2017 RISP International Workshop on Nonlinear Circuits,
Communications and Signal Processing (NCSP'17)
Guam, USA, February 28th to March 3rd, 2017

Synchronization Phenomena of Coupled Two Rings of Chaotic Circuits

Katsuki Nsakashima, Kazuki Ueta, Yoko Uwate and Nishio Yoshifumi

†Dept. of Electrical and Electronic Engineering, Tokushima University,
2-1 Minamijosanjima, Tokushima, 770-8506 Japan
Email: {nakashima, kazuki, uwate, nishio}@ee.tokushima-u.ac.jp

Abstract

Nonlinear phenomena of coupled chaotic circuits are drawing attention from many researchers. In this study, we investigate the synchronization phenomena of coupled two rings of chaotic circuits. One ring chaotic circuit generates chaotic attractors and the other ring generates three-periodic attractors. In addition, we observe the synchronization phenomena by changing the coupling strength.

1. Introduction

Synchronization phenomena have been found in various fields of natural world [1] – [3]. Especially, there are a lot of relationships of biological systems. Also, synchronization phenomena have a relationship with the human body. For example, cells of the human body are synchronized. Therefore, the vibration of same timing produces big vibration. According to synchronization phenomena, small power produces very big power.

Recently, complex networks have attracted attention and topology of complex networks is studied for influence on the system. Also, synchronization phenomena of chaotic circuits are studied from various viewpoints. Synchronization phenomena of chaotic circuits are the same as the vibration of the natural world. Before now simple system has already been studied. For example, it is only ring structure, only ladder structure, only star structure and so on [4] – [6]. However, many researchers have not been studied about more complex systems. Therefore, we propose the coupled two rings of chaotic circuits as the minimum scale of complex systems.

In this study, we investigate the synchronization phenomena of coupled two rings of chaotic circuits. We couple three chaotic circuits on the ring structure and we propose a system model that the two rings are coupled via a resistor. One ring chaotic circuit generates chaotic attractors and the other ring generates three-periodic attractors. Then, we focus on synchronization phenomena of the chaotic circuits.

2. System model

The chaotic circuit is shown in Fig. 1 and the system model is shown in Fig. 2. This chaotic circuit consists of two inductors L_1 and L_2 , one capacitor C , negative resistor $-r$ and two diodes.

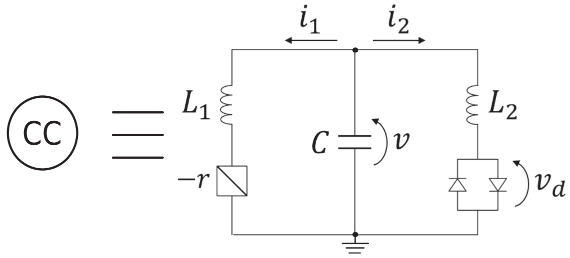

Figure 1: Chaotic circuit.

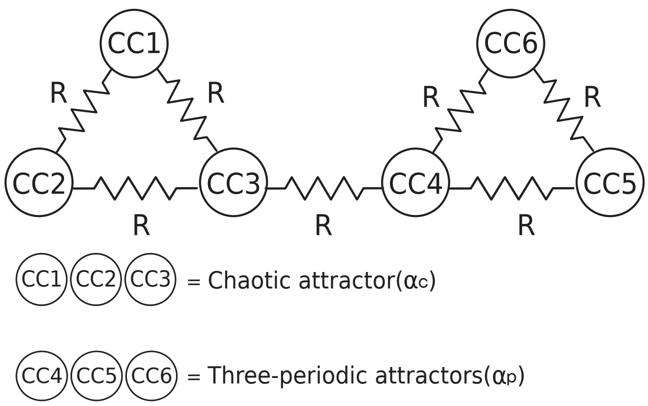

Figure 2: System model.

The circuit equations of chaotic circuits are given as follows:

$$\begin{cases} L_1 \dfrac{di_1}{dt} = v + ri_1, \\ L_2 \dfrac{di_2}{dt} = v - v_d, \\ C \dfrac{dv}{dt} = -i_1 - i_2. \end{cases} \quad (1)$$

The current-voltage characteristics of nonlinear resistor are given as follows:

$$v_d = \frac{r_d}{2}\left(\left|i_2 + \frac{V}{r_d}\right| - \left|i_2 - \frac{V}{r_d}\right|\right). \quad (2)$$

By changing the parameters as follows:

$$\begin{cases} i_1 = \sqrt{\dfrac{C}{L_1}} V x_n, \;\; i_2 = \dfrac{\sqrt{L_1 C}}{L_2} V y_n, \;\; v = V z_n, \\ \alpha = r\sqrt{\dfrac{C}{L_1}}, \;\; \beta = \dfrac{L_1}{L_2}, \;\; \delta = r_d \dfrac{\sqrt{L_1 C}}{L_2}, \\ \gamma = \dfrac{1}{R}, \;\; t = \sqrt{L_1 C_2}\tau. \end{cases} \quad (3)$$

The normalized equations of chaos circuits are given as follows:

$$\begin{cases} \dfrac{dx_i}{d\tau} = \alpha x_i + z_i, \\ \dfrac{dy_i}{d\tau} = z_i - f(y_i), \\ \dfrac{dz_i}{d\tau} = -x_i - \beta y_i - \displaystyle\sum_{i,j=1}^{6} \gamma_{ij}(z_i - z_j), \\ \qquad (i, j = 1, 2, \cdots, 6). \end{cases} \quad (4)$$

Where γ is the coupling strength. $f(y_i)$ is described as follows:

$$f(y_i) = \frac{1}{2}\left(\left|y_i + \frac{1}{\delta}\right| - \left|y_i - \frac{1}{\delta}\right|\right). \quad (5)$$

We define α_c to generate the chaotic attractor, and α_p is defined to generate the three-periodic attractors.

3. Simulation results

We set the parameters of the system as $\alpha_c = 0.460$, $\alpha_p = 0.412$, $\beta = 3.0$ and $\delta = 470.0$. We investigate synchronization phenomena by changing the coupling strength γ between the circuits. Figure 3 shows attractor of each chaotic circuit and Fig. 4 shows the voltage of different waveform when we set the coupling strength as $\gamma = 0.001$. Then, CC5 and CC6 of three-periodic attractors are synchronized.

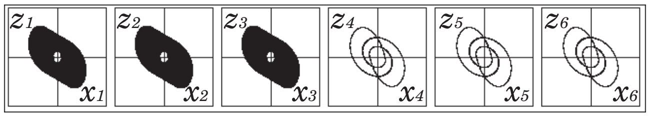

Figure 3: Attractor ($\gamma = 0.001$).

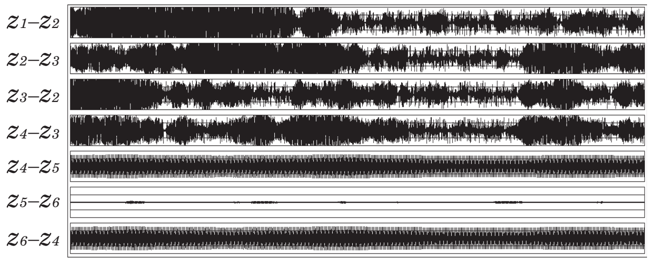

Figure 4: Different waveform ($\gamma = 0.001$).

Figure 5 shows attractor of each chaotic circuit and Fig. 6 shows the voltage of different waveform when we set the coupling strength as $\gamma = 0.1$. In case of $\gamma = 0.1$, CC1, CC2 and CC3 are not synchronized because this ring circuits generate chaotic attractors. In this case, we observe the chaotic propagation. Therefore, CC4, CC5 and CC6 of three-periodic attractors change to chaotic behavior.

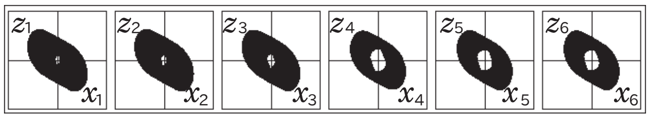

Figure 5: Attractor ($\gamma = 0.1$).

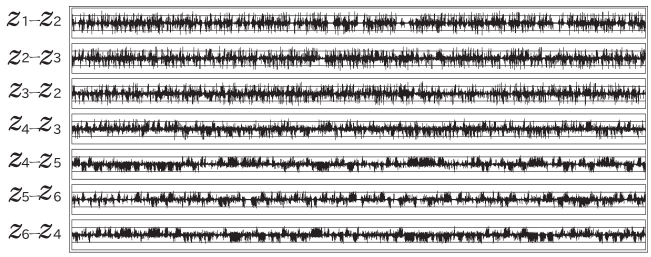

Figure 6: Different waveform ($\gamma = 0.1$).

Figure 7 shows attractor of each chaotic circuit and Fig. 8 shows the voltage of different waveform when we set the coupling strength as $\gamma = 0.2$. In case of $\gamma = 0.2$, we also observe the chaotic propagation. When CC5 and CC6 alternate between synchronous and asynchronous states, the cycle of synchronous and asynchronous states of CC4 - CC5 and CC4 - CC6 are symmetric. Then, Synchronous and asynchronous states change with the simulation time.

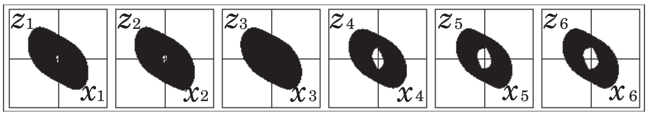

Figure 7: Attractor ($\gamma = 0.2$).

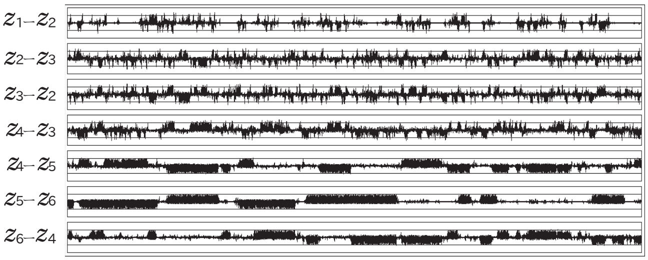

Figure 8: Different waveform ($\gamma = 0.2$).

Figure 9 shows attractor of each chaotic circuit and Fig. 10 shows the voltage of different waveform when we set the coupling strength as $\gamma = 0.23$. As is the case in $\gamma = 0.2$, we observe the chaotic propagation. Synchronous and asynchronous states change with the simulation time. Here, we make a comparison the circuits between the left side ring and the right side ring. From Fig. 10, we can say that the circuits of the left side ring are more synchronized than the other ring.

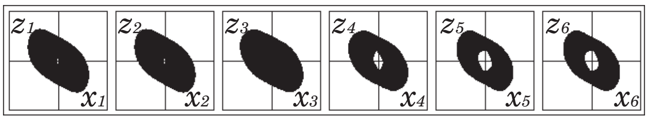

Figure 9: Attractor ($\gamma = 0.23$).

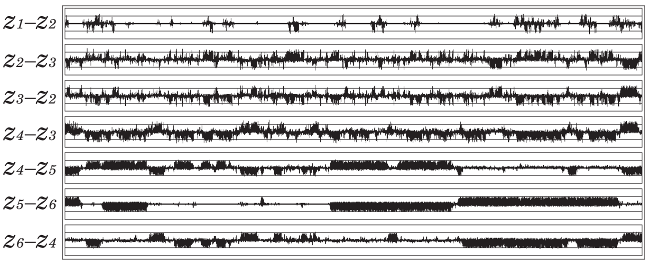

Figure 10: Different waveform ($\gamma = 0.23$).

Finally, we investigate the synchronization rate. We compare with the synchronization rate of chaotic solutions(CC1 and CC2) and periodic solutions(CC5 and CC6). In this study, definition of synchronization is given as follows:

$$|Z_i - Z_j| < 0.1 \qquad (i, j = 1, 2, \cdots, 6). \tag{6}$$

Figure 11 shows definition of synchronization. We define the inside of the red line as synchronization.

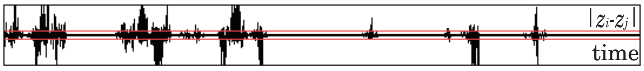

Figure 11: Definition of synchronization.

Figure 12 shows simulation results of synchronization rate. The horizontal axis shows the coupling strength γ. The vertical axis shows the synchronization rate. From Fig. 12, the synchronization rate of the circuits of the right side ring(CC5 and CC6) are decreased as increasing γ. However, the synchronization rate of the circuits of the left side ring(CC1 and CC2) are increased. We can confirm that the synchronization rate of the circuits of the left side ring becomes higher from the coupling strength $\gamma = 0.15$.

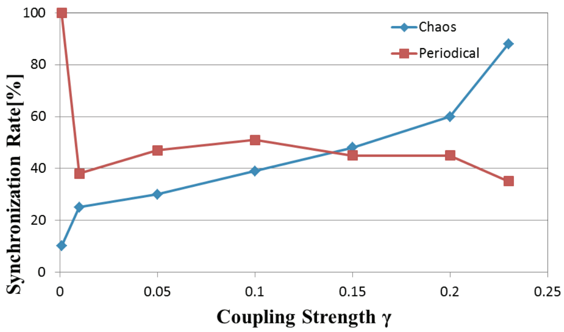

Figure 12: Synchronization rate.

4. Conclusion

In this study, we have proposed a system model using two ring circuits that are coupled by a resistor. One ring chaotic circuit generates chaotic attractors and the other ring generates three-periodic attractors. We have investigated synchronization phenomena by changing the coupling strength γ. By the computer simulations, we have observed different waveform. As a result, we confirmed that synchronous and asynchronous state of chaotic solutions changes in the simulation time when the coupling strength γ set to 0.2 and 0.23.

In the future works, we will investigate the reason for changing synchronous and asynchronous states in the simulation time. Also, we investigate the reason why the chaotic solutions are synchronized when the coupling strength changes.

Acknowledgment

This work was partly supported by JSPS Grant-in-Aid for Challenging Exploratory Research 26540127.

References

[1] C. M. Gray, “Synchronous Oscillators in Neural Systems: Mechanisms and Functions,” J. Computational Neuroscience, vol. 1, pp. 11-38, Feb. 1994.

[2] G. Abramson, V. M. Kenkre and A. R. Bishop, “Analytic Solutions for Nonlinear Waves in Coupled Reactig Systems,” Physical A: Statistial Mechanics and its Applications, vol. 305, no.3-4, pp. 427-436, 2002.

[3] S. Cooper, “Rethinking Synchronization of Mammmalian Cells for Cell Cycle Analysis,” CMLS, Cell. Mol. Life Sci. 60 (2003) 0019, 2003.

[4] Yoshifumi Nishio, Katsunori Suzuki, Shinsaku Mori and Akio Ushida ”Synchronization in Mutually Coupled Chaotic Circuits” Proceedings of European Conference on Circuit Theory and Design (ECCTD’93), vol. 1, pp. 637-642, Aug. 1993.

[5] Yoshifumi Nishio and Akio Ushida ”On Synchronization Phenomena in Coupled Chaotic Circuits Networks” Proceedings of IEEE International Symposium on Circuits and Systems (ISCAS’96), vol. 3, pp. 92-95, May. 1996. (Invited paper)

[6] M. S. Papadopoulou, I. M. Kyprianidis, I. N. Stouboulos ”Chaotic Dynamics of Coupled Nonlinear Circuits in Ring Connection ” Chaotic Modeling and Simulation (CMSIM) 1: 177-184, 2012