2017 RISP International Workshop on Nonlinear Circuits,
Communications and Signal Processing (NCSP'17)
Guam, USA, February 28th to March 3rd, 2017

Synchronization in Two Rings of Coupled Three van der Pol Oscillators

Daiki Nariai, Tran Minh Hai, Yoko Uwate and Yoshifumi Nishio

†Dept. of Electrical and Electronic Engineering, Tokushima University,
2-1 Minamijosanjima, Tokushima, 770-8506 Japan
Email: {nariai, minhhai, uwate, nishio}@ee.tokushima-u.ac.jp

Abstract

In this study, we investigate synchronization phenomena in two rings of van der Pol oscillators coupled by resistors. We propose novel coupled oscillatory system such as two rings of van der Pol oscillators coupled by resistors. We focus on the coupling strength of coupled van der Pol oscillators. From computer simulations, we investigate how synchronization phenomena change by changing the coupling strength. In this results, we observe various synchronization phenomena.

1. Introduction

Synchronization phenomena of coupled oscillators are the most familiar phenomena. Synchronization phenomena have been studied in various fields since a long time ago, such as in electrical systems, in mechanical systems, in biological systems and basically everywhere. Among them, synchronization phenomena of van der Pol oscillator are similar to natural phenomena by changing frequency. The coupled system of van der Pol oscillators is simple and easy to handle. Many researchers have proposed various coupled oscillatory networks of van der Pol oscillators [1] - [3]. We focus on the coupling strength of coupled oscillatory networks consisted of two kinds of van der Pol oscillators.

The van der Pol oscillator is a simple circuit. It is consisted of resistor, inductor, capacitor and nonlinear resistor. It was invented by electrical engineer Balthasar van der Pol. Equation of van der Pol is second-order differential equation.

In this study, we propose a novel coupled oscillatory system such as two rings of van der Pol oscillators coupled by resistors. First ring is consisted of three van der Pol oscillators connected by resistors. Second ring is consisted of three van der Pol oscillators connected by inductors and resistors. By computer simulations, we investigate synchronization phenomena observed in the proposed circuit system by changing the coupling strength.

2. System model

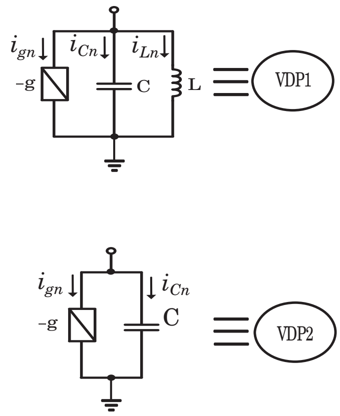

Figure 1: Circuit of van der Pol oscillators.

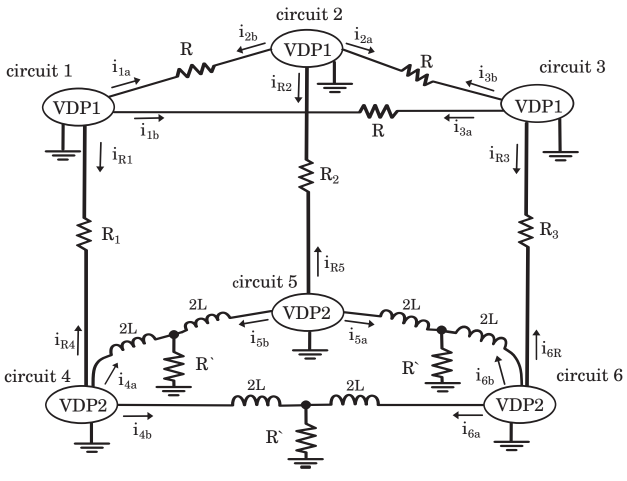

Figure 2: Circuit model.

Figure 1 shows two circuits which were used in my research. We use six van der Pol oscillators (three VDP1 and three VDP2). Figure 2 shows a system model with van der Pol oscillators (VDP1 and VDP2). We use two ring circuits of van der Pol oscillators, three VDP1 of first ring are connected by resistors, three VDP2 of second ring are connected by inductors and resistors. First and second ring are connected by resistors (R_1, R_2, R_3). We observe synchronization phenomena of adjacent oscillators. We investigate synchronization phenomena how to change by changing the value of resistors.

The circuit equations of first ring are given as follows:

$$\begin{cases} -i_{gn} - i_{cn} - i_{Ln} &= i_{an} + i_{bn} + i_{rn} \\ L\frac{di_{Ln}}{dt} &= v_n \\ v_n - v_i &= (i_{an} - i_{b(i)})R \\ v_n - v_j &= (i_{bn} - i_{a(j)})R \\ v_n - v_{n+3} &= (i_{rn} - i_{r(n+3)})R. \end{cases} \quad (1)$$

The circuit equations of second ring are given as follows:

$$\begin{cases} -i_{gn} - i_{cn} &= i_{an} + i_{bn} + i_{rn} \\ v_n - 2L\frac{di_{an}}{dt} &= -(i_{an} + i_{b(i)})R' \\ v_n - 2L\frac{di_{bn}}{dt} &= -(i_{bn} + i_{a(j)})R' \\ v_n - v_{n-3} &= (i_{rn} - i_{r(n-3)})R \end{cases} \quad (2)$$

where n denotes the number of circuit and $n = 1,2,3,4,5,6$. i denotes the number of circuit and $i = 2,3,1,5,6,4$. j denotes the number of circuit and $j = 3,1,2,6,4,5$.

Nonlinear resistor defined as follows:

$$i_{gn} = -g_1 v_n + g_3 v_n^3 \quad (3)$$

By changing the variables and parameters.

$$\begin{aligned} &t = \sqrt{LG}\tau, v_n = \sqrt{\frac{g_1}{3g_3}}x_n, i_n = \sqrt{\frac{g_1 C}{3g_3 L}}y_n, \\ &\varepsilon = g_1\sqrt{\frac{C}{L}}, \alpha = \frac{1}{R}\sqrt{\frac{L}{C}}, \\ &\beta = R'\sqrt{\frac{C}{L}}, \gamma_n = \frac{1}{R_n}\sqrt{\frac{L}{C}}. \end{aligned} \quad (4)$$

The normalized equations of first ring are given as follows:

$$\begin{cases} \dot{x}_n &= \varepsilon(x_n - x_n^3) - y_n - \gamma(x_n - x_{n+3}) \\ &\quad +\alpha(-x_n + x_i + x_j) \\ \dot{y}_n &= x_n. \end{cases} \quad (5)$$

The normalized equations of second ring are given as follows:

$$\begin{cases} \dot{x}_n &= \varepsilon(x_n - x_n^3) - y_{an} \\ &\quad -y_{bn} + \gamma(x_n - x_{n-3}) \\ \dot{y}_{an} &= x_n - \beta(y_{an} + y_{b(i)}) \\ \dot{y}_{bn} &= x_n - \beta(y_{bn} + y_{a(j)}) \end{cases} \quad (6)$$

where n denotes the number of VDP1 and VDP2, $n = 1,2,3,4,5,6$. The parameters ε, α, β, and γ denote the coupling strengh of the inductor, resistor R, resistor R' and resistor R_n.

3. Simulation Results

First, we define the synchronization condition by the following equation:

$$|x_a - x_b| < 0.05. \quad (7)$$

The simulation results of the system model are shown from Fig. 3 to Fig. 6. The value of the parameters are set to $\varepsilon = 0.05$, $\alpha = 0.05$, $\beta = 0.05$.

In case of $\gamma_1 = \gamma_2 = \gamma_3 = 0.02$, we conduct simulation, each one at a different initial value. In Fig. 3, synchronization phenomena are observed in circuit 1 - circuit 4. However, in Fig. 4, synchronization phenomena are observed in circuit 2 - circuit 5.

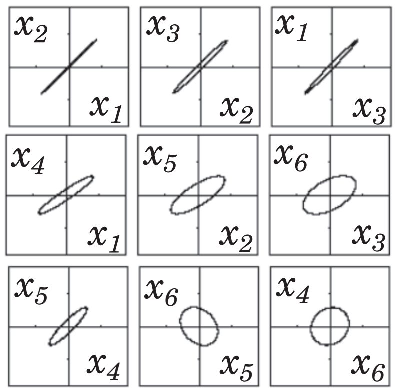

Figure 3: Phase difference ($\gamma_1 = \gamma_2 = \gamma_3 = 0.02$)

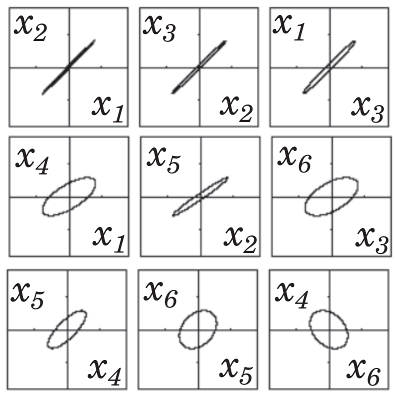

Figure 4: Phase difference ($\gamma_1 = \gamma_2 = \gamma_3 = 0.02$)

By chaging γ_1, γ_2 and γ_3, we can control synchronization phenomena regardless of initial value.

In case of $\gamma_1 = 0.001, \gamma_2 = 0.0001, \gamma_3 = 0.02$, in-phase synchronization phenomena are observed in oscillators of first ring, 3-phase synchronization phenomena are observed in oscillators of second ring. In this result, when we increase γ_3, three oscillators of first ring become in phase synchronization phenomena, oscillators of second ring become 3-phase synchronization phenomena.

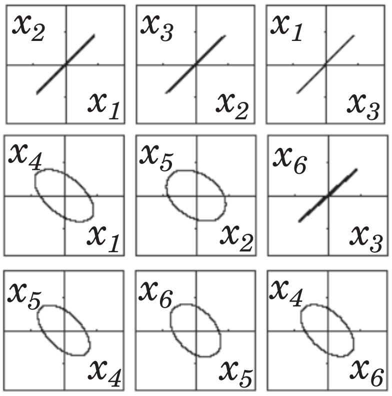

Figure 5: Phase difference ($\gamma_1 = 0.001, \gamma_2 = 0.001, \gamma_3 = 0.02$)

The time waveform of the voltage of each VDP2 after sufficient time has elapsed are shown in Fig. 6. This result show three oscillators of second ring become 3-phase synchronization phenomena.

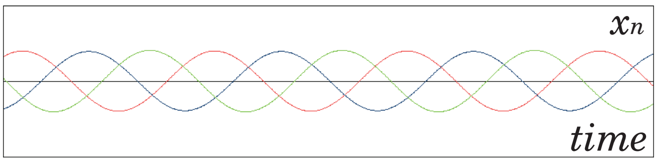

Figure 6: Time waveform of the voltage.

In case of $\gamma_1 = 0.02, \gamma_2 = 0.005, \gamma_3 = 0.02$, synchronization phenomena are observed in circuit 4 - circuit 6 without reference to initial value. In this result, when we increase two of γ_1 and γ_3, three oscillators of first ring and two of three oscillators of second ring become synchronization phenomena.

We investigate various synchronization phenomena by changing the coupling strengths. In this result, when we demand two oscillators of second ring to synchronize, we strengthen up two of γ_1, γ_2 and γ_3. When we demand three oscillators of second ring to become 3-phase synchronization phenomena, we strengthen up two of γ_1, γ_2 and γ_3. Therefore, we can control synchronization phenomena by coupling strengths.

Next, we investigate the synchronization rate this circuit model. We change the value of γ_2 from 0 to 0.03 at intervals of 0.001. We fix the value of other coupling strengths

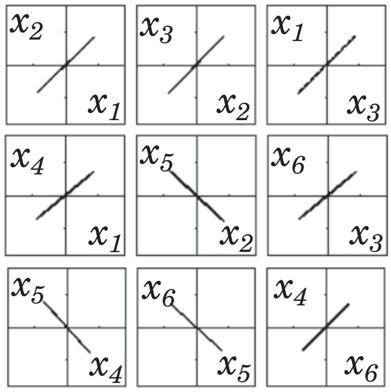

Figure 7: Phase difference ($\gamma_1 = 0.02, \gamma_2 = 0.005, \gamma_3 = 0.02$)

and initial value. Figure 8 shows synchronization rate of first ring. Figure 9 shows synchronization rate of oscillators between oscillators of first ring and oscillators of second ring. Figure 10 shows synchronization rate of second ring.

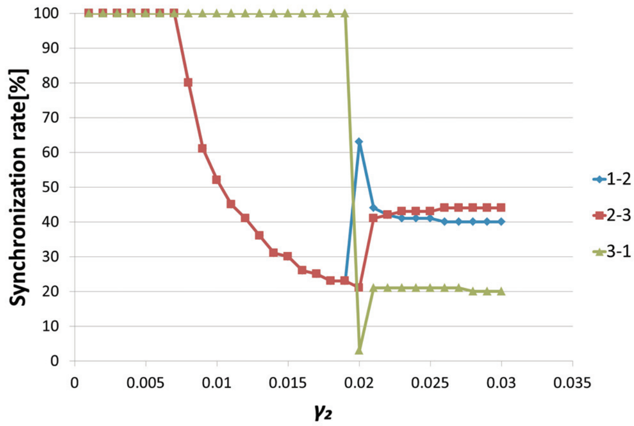

Figure 8: Synchronization rate.

In Fig. 8, when γ_2 is from 0 to 0.007, circuit 1 - circuit 2, circuit 2 - circuit 3 and circuit 3 - circuit 1 become in-phase synchronization phenomena. When γ_2 is 0.008, synchronization rate of circuit 1 - circuit 2 and circuit 2 - circuit 3 begin to decrease. When γ_2 is 0.02, synchronization rate of synchronization rate of synchronization rate of circuit 1 - circuit 2 is decrease sharply. When γ_2 is from 0.021 to 0.03, synchronization rate of circuit 1 - circuit 2, circuit 2 - circuit 3 and circuit 3 - circuit 1 are steady - state value.

In Fig. 9, when γ_2 is from 0.012, synchronization rate of circuit 1 - circuit 4 and circuit 3 - circuit 6 begin to decrease. When γ_2 is from 0.015, synchronization rate of circuit 2 - circuit 5 begin to increase. When γ_2 is from 0.021 to 0.03, synchronization rate of circuit 1 - circuit 4 and circuit 3 -

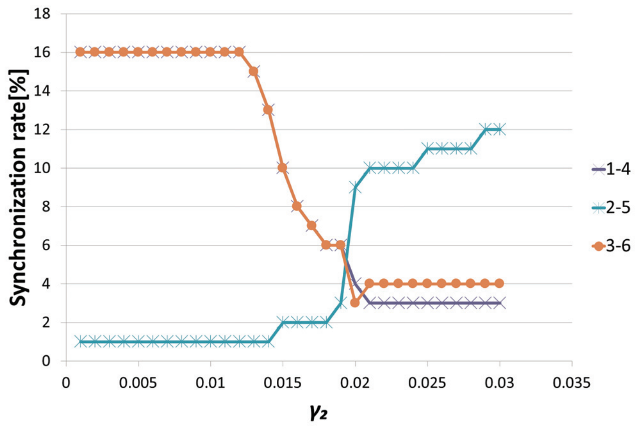

Figure 9: Synchronization rate.

circuit 6 are steady - state value.

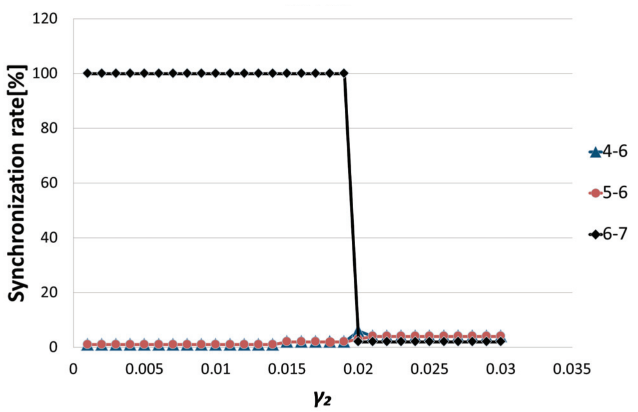

Figure 10: Synchronization rate.

In Fig. 10, when γ_2 is even slightly smaller than γ_1 and γ_3, circuit 4 - circuit 5 become in-phase synchronization phenomena. However when γ_1, γ_2, γ_3 equal, circuit 4 - circuit 5 do not become in-phase synchronization. When γ_2 is even bigger than γ_1 and γ_3, any oscillators do not become in-phase synchronization phenomena.

4. Conclusion

We have proposed a system model using two rings of coupled three van der Pol oscillators coupled by resistors or inductors. We can control the synchronization phenomena by changing the coupling strengths. When three coupling strengths (γ_1, γ_2, γ_3) equal, synchronization phenomena are observed by changing initial value. However when we strengthened up one of γ_1, γ_2 and γ_3, oscillators of first ring become in-phase synchronization phenomena, oscillators of second ring become 3-phase synchronization phenomena. when we increase two of γ_1, γ_2 and γ_3, three oscillators of first ring and two oscillators of second ring become synchronization phenomena. In the future, we investigate synchronization phenomena using other parameters and analyze the proposed circuit model.

Acknowledgment

This work was partly supported by JSPS Grant-in-Aid for Challenging Exploratory Research 26540127.

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