mori, "Multimode Oscillations in a Modified van der Pol Oscillator Conraining a Positive Nonlinear Conductance", Proc. IEEE, vol. 69, pp. 394-395, 1981.

[8] N. Inaba, T. Saito and S. Mori, "Chaotic Phenomena in a Curcuit with a Negative Resistance and an Ideal Switch of Diodes", Trans. of IEICE, vol.E70, no. 8, pp. 744-754, 1987.