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Synchronization Phenomena in Complex Networks of Coupled Chaotic Circuits with Different Degree Distribution

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1. Abstract

In this study, we investigate synchronization phenomena in complex networks of coupled chaotic circuits with different degree distributions. In addition, we investigate synchronization phenomena of each degree distribution by changing the path length. Furthermore, we research difference of synchronization phenomena of each network.

2. Introduction

Synchronization phenomena can be observed everywhere in our life. For example, we can confirm metronome, flashing firefly lights, beating rhythm of the heart and so on. Especially, synchronization phenomena of oscillatory network are interesting. In addition, complex networks attract attention from various fields. The feature of networks is characterized by the degree distribution, the path length and the clustering coefficient.

Complex networks of chaotic circuits have been studied [1]. However, many researchers have not been researched more about synchronization phenomena in complex networks of coupled chaotic circuits which compare degree distribution.

In this study, we compare synchronization phenomena of networks with three types of degree distribution. Complex networks of real world have characters like scale free property, cluster property, small world property [2][3]. So, we use degree distribution models based on the power law and the normal distribution. Scale free network follows the power law. Random network follows the normal distribution. Furthermore, we use the soaring distribution which is different from the others. We compare synchronization rate of network based on three types of degree distribution by changing the coupling strength. And we investigate synchronization phenomena of network with different path length by changing the coupling strength.

3. Circuit model

The chaotic circuit model which called Nishio-Inaba circuit is shown in Fig. 1. This circuit consists of a negative resister, a capacitor, two inductors and dual-directional diodes. And this circuit equation is shown Eq. (1).

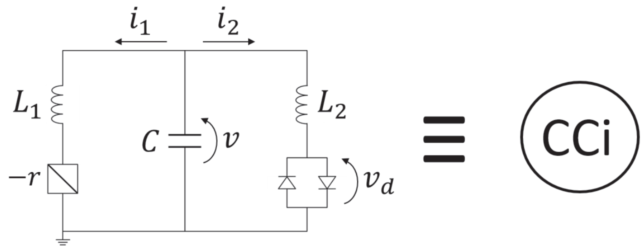

Figure 1: Circuit model.

$$\begin{cases} L_1 \dfrac{di_1}{dt} = \alpha v + r i_1 \\ L_2 \dfrac{di_2}{dt} = v - v_d \\ C \dfrac{dv}{dt} = -i_1 - i_2, \end{cases} \tag{1}$$

The characteristic of nonlinear resistance which consists of dual diodes is following Eq. (2).

$$v_d = \frac{r_d}{2}\left(\left|i_2 + \frac{V}{r_d}\right| - \left|i_2 - \frac{V}{r_d}\right|\right). \tag{2}$$

By changing the variables and parameters,

$$\begin{cases} i_1 = \sqrt{\dfrac{C}{L_1}} V x_n, \ \ i_2 = \dfrac{\sqrt{L_1 C}}{L_2} V y_n, \ \ v = V z_n \\ \alpha = r\sqrt{\dfrac{C}{L_1}}, \ \ \beta = \dfrac{L_1}{L_2}, \ \ \gamma = r_d \dfrac{\sqrt{L_1 C}}{L_2}, \\ \delta = \dfrac{1}{R}\sqrt{\dfrac{L_1}{C}}, \ \ t = \sqrt{L_1 C_2}\tau, \end{cases} \tag{3}$$

The normalized equation of this circuit is given as follows:

$$\begin{cases} \dfrac{dx}{d\tau} = \alpha x + z \\ \dfrac{dy}{d\tau} = z - f(y) \\ \dfrac{dz}{d\tau} = -x - \beta y, \end{cases} \tag{4}$$

where $f(y)$ is described as follows :

$$f(y_i) = \frac{1}{2}\left(\left|y_i + \frac{1}{\delta}\right| - \left|y_i - \frac{1}{\delta}\right|\right). \tag{5}$$

4. System model

In this study, we create network which ten chaotic circuits connect by resistances. Each network is composed by each degree distribution. Network model which used in this research is shown in Fig. 2.

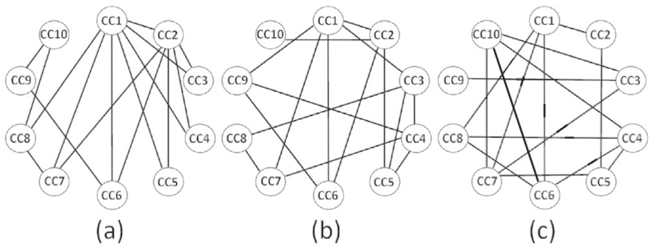

Figure 2: Network model.

The normalized circuit equation of these network models is given by the following equations.

$$\begin{cases} \dfrac{dx_i}{d\tau} = \alpha x_i + z_i \\ \dfrac{dy_i}{d\tau} = z_i - f(y) \\ \dfrac{dz_i}{d\tau} = -x_i - \beta y_i - \displaystyle\sum_{i,j=1}^{N} \delta_{ij}(z_i - z_j) \\ \qquad\qquad (i, j = 1, 2, \cdots, N). \end{cases} \tag{6}$$

The parameter δ corresponds the coupling strength between the circuits. The parameter α is chaos degree and the parameter N is the number of circuits.

Figure 3 (a), (b) and (c) show the degree distribution which used network in this study. The model (a) imitates the degree distribution of the power law. The model (b) imitates the degree distribution of a random network. The model (c) is the degree distribution of soaring. We fix that the number of circuits in each network model is 10 and the number of edges in each network model is 16.

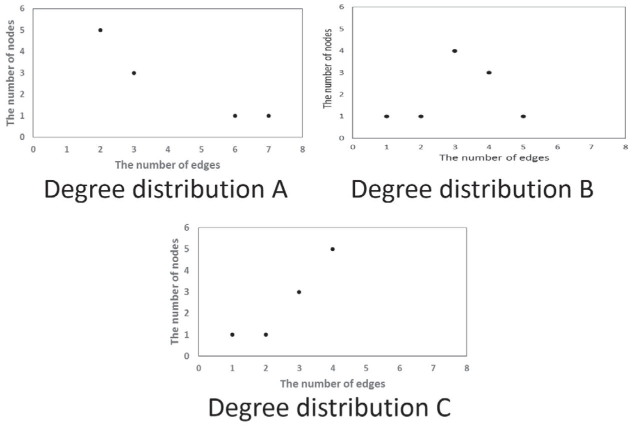

Figure 3: Degree distribution.

The path length of each network model is nearly the same value. In addition to these networks model, we make other model with the different path length. Those network models are shown Fig. 4.

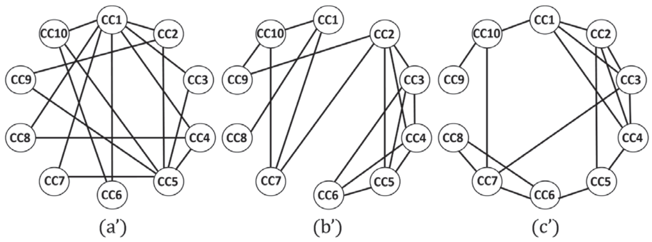

Figure 4: Network model with the different path length.

These network models in Fig. 4 also created based on each degree distribution. The path length of model (a) is longer than (a'). The path length of model (b) and (c) are shorter than model (b') and (c') each other. The path length of each network is shown in Table 1.

Table 1: Path length of each network model.

	Path length
(a)	1.822
(a')	1.689
(b)	1.844
(b')	2.156
(c)	1.889
(c')	2.0

5. Simulation Result

Definition of synchronization in this study is determined a voltage difference waveform. We define synchronization as the following Eq. (7).

$$|Z_j - Z_i| < 0.1 \qquad (i, j = 1, 2, \cdots, 10) \tag{7}$$

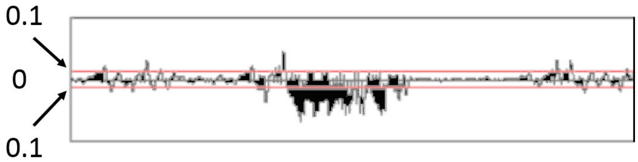

Figure 5: A difference waveform.

Figure 5 is a difference waveform which was observed in this research. The two lines shown in Fig. 5 corresponds threshold which is given Eq. (7). It is determined that wave within two line which is the threshold is synchronization. Therefore, we propose and investigate the synchronization probability denoted the synchronization rate during a certain time interval. in this research, we fix a certain time interval as (τ=1,000,000 and step=0.02τ) and investigate the synchronization rate in the entire network of 10 coupled chaotic circuits.

5.1 Synchronization rate of network model with different chaos degree

In this simulation, we use three parameters of chaos degree. Attractor of these parameters shows Fig. 6. Synchronization rate of network model (a), (b) and (c) with different parameter is shown in Figs. 7, 8 and 9. Synchronization rate of each network model at α=0.350 is highest. And synchronization rate of each network model at α=0.460 and α=0.490 are almost the same. In the coupling δ=0.01, 0.02, network model (a) is most synchronize. However, when the coupling strength increase, network model (b) is most synchronize.

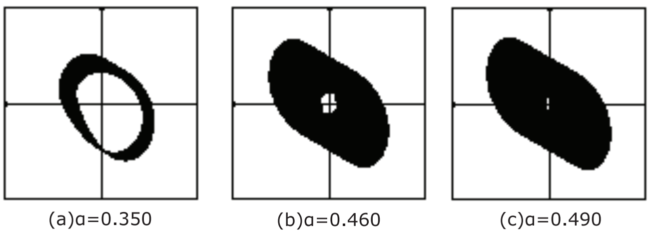

Figure 6: Attractor of different chaos degree.

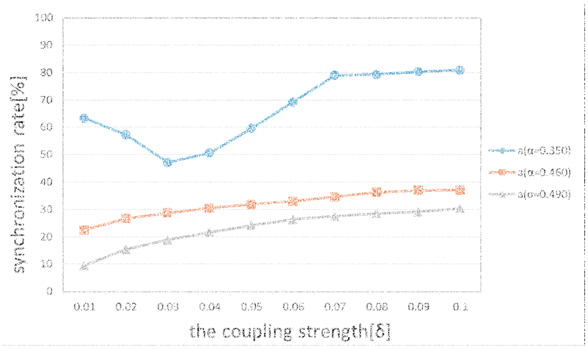

Figure 7: Synchronization rate of network model (a) with different chaos degree.

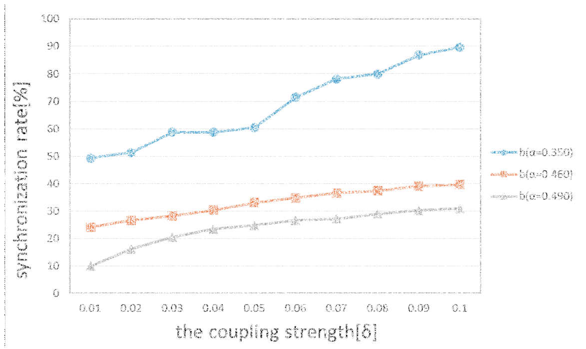

Figure 8: Synchronization rate of network model (b) with different chaos degree.

5.2 Synchronization rate of network with different path length

Figures 10, 11 and 12 show synchronization rate of network with different path length. The parameter α is fixed at 0.460.

First, we compare synchronization rate of network model (a) and (a'), synchronization rate of network model (a) is lower than network model (a') in the weak coupling strength. However, in the coupling strength $\delta = 0.08$ to $\delta = 0.1$, network model (a) more synchronized than network model (a').

Second, we compare synchronization rate of network model (b) and (b'), (c) and (c'). In the weak coupling strength, synchronization rate of network model (b') is lower than network model (b). In the case of the coupling strength $\delta = 0.08$ to $\delta = 0.1$, network model (b') more synchronized than network model (b). When the weak coupling strength, synchronization rate of network model (c') is lower than network model (c). However synchronization rate of network model (c') is higher than network model (c) in the coupling strength $\delta = 0.08$ to $\delta = 0.1$.

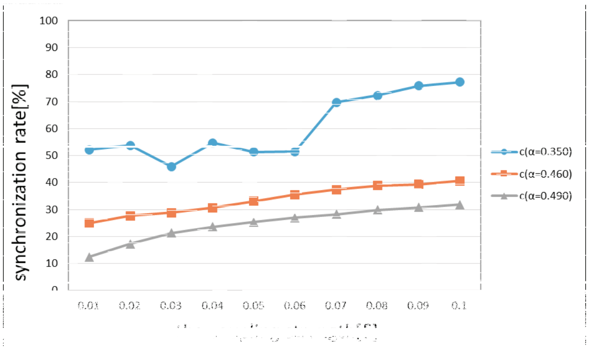

Figure 9: Shynchronization rate of network model (c) with different chaos degree.

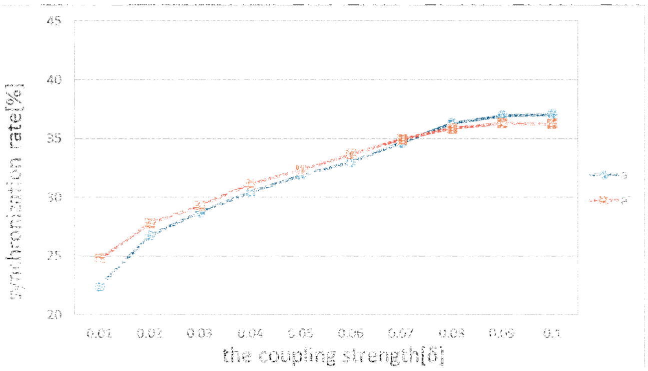

Figure 10: Shynchronization rate with network model (a) and (a').

6. Conclusion

In this study, we have investigated synchronization rate of several network topology structures by changing the coupling strength. Furthermore, we investigate synchronization phenomena with the different path length. The network topology structures are following degree distribution of three types.

When we compare network of each degree distribution with different chaos degree, we have confirmed synchronization rate of each degree distribution at $\alpha = 0.350$ is higher than other chaos degree. And, we have confirmed that synchronization rate of the models which imitate a random network is better than that of power law and soaring as the coupling strength increase.

When we compare synchronization rate in the different path length, we have confirmed synchronization rate of network which have long path length is lower than short one in weak the coupling strength. However, as the coupling strength increase, network of long path length more synchronized than the short one.

For the future works, we would like to develop the network model by adding a weight to the coupling. And we investigate synchronization phenomena of larger network.

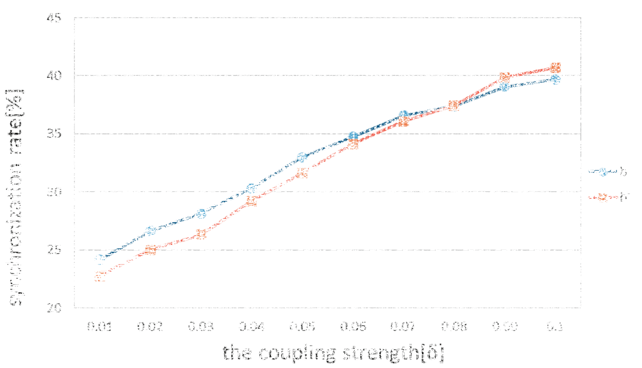

Figure 11: Shynchronization rate with network model (b) and (b').

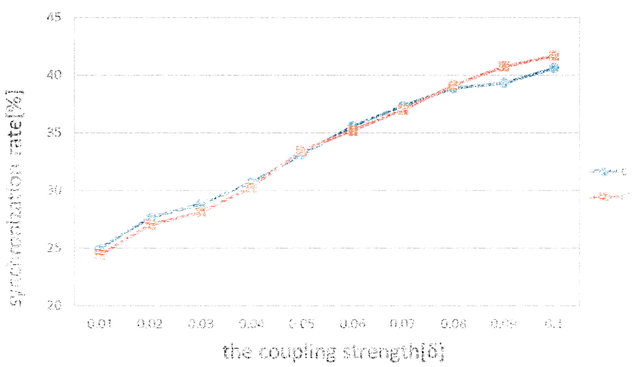

Figure 12: Shynchronization rate with network model (c) and (c').

Ackowledgment

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References

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