Spread of Chaotic Behavior in Scale-free and Random Networks

Takahiro Chikazawa, Yoko Uwate and Yoshifumi Nishio
Dept. of Electrical and Electronic Engineering, Tokushima University
2-1 Minami-Josanjima, Tokushima 770–8506, Japan
Email: {chikazawa,uwate,nishio}@ee.tokushima-u.ac.jp

***Abstract*—In this study, we investigate the spread of chaotic behavior to complex network. We propose three types random network models with different network topology. In each network model, the proposed network is composed of coupled chaotic circuits when one circuit is set to generate chaotic attractor and the other circuits are set to generate three-periodic attractors. By using computer simulations, we investigate ratio of propagation and spread of chaotic behavior by changing the initial chaos position. We compare their differences of simulation results in scale-free and random networks. From the simulation result, in scale-free network, the ratio of propagation is effected by clustering coefficient and path length.**

I. Introduction

In the complex social network, various type of propagation have attracted a great deal of attention from various fields. The pandemic outbreak of viral infection and the traffic jam of the transportation network are mentioned as an example of propagation in the real network. It is important to investigate the chaos propagation and the spread of chaotic behavior under some difficult situations for the circuits. For example of some difficult situations, network is briefly given external stimulation and frustration is occurred in the network. Therefore we consider that it is necessary to investigate that behavior of unlike the others influence the whole system. In the biology, mechanism of the communication in nerve cells and viral infection have not figure out yet. Furthermore, it is difficult to analyze propagation mechanism. So, we can prevent the unknown virus spreading if we comprehend the mechanism of the communication in nerve cells and viral infection. Additionally, it is important to investigate propagation phenomena observed from coupled chaotic circuits for future engineering applications. However, there are not many studies of large-scale network of continuous-time real physical systems such as electrical circuits [1]-[3].

In our research group, the chaos propagation is already studied in simple networks such as ring topology [4]. Moreover, we investigate the spread of chaotic behavior in scale-free network [5]. In these previous studies, the spread of chaotic behavior becomes weak when the initial chaos position is set to the high degree node.

In this study, we propose random networks using of chaotic circuits coupled by the resistors as different topology network. In these model, one circuit is set to generate chaotic attractor and the other circuits are set to generate three-periodic attractors. First, in each model, we investigate ratio of propagation by changing the initial chaos position according to degree distribution. Next, we investigate ratio of propagation by comparing scale-free and random networks. Furthermore, we investigate the spread of chaotic behavior in three types random networks.

978-1-5386-0524-0/17/$31.00 ©2017 IEEE

II. Circuit model

The chaotic circuit is shown in Fig. 1. This circuit consists of a negative resistor, two inductors, a capacitor and dual-directional diodes. This chaotic circuit is called Nishio-Inaba circuit.

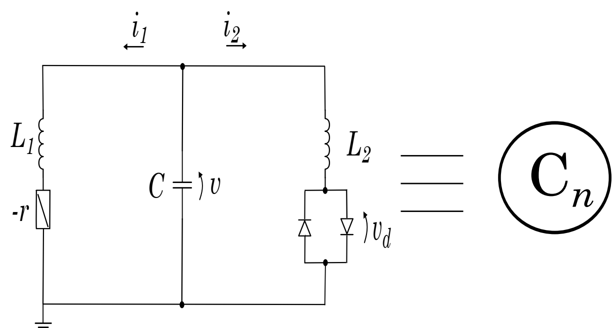


Fig. 1. Chaotic circuit.

The circuit equations of this circuit are described as follows:

$$\begin{cases} L_1 \dfrac{di}{dt} = v + ri \\ L_2 \dfrac{di}{dt} = v - v_d \\ C \dfrac{dv}{dt} = -i_1 - i_2, \end{cases} \tag{1}$$

The characteristic of nonlinear resistance is described as follows:

$$v_d = \frac{r_d}{2}\left(\left|i_2 + \frac{V}{r_d}\right| - \left|i_2 - \frac{V}{r_d}\right|\right). \tag{2}$$

By changing the variables and parameters,

$$\begin{cases} i_1 = \sqrt{\dfrac{C}{L_1}}Vx_n, \quad i_2 = \dfrac{\sqrt{L_1C}}{L_2}Vy_n, \quad v = Vz_n \\ \alpha = r\sqrt{\dfrac{C}{L_1}}, \quad \beta = \dfrac{L_1}{L_2}, \quad \delta = r_d\dfrac{\sqrt{L_1C}}{L_2}, \\ \gamma = \dfrac{1}{R}\sqrt{\dfrac{L_1}{C}}, \quad t = \sqrt{L_1C_2}\tau, \end{cases} \tag{3}$$

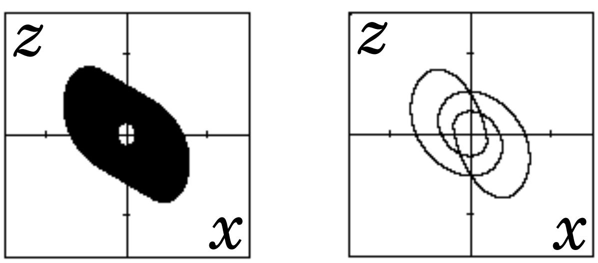
In the proposed system, each circuit is connected to only adjacent circuits by the resistors.The normalized circuit equations of the system are given as follows:

$$\begin{cases} \dfrac{dx_i}{d\tau} = \alpha x_i + z_i \\ \dfrac{dy_i}{d\tau} = z_i - f(y_i) \\ \dfrac{dz_i}{d\tau} = -x_i - \beta y_i - \displaystyle\sum_{j\in S_n} \gamma(z_i - z_j) \\ \qquad\qquad (i, j = 1, 2, \cdots, N). \end{cases} \tag{4}$$

In Eq. (4), N is the number of coupled chaotic circuits and γ is the coupling strength. $f(y_i)$ is described as follows:

$$f(y_i) = \frac{1}{2}\left(\left|y_i + \frac{1}{\delta}\right| - \left|y_i - \frac{1}{\delta}\right|\right). \tag{5}$$

In this chaotic circuit, we define α_c to generate the chaotic attractor (see Fig. 2(a)) and α_p is defined to generate the three-periodic attractors (see Fig. 2(b)). For the computer simulations, we calculate Eq. (4) using the fourth-order Runge-Kutta method with the step size h = 0.01. In this study, we set the parameters of the system as $\alpha_c = 0.460$, $\alpha_p = 0.412$, $\beta = 3.0$ and $\delta = 470.0$.



(a)Chaotic attractor (b)Three-periodic attractor

Fig. 2. Attractors of chaotic circuit.

III. Compare different type network model

A. Proposed system model

Topological structures in complex networks of N nodes and E edges can be evaluated by the typical three types structural metrics (degree, clustering coefficient and path length). First, degree (k) is the number of edges which is connected on a node. Second, clustering coefficient (C) shows the number of actual links between neighbors of a node divided by the number of possible links between those neighbors. Third, path length(L) shows the shortest path in the network between two nodes. These are given as follows;

$$C = \frac{1}{N}\sum_{n=1}^{N} C_n = \frac{1}{N}\sum_{n=1}^{N}\frac{2E_n}{k_n(k_n-1)}. \tag{6}$$

$$L = \frac{2}{N(N-1)}\sum_{m=1}^{N-1}\sum_{n=m+1}^{N} l(m,n). \tag{7}$$

Figures 3 and 4 show the proposed different type network models. Model-A is already proposed in previous study [5]. The characteristic of Model-A is the scale-free network topology. The characteristic of Model-B is the random network topology. In each model, each chaotic circuit is coupled by one resistor R. We use 25 coupled chaotic circuits and 34 resistors in each network model.

Figure 5 shows degree distribution of each network. In this graph, the vertical axis denotes the number of nodes and the horizontal axis denotes the value of degree. Moreover, the feature quantities of the proposed each network are summarized in Table I.

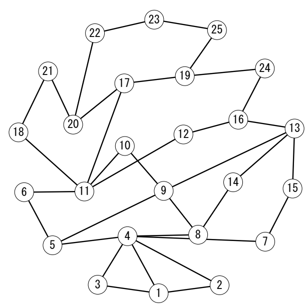


Fig. 3. Model-A (Scale-free network).

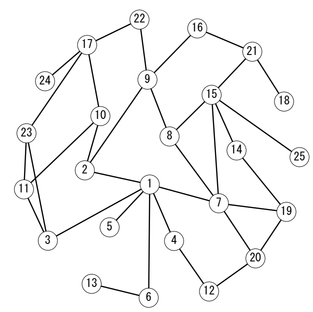


Fig. 4. Model-B (Random network).

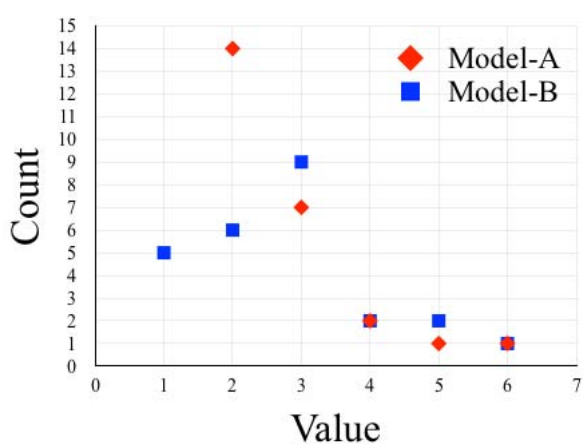


Fig. 5. Degree distribution of each network..

TABLE I
FEATURE QUANTITIES OF PROPOSED EACH NETWORK.

Feature	Model-A	Model-B
The number of nodes	25	25
The number of edges	34	34
Avg. degree	2.720	2.720
Avg. path length	3.503	3.110
Avg. clustering coefficient	0.112	0.092

B. Ratio of propagation

We investigate the ratio of propagation when we change the value of degree in the initial chaos position according to degree distribution. Moreover, in scale-free and random networks, we compare their differences of simulation results. For example, in model-A, when the value of degree k in the initial chaos position is 4, we set the chaotic attractor in 9th or 13th node. Also, in model-B, when the value of degree k in the initial chaos position is 4, we set the chaotic attractor in 9th or 17th node.

The simulation results of ratio of propagation according to degree distribution are shown in Figs. 6 and 7. In addition, we investigate the ratio of propagation in the static state when we fix coupling strength as $\gamma = 0.001, 0.005$. Here, we define the ratio of propagation as number of chaotic circuits of whole network at steady state. Further, the simulation result of Fig. 6 has been reported [5].

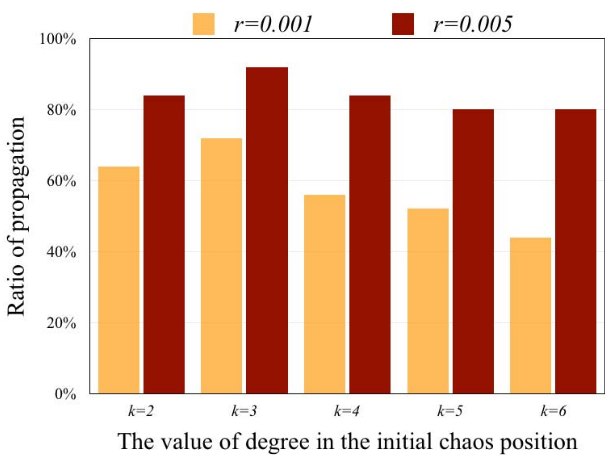


Fig. 6. Ratio of propagation according to degree distribution in Model-A (scale-free network).

From the result, in scale-free network, chaos propagation become to more difficult, when we increase the value of degree in initial chaos position. On the other hand, in random network, even though we increase the value of degree in initial chaos position, the ratio of propagation changes only a little.

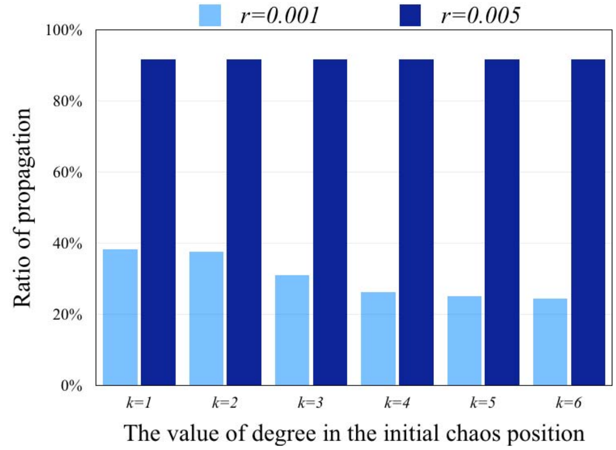


Fig. 7. Ratio of propagation according to degree distribution in Model-B (Random network).

IV. Spread of chaotic behavior in random network

A. Proposed system model

In this research, we propose two types random network models different from Model-B (see Figs. 8 and 9). Model-B, C and D is used same coupled chaotic circuits. However, we change the number of edge compare to Model-B. Therefore, average path length become shorter and average clustering coefficient become larger than model-B. The feature quantities of the proposed each proposed network are summarized in Table II.

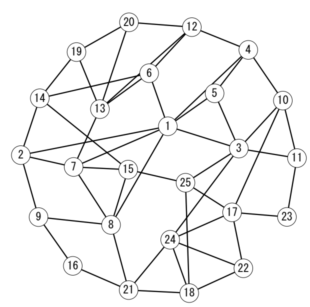


Fig. 8. Model-C (Random network).

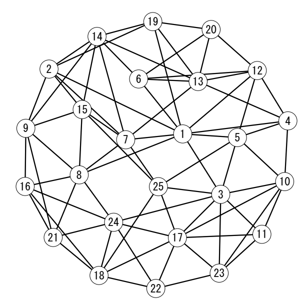


Fig. 9. Model-D (Random network).

TABLE II
FEATURE QUANTITIES OF PROPOSED EACH NETWORK.

Feature	Model-C	Model-D
The number of nodes	25	25
The number of edges	50	75
Avg. degree	4.000	6.000
Avg. path length	2.617	2,173
Avg. clustering coefficient	0,234	0.509

B. Simulation result

We investigate the spread of chaotic behavior in each random network by increasing the coupling strength. Moreover, when we set the initial chaos position in same condition, we compare each random network for spreading of chaotic behavior. We summarize the each simulation result in Figs. 10 and 11.

Figure 10 shows the spread of chaotic behavior when we we set the initial chaos position in minimum degree. In Model-B, the minimum degree is 1; hence, we set the initial chaos position in 5th, 13th, 18th, 24th or 25th node. In Model-C, the minimum degree is 2; hence, we set the initial chaos position in 16th or 23th node. In Model-D, the minimum degree is 4; hence, we set the initial chaos position in 11th, 20th or 22th node. Moreover, we increase the coupling strength when we set the step size to $\gamma = 0.0001$.

From the result, when we increasing the coupling strength, each three periodic attractor are affected from the chaotic attractors. Furthermore, the ratio of propagation in Model-D reach 90% earlier than other model.

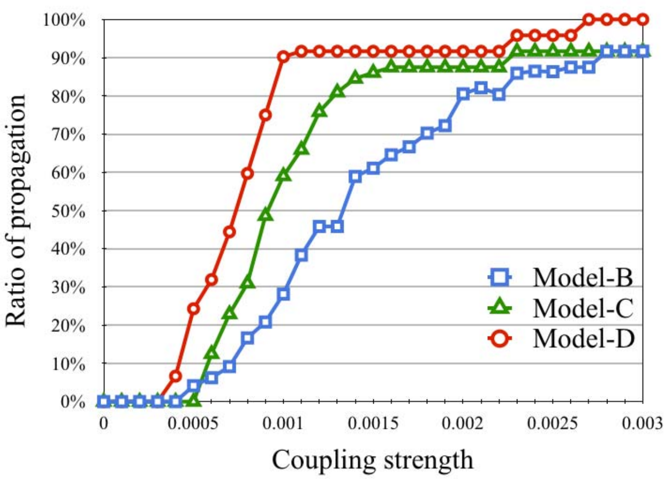


Fig. 10. Spread of chaotic behavior in random network when we set the initial chaos position in minimum degree.

Figure 11 shows the spread of chaotic behavior when we we set the initial chaos position in maximum degree. In Model-B, the maximum degree is 6; hence, we set the initial chaos position in 1st node. In Model-C, the maximum degree is 7; hence, we set the initial chaos position in 1st node. In Model-D, the maximum degree is 9; hence, we set the initial chaos position in 1st node. Moreover, we increase the coupling strength when we set the step size to $\gamma = 0.0001$.

From the result, when we increasing the coupling strength, each three periodic attractor are affected from the chaotic attractors. Maximum node as well as minimum node, the ratio of propagation in Model-D reacesh 90% earlier than other model.

From each result, as clustering coefficient become larger, the three-periodic attractors are easily affected from the chaotic attractor in random network. Additionally, there is no change in the ratio of propagation when the coupling strength exceed the fixed threshold.

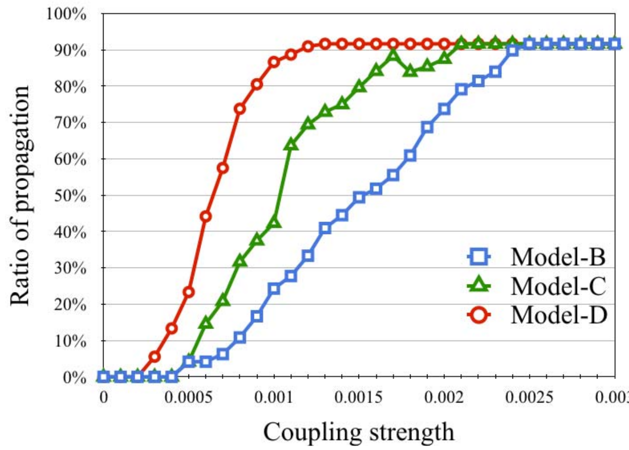


Fig. 11. Spread of chaotic behavior in random network when we set the initial chaos position in maximum degree.

V. CONCLUSIONS

In this study, we have investigated the spread of chaotic behavior in coupled chaotic circuits of our proposed models. By the computer simulations, we confirmed that the three-periodic attractors are affected from the chaotic attractors when the coupling strength increases.

First, we compare scale-free network and random networks. In scale-free network, chaos propagation is more difficult when we increasing the value of degree in the initial chaos position. By contrast, in random network, even though we change the initial chaos position, the ratio of propagation changes only a little.

Next, we investigate the spread of chaotic behavior in each random network. From the above results, we confirm the largest clustering coefficient network is easily affected from the chaotic behavior than other random network. We consider that the spread of chaotic behavior depends on average clustering coefficient in random network.

For the future works, we will focus on clustering coefficient and path length in each node. Furthermore, we compare scale-free and random networks in large scale complex networks.

REFERENCES

[1] G. Abramson, V.M. Kenkre and A.R. Bishop, "Analytic Solutions for Nonlinear Waves in Coupled Reacting Systems," Physica A: vol. 305, no. 3-4, pp. 427-436, 2002.

[2] L. Jinhu, Y. Xinghuo and C. Guanrong, "Chaos synchronization of general complex dynamical networks" Physica A: Statistical Mechanics and its Applications, vol. 334, pp.281-302, 2004

[3] A.L. Barabasi and Z. N. oltvai, "Network biology: understanding the cell's functional organization"Nature Reviews Genetics 5, pp.101-113, 2004

[4] Y. Uwate and Y. Nishio, "Chaos Propagation in a Ring of Coupled Circuits Generating Chaotic and Three-Periodic Attractors" Proceedings of IEEE Asia Pacific Conference on Circuits and Systems (APCCAS'12), pp. 643-646, Dec. 2012.

[5] T. Chikazawa, Y. Uwate and Y. Nishio,"Spread of Chaotic Behavior in Complex Network with Coupled Chaotic Circuits" IEICE Proceeding Nolta society 2017, no. B-12, p. NLS-32, Jun. 2017.