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Analysis of Firefly Algorithm Combined with Chaotic Map

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Abstract

Recently, optimization problems have drawn attention. To solve the problems, nature-inspired metaheuristic optimization algorithms such as Firefly Algorithm (FA) has been developed. FA is idealized from the social behavior of fireflies based on their flashing characteristics. Furthermore, FA combined with chaotic map is shown to be of benefit in the previous study. In our study, we investigate a different approach to insert chaotic map. We compare improved FA to the conventional FA using benchmark functions of Congress on Evolutionary Computation 2013. In our study, improved FA performs better than the conventional FA.

1. Introduction

Swarm Intelligence is one of research territory of artificial intelligence. Examples existing in nature are ant, bee, bird and fish. Nevertheless individual thing has simple and limited information, it shows advanced behavior as a whole when it becomes a group. The swarm intelligence technique is important because simple control regulation is more beneficial than complex control regulation. The applications of swarm intelligence technique are unmanned aircraft and a self-driving car. The good points of this technique are not being high altitude and being able to be downsize of robots.

Recently, optimization problems have drawn attention. The examples of optimization problems are traveling salesman problem, knapsack problem, shortest path problem, etc. These problems are very difficult to solve, because the nonlinearity of many these problems often results in local optima. To overcome this issue, metaheuristic optimization algorithms are engaged in research. These optimization algorithms attempt to idealize social behavior or natural phenomena. In social insect colonies, each individual insect seems to have its own agenda and the group in total appears to be highly organized. Nature-inspired Algorithms have been demonstrated to show effectiveness and efficiency to solve difficult optimization problems. A swarm is a group of multi-agent systems such as fireflies. Simple agents coordinate their activities to solve the complex problem to multiple forage sites in dynamic environments [1]. Several metaheuristic optimization algorithms are developed for global search. Such optimization algorithms develop more efficiency and solve larger problems. Metaheuristic algorithms have Genetic Algorithm (GA), Ant Colony Optimization (ACO), Particle Swarm Optimization (PSO), Firefly Algorithm (FA), etc. In our study, we use the FA. FA has also been applied to train neural networks [2].

FA idealizes the social behavior of fireflies based on their flashing characteristics. Therefore, FA is applicable for mixed variable and engineering optimization. For discrete problems and combinatorial optimization, discrete versions of FA have been developed with superior performance, which can be used for traveling salesman problems, graph coloring and other applications [3].

Recent applications of nonlinear dynamics, especially of chaos, have drawn attention in many fields. Chaos is seemingly a random movement of deterministic system. Chaos system has the properties of sensitivity to initial conditions. Therefore, using chaotic system in image encryption can meet security requirements [4]. Moreover, Chaos has a characteristic of an unpredictable movement. In the previous study, FA combined with chaotic map is shown to be of benefit [4]. We especially drew attention to one-dimensional chaotic map because one-dimensional chaotic map is generated from elementary equation and easy to deal with. In our study, we investigated a different approach to insert one-dimensional chaotic map into the conventional FA [5]. Moreover, we try to analysis of FA combined with chaotic maps.

This paper illustrates FA combined with chaotic map. Section 2 describes the conventional FA. Section 3 explains our proposed method. Numerical simulation and simulation results are discussed in Section 4. Section 5 is discussed the conclusion and outlines directions for further research.

2. The Conventional Firefly Algorithm (FA) [6]

First, we will introduce the behavior of fireflies. Firefly is the one of insects. The flashing light of fireflies is an amazing sight in the summer sky in the tropical and temperate regions. There are about 2000 firefly species in the world and Japan has about 40 firefly species. Moreover, most fireflies produce short and rhythmic flashes. The pattern of flashes is often unique for a particular species. Some species of fireflies can even synchronize their flashes.

The light intensity at a particular distance from the light source obeys the inverse square law. Furthermore, the air absorbs light which becomes weaker and weaker as the distance increases.

We can idealize some of the flashing characteristics of fireflies so as to develop firefly-inspired algorithms. The FA is swarm intelligence-based algorithm. Therefore, it has the similar advantages that other swarm intelligence-based algorithms have. The conventional FA was developed by Xin-She Yang in 2007. We use the following 3 idealized rules:

- All fireflies are unisex so that one firefly will be attracted to other fireflies regardless of their sex.
- Attractiveness is proportional to the their brightness, thus for any two flashing fireflies, the less brighter one will move towards the brighter one. The attractiveness is proportional to the brightness and they both decrease as their distance increases. If there is no brighter one than a particular firefly, it will move randomly.
- The brightness of a firefly is affected or determined by the landscape of the objective function.

The attractiveness of firefly β is defined by

$$\beta = (\beta_0 - \beta_{min})\mathrm{e}^{-\gamma r_{ij}^2} + \beta_{min} \tag{1}$$

where γ is the light absorption coefficient, β_0 is the attractiveness at $r_{ij} = 0$ and r_{ij} is the distance between any two fireflies i and j located at x_i and x_j respectively. β_{min} is the minimum value of β. The firefly i is attracted to another more attractive firefly j and the movement of firefly i is determined by

$$\boldsymbol{x_i} = \boldsymbol{x_i} + \beta(\boldsymbol{x_j} - \boldsymbol{x_i}) + \alpha\boldsymbol{\epsilon_i} \tag{2}$$

where α is the randomization parameter and ϵ_i is a random vector which are drawn from a Gaussian distribution.

The parameter $\alpha(t)$ is defined by

$$\alpha(t) = \alpha(0)\left(\frac{10^{-4}}{0.9}\right)^{t/t_{max}} \tag{3}$$

where t is the number of iteration. t_{max} is the maximum number of t.

3. Our proposed method

We analyze the improved FA(BFA, TFA and LFA). BFA, TFA and LFA are combined the conventional FA with Bernoulli shift map, Tent map and Logistic map. These maps are one of the one-dimensional chaotic maps, which is the simplest systems with the capability of generating chaotic motion. One-dimensional maps are introduced as follows. It generates chaotic sequences in (0, 1) assuming Eq. (5), Eq. (6) and Eq. (7). In the previous study, the author inserted 12 chaotic maps into the attractiveness of firefly β and the light absorption coefficient γ [4]. Each method is simulated by Sphere Function and Griewank's Function. In this study, we insert Bernoulli shift map, Tent map and Logistic map into the vector of random variable.

$$\boldsymbol{x_i} = \boldsymbol{x_i} + \beta(\boldsymbol{x_j} - \boldsymbol{x_i}) + \alpha(\boldsymbol{\epsilon_i} + \boldsymbol{z_i}) \tag{4}$$

The Bernoulli shift map(see Fig. 1) belongs to the class of piecewise linear maps similar to the Logistic map or the Kent map. It is formulated as follows:

$$z_{i+1} = \begin{cases} 2z_i & (0 \leq z_i \leq 0.5) \\ 2z_i - 1 & (0.5 \leq z_i \leq 1). \end{cases} \tag{5}$$

The Tent map(see Fig. 2) is similar to the Logistic map. It displays some specific chaotic effects. This map is formulated as follows:

$$z_{i+1} = \begin{cases} 2z_i & (0 \leq z_i \leq 0.5) \\ 2 - 2z_i & (0.5 \leq z_i \leq 1). \end{cases} \tag{6}$$

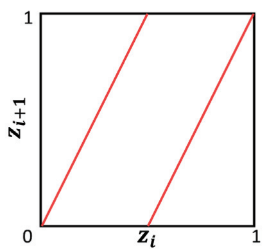

Figure 1: Bernoulli shift map.

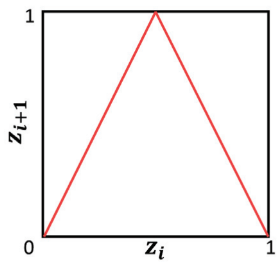

Figure 2: Tent map.

Although Logistic map(see Fig. 3) is a simple equation, it is an interesting map that shows a complicated chaotic behavior. This map is formulated as follows:

$$z_{i+1} = az_i(1 - z_i), \quad a = 4.0 \tag{7}$$

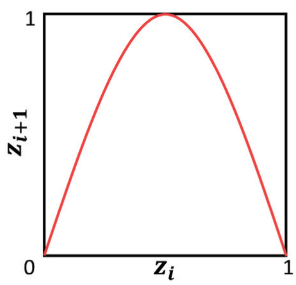

Figure 3: Logistic map.

4. Numerical simulation

We compare BFA, TFA and LFA to the conventional FA using 7 benchmark functions of Congress on Evolutionary Computation(CEC) 2013 [7]. Table 1 shows the functions we used. We chose 2 unimodal functions(f_1 and f_2), 3 basic multimodal functions(f_3, f_4 and f_5) and 2 composition functions(f_6 and f_7). The function's graphs(from Fig. 4 to Fig. 10) we used are showed in last page.

In this simulation, the optimal solutions x^* of these benchmark functions are shifted from 0 and the global optima $f(x^*)$ are not equal to 0. In addition, we assign the search range of these function is $[-100, 100]^D$(D:Dimension), the number of firefly N is 30. Each numerical experiment is run 50 times. The average length of design variables L is 100. Furthermore, we use $\beta_0 = 1.0$, $\beta_{min} = 0.2$, $\gamma = \frac{1}{\sqrt{L}}$, $D = 30$, $\alpha(0) = 0.5$ and $t_{max} = 1500$.

Table 2 shows the average, minimum and maximum error value.

We chose these functions because these functions is popular function for optimization problems. Sphere Function is no dependency between variables. Weierstrass Function is real-valued function advanced in 1872 by Karl Weierstrass. This function is continuous. However it is almost Nondifferentiable. Historically, this function is very important as example of pathological functions. Griewank's Function is almost gentle globally. However it has many local minima. Griewank's Function is said to be suitable for Simulated Annealing(SA) search. Composition Function 2 is generated from Schwefel's Function. Moreover Composition Function 5 is generated from Rotated Schwefel's Function, Rotated Rastrigin's Function and Rotated Weierstrass Function. Moreover we tried to use these functions for FA.

Showing in average error value of Table 2, BFA, TFA and LFA performs better than the conventional FA in several functions. In the unimodal functions(f_1 and f_2), TFA is the best performance. In the basic multimodal functions(f_3), however, the result changed relatively little, we assume BFA's average error value is effective. From examining the findings, Rotated Rosenbrock's Function may be not affected to a large degree by our proposed algorithms. In the basic multimodal functions(f_4), LFA is the best performance. In the basic multimodal functions(f_5), BFA is the best performance. Rotated Weierstrass Function(f_4) and Rotated Griewank's Function(f_5) has some local optima. These functions may be easy to be affected by Our proposed algorithms. In the composition functions(f_6), TFA and LFA is better than FA. In the composition functions(f_7), the result changed relatively little. Further, FA, BFA and LFA's average error value is same value.

In f_2, f_4 and f_5, our proposed 3 algorithms perform better than the conventional FA. Therefore, FA combined with chaotic map especially have good effects in Discus Function, Weierstrass Function and Griewank's Function. In this simulation, we also compare with BFA, TFA and LFA. BFA performs best in 2 functions(f_3 and f_5). TFA performs best in 3 functions(f_1, f_2 and f_6). LFA performs best in 1 function(f_4). Hence, TFA is the best algorithm in our proposed 3 algorithms.

Next, we mainly compare the average error values of simulation results. In results, BFA performs better than FA on 5 functions(f_1, f_2, f_3, f_4 and f_5). We assume BFA is strong for the unimodal functions and the basic multimodal functions. TFA performs better than FA on 5 functions(f_1, f_2, f_4, f_5 and f_6). LFA performs better than FA on 4 functions(f_2, f_4, f_5 and f_6).

Focusing attention on the form of map, the form of Bernoulli shift map and Tent map is rectilinear. However, the form of Logistic map is rounded. In f_1, rectilinear map(BFA and TFA) performs better than FA. In f_6, similar form's map(TFA and LFA) performs better than FA. Depending on the function, the form of map may influence the algorithm. LFA and FA's average of error value is same result in 3 functions(f_1, f_3 and f_7). Hence, we assume the map of rounded form is apt not to develop.

As a result, we assumed FA combined with chaotic map(BFA, TFA and LFA) is effective, compared to the conventional FA. However, the map of rectilinear is better than the rounded form. We are able to be simple to apply because one-dimensional chaotic map is easy equation.

5. Conclusion

This paper introduced analysis of the improved Firefly Algorithm(BFA, TFA and LFA). We tried to improve the conventional FA using Bernoulli shift map, Tent map and Logistic map. We compared average error values of simulation results. BFA, TFA and LFA performed better than the conventional FA. In light of the evidence, the form of map may influence the algorithm depending on the function.

In the future work, we will investigate BFA, TFA and LFA using more functions and insert other one-dimensional chaotic maps. Moreover, we will reseach the relationship to the form of map.

Acknowledgment

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References

[1] J. Senthilnath, S.N. Omkar and V. Mani, *"Clustering Using Firefly Algorithm: Performance Study"*, Swarm and Evolutionary Computation, 1, pp.164-171 (2011).

[2] S. Nandy, P.P. Sarker and A. Das, *"Analysis of Nature-Inspired Firefly Algorithm Based Back-Propagation Neural Network Training"*, International Journal of Computer Applications, 43(22), pp.8-16 (2012).

[3] X.-S. Yang and X. He, *"Firefly Algorithm: Recent Advances and Applications"*, Int. J. Swarm Intelligence, 1, 1, pp.36-50 (2012).

[4] A.H. Gandomi, X.-S. Yang, S. Talatahari and A.H. Alavi, *"Firefly Algorithm with Chaos"*, Commun Nonlinear Sci Numer Simulat, 18, pp.89-98 (2013).

[5] M. Moriyama, M. Takeuchi, Y. Uwate and Y. Nishio, *"Firefly Algorithm Combined with Chaotic Map"*, IEEE Workshop on Nonlinear Circuit Networks (NCN'16), pp.50-52 (2016).

[6] X.-S. Yang, *"Nature-Inspired Metaheuristic Algorithms Second Edition"*, Luniver Press (2010).

[7] J.J. Liang, B.Y. Qu, P.N. Suganthan and Alfredo G. Hernndez-Daz, *"Problem Definitions and Evaluation Criteria for the CEC2013 Special Session on Real-Parameter Optimization"*, Technical Report 201212, Computational Intelligence Laboratory, Zhengzhou University, Zhengzhou, China, and Technical Report, Nanyang Technological University, Singapore (2013).

Table 1: 2013 CEC Benchmark Functions

No.	Name	$f(x^*)$
f_1	Sphere Function	-1400
f_2	Rotated Discus Function	-1100
f_3	Rotated Rosenbrock's Function	-900
f_4	Rotated Weierstrass Function	-600
f_5	Rotated Griewank's Function	-500
f_6	Composition Function 2 (n=3, Unrotated)	800
f_7	Composition Function 5 (n=3, Rotated)	1100

Table 2: Simulation results

f		FA	BFA	TFA	LFA
f_1	avg	6.45×10^{-4}	$\mathbf{6.26 \times 10^{-4}}$	$\mathbf{6.11 \times 10^{-4}}$	6.45×10^{-4}
	min	4.32×10^{-4}	3.27×10^{-4}	3.91×10^{-4}	2.60×10^{-4}
	max	9.66×10^{-4}	1.07×10^{-3}	8.97×10^{-4}	1.00×10^{-3}
f_2	avg	1.22×10^{5}	$\mathbf{1.19 \times 10^{5}}$	$\mathbf{1.14 \times 10^{5}}$	$\mathbf{1.20 \times 10^{5}}$
	min	7.55×10^{4}	6.61×10^{4}	7.12×10^{4}	7.29×10^{4}
	max	2.14×10^{5}	1.86×10^{5}	1.68×10^{5}	1.85×10^{5}
f_3	avg	2.73×10^{1}	$\mathbf{2.72 \times 10^{1}}$	2.73×10^{1}	2.73×10^{1}
	min	2.54×10^{1}	2.53×10^{1}	2.59×10^{1}	2.59×10^{1}
	max	2.85×10^{1}	2.83×10^{1}	2.90×10^{1}	2.85×10^{1}
f_4	avg	1.04×10^{1}	$\mathbf{9.86 \times 10^{0}}$	$\mathbf{1.03 \times 10^{1}}$	$\mathbf{9.70 \times 10^{0}}$
	min	7.07×10^{0}	3.68×10^{0}	5.69×10^{0}	4.08×10^{0}
	max	1.59×10^{1}	1.73×10^{1}	1.50×10^{1}	1.59×10^{1}
f_5	avg	5.63×10^{-1}	$\mathbf{4.06 \times 10^{-1}}$	$\mathbf{5.39 \times 10^{-1}}$	$\mathbf{5.15 \times 10^{-1}}$
	min	4.15×10^{-2}	7.28×10^{-2}	3.08×10^{-2}	7.00×10^{-2}
	max	2.23×10^{0}	1.70×10^{0}	1.93×10^{0}	2.24×10^{0}
f_6	avg	3.31×10^{3}	3.46×10^{3}	$\mathbf{3.11 \times 10^{3}}$	$\mathbf{3.26 \times 10^{3}}$
	min	6.08×10^{2}	1.36×10^{3}	1.48×10^{3}	9.72×10^{2}
	max	6.27×10^{3}	6.13×10^{3}	6.21×10^{3}	6.11×10^{3}
f_7	avg	$\mathbf{2.33 \times 10^{2}}$	$\mathbf{2.33 \times 10^{2}}$	2.34×10^{2}	$\mathbf{2.33 \times 10^{2}}$
	min	2.01×10^{2}	2.19×10^{2}	2.14×10^{2}	2.21×10^{2}
	max	2.53×10^{2}	2.50×10^{2}	2.53×10^{2}	2.43×10^{2}

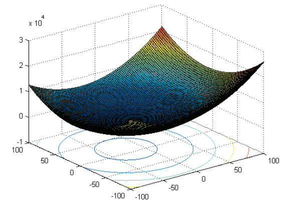

Figure 4: f_1.

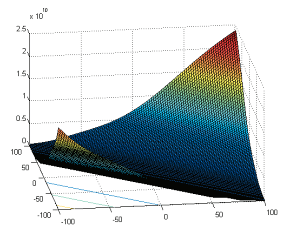

Figure 5: f_2.

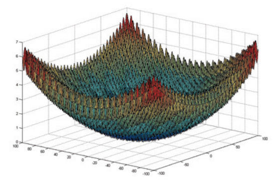

Figure 8: f_5.

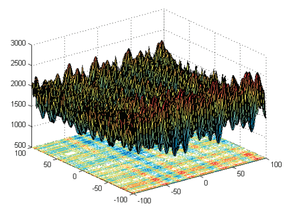

Figure 9: f_6.

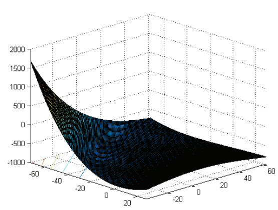

Figure 6: f_3.

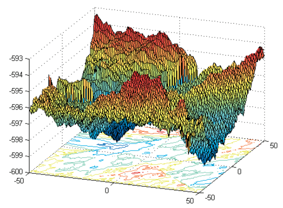

Figure 7: f_4.

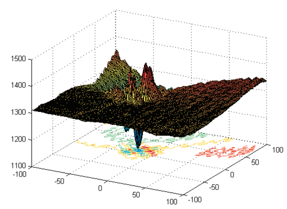

Figure 10: f_7.