2017 RISP International Workshop on Nonlinear Circuits,
Communications and Signal Processing (NCSP'17)
Guam, USA, February 28th to March 3rd, 2017

Investigation of Spreading Chaotic Behavior in Coupled Chaotic Circuit Networks with Various Features

Takahiro Chikazawa, Yoko Uwate and Yoshifumi Nishio

Dept. of Electrical and Electronic Engineering, Tokushima University,
2-1 Minamijosanjima, Tokushima, 770-8506 Japan
Email: {chikazawa, uwate, nishio}@ee.tokushima-u.ac.jp

Abstract

In this study, we investigate the spread of chaotic behavior in coupled chaotic circuits networks. We propose two types complex networks with hub. In each network model, the structure is composed of coupled chaotic circuits when one circuit is set to generate chaotic attractor and the other circuits are set to generate three-periodic attractors. By using computer simulations, when the coupling strength increase, we have observed that the chaotic attractor spread to the other circuits. Moreover, we investigate ratio of propagation in each proposed network by changing the initial position of chaotic attractor. The initial position of chaotic attractor changes according to feature quantities of node.

1. Introduction

In our society, there are many networks. These network models have various types of feature quantities. Examples of feature quantities are path length, degree distribution, and clustering coefficient. Recently, network models become more complex and larger scale. Furthermore, various types of propagation have attracted a great deal of attention from various fields [1]. For example, the traffic jam of the transportation network and the viral infection are mentioned. Hence, it is more difficult to analyze the phenomena in the networks. Therefore, we consider that we can analyze various complicated phenomena of complex networks by investigating the spread of chaotic behavior. Additionally, it is important to investigate propagation phenomena observed from coupled chaotic circuits for future engineering applications.

In our previous studies, the synchronization phenomena and the chaos propagation has been investigated by many researches. The synchronization phenomena in complex networks has been reported to various fields [2]. On the other hand, the chaos propagation have been investigated in simple networks of chaotic circuits such as a ring combination [3].

In this paper, we investigate the spread of chaotic behavior in complex network with coupled chaotic circuits. We propose two types complex networks with hub. One is used 25 nodes and 35 edges. The other is used 50 nodes and 85 edges. In our proposed network model, one circuit is set to generate chaotic attractor and the other circuits are set to generate three-periodic attractors. We investigate ratio of spreading chaotic behavior in each networks. We change the initial position of the chaos attractor according to network feature.

2. Circuit model

The chaotic circuit is shown in Fig. 1. This circuit consists of a negative resistor, two inductors, a capacitor and dual-directional diodes. This chaotic circuit is called Nishio-Inaba circuit [4].

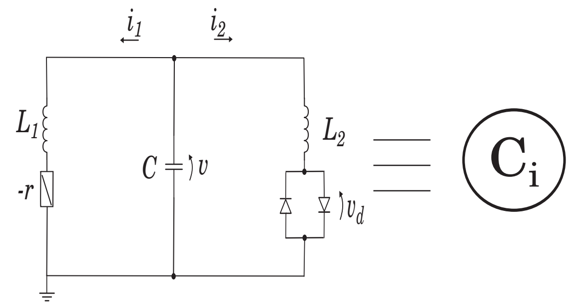


Figure 1: Chaotic circuit.

The circuit equations of this circuit are described as follows:

$$\begin{cases} L_1 \dfrac{di}{dt} = v + ri \\ L_2 \dfrac{di}{dt} = v - v_d \\ C \dfrac{dv}{dt} = -i_1 - i_2, \end{cases} \tag{1}$$

The characteristic of nonlinear resistance is described as follows:

$$v_d = \frac{r_d}{2}\left(\left|i_2 + \frac{V}{r_d}\right| - \left|i_2 - \frac{V}{r_d}\right|\right). \tag{2}$$

By changing the variables and parameters,

$$\begin{cases} i_1 = \sqrt{\frac{C}{L_1}}Vx_n, \;\; i_2 = \frac{\sqrt{L_1 C}}{L_2}Vy_n, \;\; v = Vz_n \\ \alpha = r\sqrt{\frac{C}{L_1}}, \;\; \beta = \frac{L_1}{L_2}, \;\; \delta = r_d\frac{\sqrt{L_1 C}}{L_2}, \\ \gamma = \frac{1}{R}\sqrt{\frac{L_1}{C}}, \;\; t = \sqrt{L_1 C_2}\tau, \end{cases} \tag{3}$$

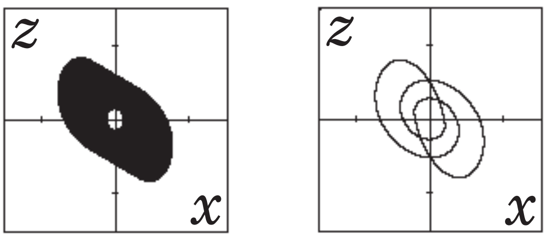
In the proposed system, each circuit is connected to only adjacent circuits by the resistors. The normalized circuit equations of the system are given as follows:

$$\begin{cases} \frac{dx_i}{d\tau} = \alpha x_i + z_i \\ \frac{dy_i}{d\tau} = z_i - f(y_i) \\ \frac{dz_i}{d\tau} = -x_i - \beta y_i - \sum_{j \in S_n} \gamma(z_i - z_j) \\ \qquad\qquad (i, j = 1, 2, \cdots, N). \end{cases} \tag{4}$$

In Eq. (4), N is the number of coupled chaotic circuits and γ is the coupling strength. $f(y_i)$ is described as follows:

$$f(y_i) = \frac{1}{2}\left(\left|y_i + \frac{1}{\delta}\right| - \left|y_i - \frac{1}{\delta}\right|\right). \tag{5}$$

In this chaotic circuit, we define α_c to generate the chaotic attractor (see Fig. 2(a)) and α_p is defined to generate the three-periodic attractors (see Fig. 2(b)). For the computer simulations, we calculate Eq. (4) using the fourth-order Runge-Kutta method with the step size h = 0.01. In this study, we set the parameters of the system as $\alpha_c = 0.460$, $\alpha_p = 0.412$, $\beta = 3.0$ and $\delta = 470.0$.

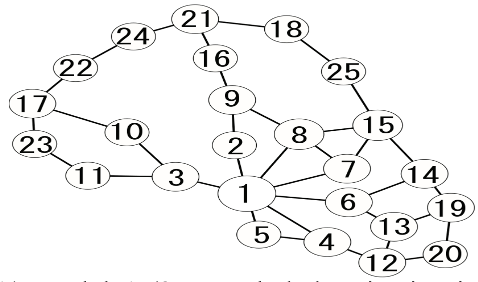


(a)Chaotic attractor (b)Three-periodic attractor

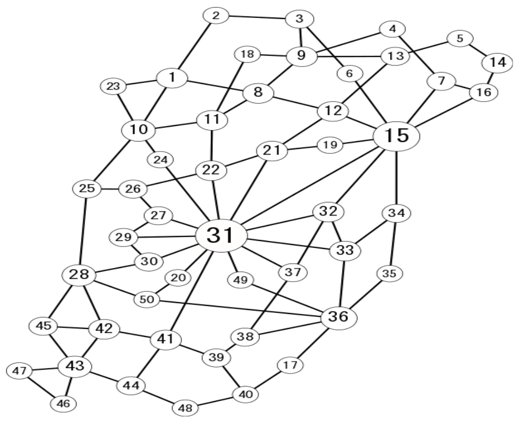
Figure 2: Attractors of chaotic circuit.

3. Network model

Figure 3 shows the proposed two types complex networks. In our proposed network model, each chaotic circuit is coupled by one resistor R. We use 25 coupled chaotic circuits and 35 edges in Fig. 3(A). The other model use 50 coupled chaotic circuits and 85 edges in Fig. 3(B). The feature quantities of the proposed each network are summarized in Table 1. Moreover, Fig. 4 shows degree distribution of each network. In this graph, vertical axis denotes the number of nodes and horizontal axis denotes the value of degree.



(A) Model-A (25 coupled chaotic circuits).



(B) Model-B (50 coupled chaotic circuits).

Figure 3: Network model.

Table 1: Feature quantities of proposed each network.

Feature	(A)	(B)
The number of nodes	25	50
The number of edges	35	85
Avg. degree	2.8	3.4
Avg. path length	3.34	3.51
Avg. clustering coefficient	0.104	0.157

In this study, we define hub as the node which is the highest value of degree. In Model-A, we define hub as 1st node. In Model-B, we define hub as 31st node.

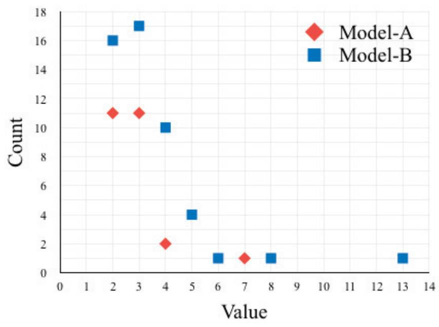


Figure 4: Degree distribution of each network.

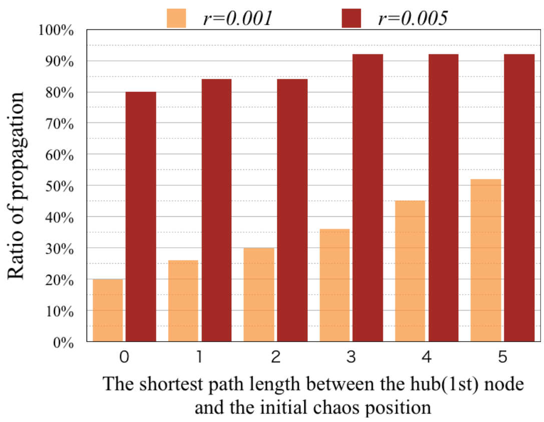


Figure 5: Ratio of propagation according to the shortest path length between the hub node and the initial chaos position in model-A.

4. Simulation results

In this section, we investigate the spread of chaotic behavior by increasing the coupling strength and changing the initial chaos position. In the previous studies, chaos propagation have been investigated simple network system of coupled chaotic circuits [5]. Moreover, the three-periodic attractors are affected from the chaotic attractors when the coupling strength are increasing.

In addition, we use two type network in this study. The number of node and edge in each network is different. However, one circuit is set to generate chaotic attractor and the other circuits are set to generate three-periodic attractors in each network.

4.1 Model-A (25 coupled chaotic circuits)

We investigate ratio of propagation when we change the shortest path length between the hub(1st) node and the initial chaos position. The simulation results of ratio of propagation according to the shortest path length are shown in Fig. 5. For example, in model-A, when the shortest path length between the hub(1st) node and the initial chaos position is 1, we set the chaotic attractor in 2nd, 3rd, 4th, 5th, 6th, 7th or 8th node. Also, when the shortest path length is 4, we set the chaotic attractor in 18th, 21st or 22nd node. Furthermore, we average the ratio of propagation in each node under the same condition. In addition, we investigate the ratio of propagation in the static state when we fix coupling strength as $\gamma = 0.001, 0.005$. Here, we define the ratio of propagation as number of chaotic circuits of whole network at steady state.

From the result, ratio of propagation in large coupling strength is higher than small coupling strength. When we increase the path length between the hub(1st) node and the initial chaos position, each three-periodic attractors are easily affected from the chaotic attractors. Moreover, when we set the initial chaos position in hub(1st) node, ratio of propagation is the lowest than other node.

4.2 Model-B (50 coupled chaotic circuits)

Next, we construct a larger scale complex network and we investigate the ratio of propagation when we change the shortest path length between the hub(31st) node and the initial chaos position. The simulation results of ratio of propagation according to the shortest path length are shown in Fig. 6. For example, in model-B, when the shortest path length between the hub(1st) node and the initial chaos position is 2, we set the chaotic attractor in 6th, 7th, 10th, 11th, 12th, 16th, 19th, 26th, 28th, 34th, 36th, 38th, 39th, 42nd, 44th or 50th node. Furthermore, we average the ratio of propagation in each node under the same condition. In addition, we investigate the ratio of propagation in the static state when we fix coupling strength as $\gamma = 0.001, 0.005$. Here, we define the ratio of propagation as number of chaotic circuits of whole network at steady state.

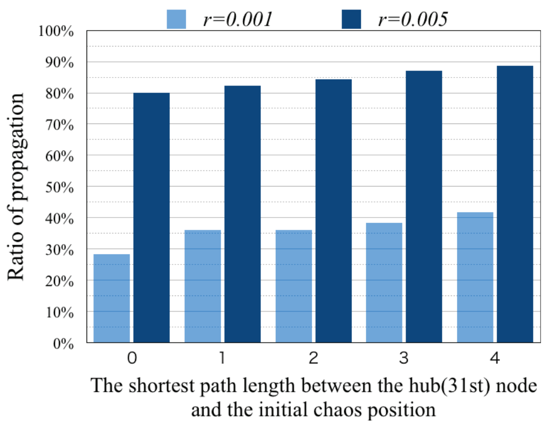


Figure 6: Ratio of propagation according to the shortest path length between the hub node and the initial chaos position in model-B.

From the result, the highest ratio of propagation is 88.67% when the coupling strength $\gamma = 0.005$ and the shortest path length is 4. As with the model-A, when we increase the path length between the hub(31st) node and the initial chaos position, each three-periodic attractors are easily affected from the chaotic attractors. As the path length in the initial chaos position increases, the spread of chaotic behavior become more easy.

Next, we investigate the spread of chaotic behavior in detailed condition. When we fix the shortest path length as 2, the ratio of propagation according to the value of degree in each node are summarized in Fig. 7. For example, in this condition, when the value of degree in the initial chaos position is 2, we set the chaotic attractor in 6th or 19th node. Moreover, when we fix the shortest path length as 3, the ratio of propagation according to the value of degree in each node are summarized in Fig. 8. For example, in this condition, when the value of degree in the initial chaos position is 2, we set the chaotic attractor in 4th, 17th, 23rd, 24th, 35th or 48th node. In addition, we fix coupling strength as $\gamma = 0.001$ in each result.

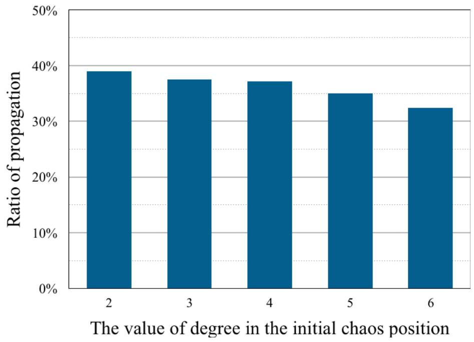


Figure 7: Ratio of propagation according to degree distribution when we fix the shortest(31st) node as 2.

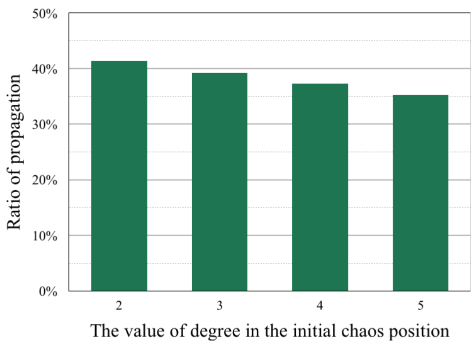


Figure 8: Ratio of propagation according to degree distribution when we fix the shortest(31st) node as 3.

From each result, when we increase the value of degree in initial chaos position, ratio of propagation become low. In other words, the spread of chaotic behavior become more difficult by increasing the value of degree in the initial chaos position. Consequently, three-periodic attractors are easily affected from the chaotic attractors when we set the initial chaos position in faraway nod from hub and smaller value of degree.

5. Conclusion

In this study, we have investigated spreading chaotic behavior in complex networks. By the computer simulations, we investigated ratio of propagation by changing the initial chaos position. We confirmed that chaos propagation is more difficult when we increase the and the value of degree in the initial chaos position. We consider that the spread of chaotic behavior affect the low degree node. Furthermore, the low degree node has important role in complex network.

Acknowledgment

This work was partly supported by JSPS Grant-in-Aid for Scientific Research 16K06357.

References

[1] A.L. Barabasi and Z. N. Oltvai, “Network Biology: Understanding the Cell’s Functional Organization,” Nature Reviews Genetics, Vol. 5, pp.101-113, 2004.

[2] N.F. Rullckov and M.M. Sushchik, “Robustness of Synchronized Chaotic Oscillations,” Int. J. Bifurcation and Chaos, vol. 7, no. 3, pp. 625-643, 1997.

[3] Y. Uwate and Y. Nishio, “Chaos Propagation in a Ring of Coupled Circuits Generating Chaotic and Three-Periodic Attractors,” Proceedings of IEEE Asia Pacific Conference on Circuits and Systems (APCCAS’12), pp. 643-646, Dec. 2012.

[4] Y. Nishio, N. Inaba, S. Mori and T. Saito, “Rigorous Analyses of Windows in a Symmetric Circuit,” IEEE Transactions on Circuits and Systems, Vol. 37, no. 4, pp. 473-487, Apr. 1990.

[5] T. Chikazawa, Y. Uwate and Y. Nishio, “Chaos Propagation in Coupled Chaotic Circuits with Multi-Ring Combination,” Proceedings of IEEE Asia Pacific Conference on Circuits and Systems (APCCAS’16), pp. 65-68, Oct. 2016.