Chaotic Circuits Network with Scale-Free Coupling Distribution

Yoko Uwate
Dept. of Electrical and Electronic Engineering,
Tokushima University
2-1 Minami-Josanjima, Tokushima 770-8506, Japan
uwate@ee.tokushima-u.ac.jp

Yoshifumi Nishio
Dept. of Electrical and Electronic Engineering,
Tokushima University
2-1 Minami-Josanjima, Tokushima 770-8506, Japan
nishio@ee.tokushima-u.ac.jp

***Abstract*—In this study, we investigate synchronization observed in chaotic circuits network with scale-free coupling distribution. The network topology is inspired from real brain network. Here, we consider the network which consists of two modules including connector and provincial hubs. We confirm several clustering patterns from the proposed system.**

***Keywords*—brain network; chaotic circuits network; clustering**

I. Introduction

We have focused on synchronization phenomena observed from nonlinear oscillatory circuits networks [1], [2]. This is because the results of synchronization in complex networks are useful deeper understanding of control methods in power networks, communication systems and so on. Also, they can be used as an alternative approach, apart from existing ones, for describing mode-locking phenomena in biological networks. Therefore, we consider the schematic brain networks as one of complex biological networks. Because, we would like to propose modeling of synchronization in brain by using coupled electrical oscillatory circuits, in order to make clear the mechanism of functional operation in brain. Structural and functional brain networks are explored using graph theory and the brain network structures have been made clear [3]. Furthermore, several research groups have reported that the synaptic connectivity in local cortical circuits has scale-free distribution [4]. We also apply this scale-free characteristics of coupling distribution for the proposed system in this study. First, a symmetric diagram of a brain network is used as a simple network model. Then, we extend the proposed network to asymmetric circuits network topologies. By using computer simulations, we confirm that the chaotic circuits network with scale-free coupling distribution can produce several kinds clustering patterns.

II. Proposed System

A. Network Model

A network model composed of 51 nodes is shown in Fig. 1. There are two important hubs in this network, “Connector hub” and “Provincial hub”. The both hubs are high-degree nodes. “Connector hub” shows a diverse connectivity by connecting two sub-networks. “Provincial hub” primarily connects nodes in the same sub-network. There are two modules and each module consists 24 chaotic circuits. The node is expressed by Nishio-Inaba chaotic circuits as shown in Fig. 2. The chaotic circuits are coupled by a resistor.

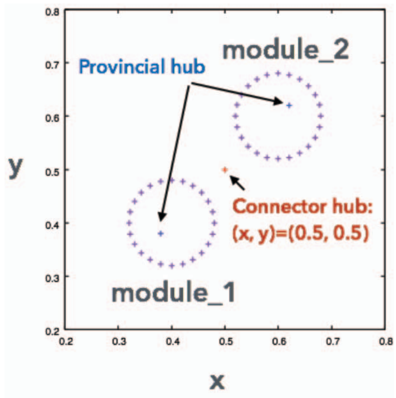


Fig. 1. Network model.

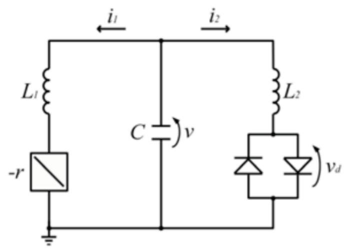


Fig. 2. Nishio-Inaba chaotic circuit.

Next, we consider link probability in the network. The link probability is expressed by the following equation.

$$p_{i,j} = \theta \cdot d_{i,j}{}^{-\alpha} + (1-\theta) \cdot d_{i,j}{}^{-\beta} \quad (1)$$

By using Eq. (1), the network coupling is shown in Fig. 3. Here, the link probability of the connector and provincial hubs have two times. For the computer simulations, we calculate circuit equations using the fourth-order Runge- Kutta method.

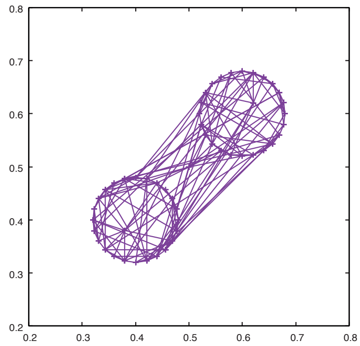


Fig. 3. Coupling distribution in symmetric network.

B. Synchronization Phenomena

In order to investigate synchronization, we calculate the five types of the phase differences as follows.

1) phase_12: connector hub to provincial hub-1
2) phase_13: connector hub to provincial hub-2
3) phase_23: provincial hub-1 to provincial hub-2
4) phase_2g: circuits in module-1
5) phase_3g: circuits in module-2

Figure 4 shows the simulation result of phase difference by changing the coupling strength *g*. By increasing the coupling strength, all chaotic circuits are synchronized. Namely, in case of symmetric network topology, we observe one clustering phenomena.

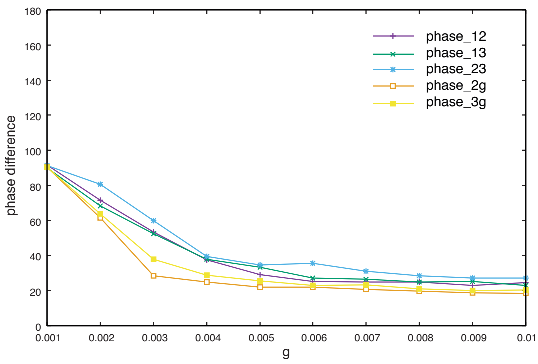
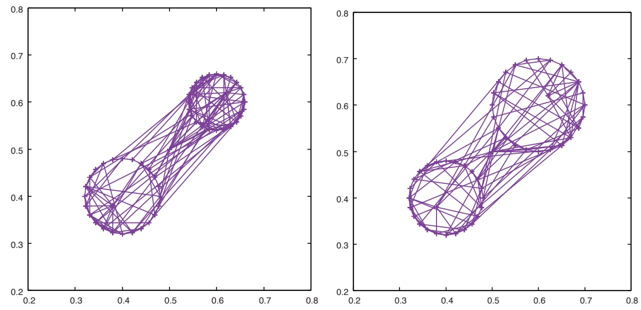


Fig. 4. Phase difference in symmetric network.

III. Asymmetric Topologies

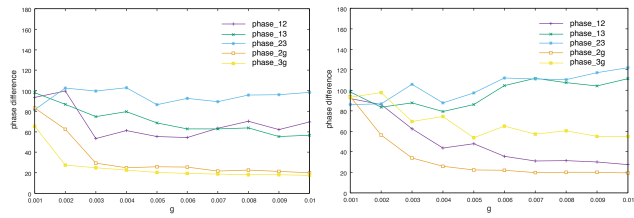
Next, we consider asymmetric networks as shown in Fig. 5. In Fig. 5(a), the radius of module-2 is smaller than module-1. While, in Fig. 5(b), the radius of module-2 is larger than module-1. The link probability is applied by using Eq. (1).

The simulation results of the phase differences are shown in Fig. 6. By increasing the coupling strength, we observe clustering pattern. Figure 7 shows the clustering pattern by using phase differences. In Fig. 7 (a), there are two clusters and two modules do not synchronize with the connector hub. In Fig. 7 (b), there is one cluster and module-1 synchronize with connector hub. From these results, we can see that the network symmetrically has important role for synchronization and clustering pattern in complex networks.



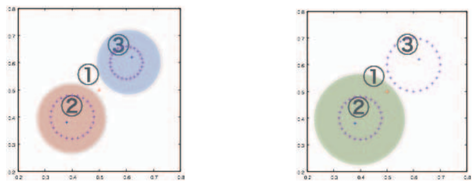
(a) pattern-1. (b) pattern-2.

Fig. 5. Coupling distribution in asymmetirc networks.



(a) pattern-1. (b) pattern-2.

Fig. 6. Phase difference in asymmetric networks.



(a) pattern-1. (b) pattern-2.

Fig. 7. Clustering in asymmetric networks.

IV. Conclusions

In this study, we have investigated synchronization observed in chaotic circuits network with scale-free coupling distribution. The network topology is inspired from real brain network. Here, we consider the network which consists of two modules including connector and provincial hubs. We have confirmed several clustering patterns from the proposed system.

References

[1] Y. Uwate and Y. Nishio, “Synchronization in Several Types of Coupled Polygonal Oscillatory Networks,” *IEEE Trans. Circuits Syst. I*, vol. 59, no. 5, pp. 1042-1050, May 2012.

[2] Y. Uwate and Y. Nishio, ”Frustrated Synchronization in Two Coupled Polygonal Oscillatory Networks,” *Proc. of ISCAS’13*, pp. 1781-1784, May 2013.

[3] E. Bullmore and O. Sporns, “Complex Brain Networks: Graph Theoretical Analysis of Structural and Functional Systems,” *Nature, Reviews, Neuro*, vol.10, pp. 186-198, Mar. 2009.

[4] R. Stoop, V. Saase, C. Wagner, B. Stoop and R. Stoop, “Beyond Scale-Free Small-World Networks: Cortical Columns for Quick Brains” *Physical Review Letter*, 110, 108105, 2013.