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Synchronization in Dynamical Polygonal Oscillatory Networks with Switching Topology

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Abstract— In this study, we propose dynamical polygonal oscillator networks with switching topology by using van der Pol oscillators. In the proposed network, the network topology is switching with time. We confirmed that new synchronization state can be occurred by stochastically switching the network topology.

1. Introduction

The synchronization phenomena observed from coupled nonlinear oscillators are suitable model to analyze the natural phenomena [1],[2]. Therefore, many researchers have proposed different coupled oscillatory networks and have discovered many interesting synchronization phenomena [3]-[6].

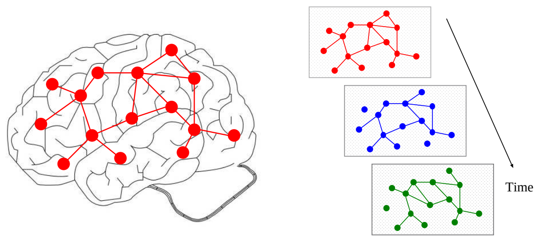
In the human brain, synchronization of neural activity between different cortical areas may proved several kinds of neural information functions depending on the network topology (see Fig. 1(a)) [7]. Recently, grid cells which are the type of neurons in the entorhinal cortex show remarkable hexagonal activity patterns [8], [9]. The polygonal structure of neuronal firing pattern has possibility important role for emerging unified computational framework [10].

Namely, it is important to study synchronization phenomena observed from the polygonal oscillatory networks with the several characteristics in the human brain as we described above, for modeling neuro-biological systems and applying its high-functional information processing to engineering applications.

Neurons in the brain are expressed by stable oscillators. Neuronal oscillators using mathematical models are often used for modeling brain networks. However, there are not many models of neuronal oscillators using electrical circuits. In order to realize the brain network by real physical systems, we need to investigate synchronization of neural activity in analog electrical circuits. Here, van der Pol oscillator which is simple oscillatory circuit model is used as neural oscillator.

We have investigated the synchronization phenomena in the coupled polygonal oscillatory networks sharing branches [11], [12]. In this system, odd number of van der Pol oscillators are connected to every corner of each polygonal network. Namely, frustration is occurred between the adjacent oscillators. By using computer simulations and theoretical analysis, we confirm that the coupled oscillators tend to synchronize to minimize the power consumption of the whole system. The phase difference of the shared oscillators is solved by finding the minimum value of the power consumption function.

Furthermore, coupling strength between neural activities are changed depending on the information processing with time (see Fig. 1(b)). However, in Ref. [11], we have considered the static network model.



(a) Sync. in neural activity. (b) Time evolution.

Figure 1: Brain network.

In this study, we propose new dynamical polygonal circuit system which is including the switching couplings, in order to understand the mechanism of high functional neural information processing in the human brain. In the proposed network, external signal is replaced with stochastic factor and we consider that the coupling strength is expressed by on/off as an extreme example. The synchronization phenomena in the proposed networks are investigated by using computer simulations. First, we investigate synchronization states in two coupled polygonal networks (triangular and quadrangular networks) with one switching coupling, in order to understand the basic phenomena. Next, the system model is extended to six coupled polygonal networks with three switching couplings for understanding more complex behavior. By using the computer simulations, we confirm that new synchronization states can be produced when the network topology is changed by switching the connection (on/off) of the edges in the proposed network.

2. Two Coupled Network - Number of Switching Coupling : 1 -

2.1. Circuit Model

Figures 2 and 4 show the conceptual circuit model and circuit realization using van der Pol oscillators. The triangular and the quadrangular oscillatory networks are coupled by sharing a branch. We call this circuit system "3-4 coupling network." In this circuit model, we consider the coupling method which two adjacent oscillators are tend to synchronize at anti-phase state.

In Fig. 2, a chain line denotes the switching coupling (SC). If the coupling edge is selected as the switching coupling, the two oscillators connect or disconnect according to the coupling probability (p). Namely, the switching coupling is connecting and disconnecting stochastically. If the coupling probability is $p = 0.0$, the coupling strength is $\gamma = 0$. The state (connecting/disconnecting) of the switching coupling is updated at every switching time (ST) in the simulation. The switching time denotes the iteration number. One example of operation of the switching coupling is shown in Fig. 3.

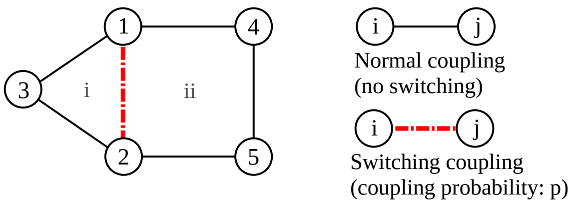


Figure 2: Conceptual circuit model for 3-4 coupling network.

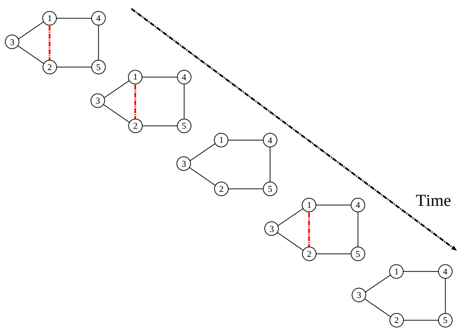


Figure 3: Example of operation of switching coupling for 3-4 coupling network.

We consider only one coupling edge can be the switching coupling in the 3-4 coupling network. Table I is summarized the network typologies and synchronization states of four patterns when the switching coupling is connecting

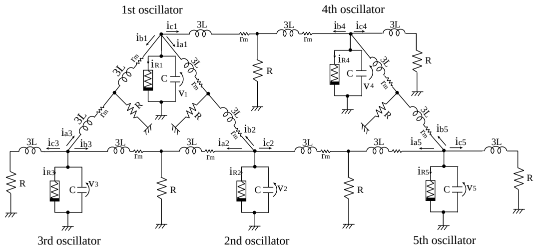
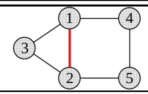
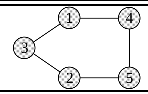
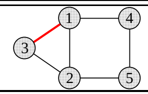
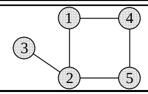
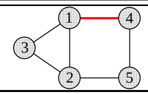
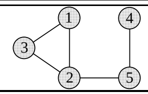
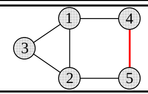
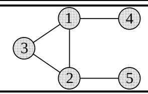


Figure 4: Circuit model for 3-4 coupling network.

and disconnecting. We consider the four patterns (pattern-A, B, C, and D) depending on the switching coupling. "state-1" and "state-2" denote the synchronization state obtained from the two types of the network topologies.

Table 1: Switching Coupling (SC) Patterns.

Pattern	state-1 (SC: on)	state-2 (SC: off)
pattern-A (SC:γ_{12})	 $\phi_{12} \approx 138.6°$	 $\phi_{12} \approx 72°$
pattern-B (SC:γ_{13})	 $\phi_{12} \approx 138.6°$	 $\phi_{12} \approx 180°$
pattern-C (SC:γ_{14})	 $\phi_{12} \approx 138.6°$	 $\phi_{12} \approx 120°$
pattern-D (SC:γ_{45})	 $\phi_{12} \approx 138.6°$	 $\phi_{12} \approx 120°$

Next, we develop the expression for the circuit equations of 3-4 coupling oscillatory networks as shown in Fig. 4. The $v_k - i_{Rk}$ characteristics of the nonlinear resistor are approximated by the following third order polynomial equation,

$$i_{Rk} = -g_1 v_k + g_3 {v_k}^3 \quad (g_1, g_3 > 0), (k = 1, 2, 3, 4). \qquad (1)$$

The normalized circuit equations governing the circuit are expressed as

[kth oscillator]

$$\begin{cases} \dfrac{dx_k}{d\tau} = \varepsilon\left(1 - \dfrac{1}{3}{x_k}^2\right)x_k - (y_{ak} + y_{bk} + y_{ck}) \\ \dfrac{dy_{ak}}{d\tau} = \dfrac{1}{3}\left\{x_k - \eta y_{ak} - \gamma_{kn}(y_{ak} + y_n)\right\} \\ \dfrac{dy_{bk}}{d\tau} = \dfrac{1}{3}\left\{x_k - \eta y_{bk} - \gamma_{kn}(y_{bk} + y_n)\right\} \\ \dfrac{dy_{ck}}{d\tau} = \dfrac{1}{3}\left\{x_k - \eta y_{ck} - \gamma_{kn}(y_{ck} + y_n)\right\} \\ \qquad\qquad (k = 1, 2, 3, 4, 5). \end{cases} \qquad (2)$$

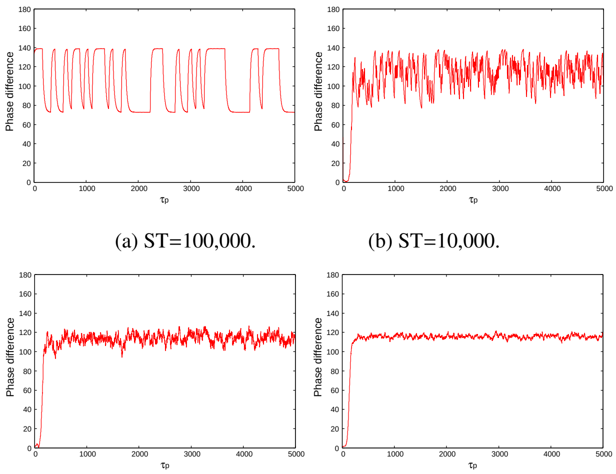
where

$$t = \sqrt{LC}\tau, \quad v_k = \sqrt{\frac{g_1}{3g_3}}x_k, \quad i_{ak} = \sqrt{\frac{g_1}{3g_3}}\sqrt{\frac{C}{L}}y_{ak},$$
$$i_{bk} = \sqrt{\frac{g_1}{3g_3}}\sqrt{\frac{C}{L}}y_{bk}, \quad \varepsilon = g_1\sqrt{\frac{L}{C}},$$
$$\gamma = R\sqrt{\frac{C}{L}}, \quad \eta = r_m\sqrt{\frac{C}{L}},$$
$$(k = 1, 2, 3, 4, 5).$$

In these equations, γ_{kn} is the coupling strength, ε denotes the nonlinearity of the oscillators and y_n denotes the current of neighbor oscillator on the coupling resistor. For the computer simulations, we calculate Eq. (2) using the fourth-order Runge-Kutta method with the step size $h = 0.005$. The parameters of this circuit model are fixed as $\varepsilon = 0.1$ and $\eta = 0.0001$.

2.2. Synchronization Phenomena

First, we investigate the synchronization state of the 3-4 coupling network with the switching topology. The switching time (ST) is changed from 100 to 10000. Figures 5 and 6 show the examples of the phase difference between 1st and 2nd oscillators for Patterns A and C at Table 1 when the coupling probability is set to p=0.5. The phase difference is calculated by using the Poincaré section : $x_k < 0$, $y_k = 0$.

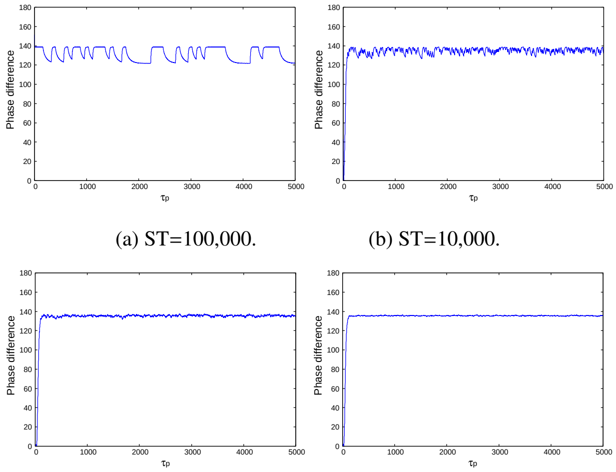


(a) ST=100,000. (b) ST=10,000.

(c) ST=1,000. (d) ST=100.

Figure 5: Example of the phase difference for Pattern A (p=0.5).

In the both cases of Patterns-A and B, the phase difference changes between two states (state-1 and state-2), when the switching time (ST) is fixed with $100,000$ as shown in Figs. 5(a) and 6(a). By decreasing the value of ST, the oscillation range of the phase difference becomes small and the phase difference converges to the certain value (see Figs. 5(d) and 6(d)). And the converging phase state shows value between the state-1 and the state-2



(a) ST=100,000. (b) ST=10,000.

(c) ST=1,000. (d) ST=100.

Figure 6: Example of the phase difference for Pattern C (p=0.5).

Figure 7 shows the simulation results of observing new phase states by changing the coupling probability p for four patterns. The phase difference shows the average value of 1000 iterations in the simulation.

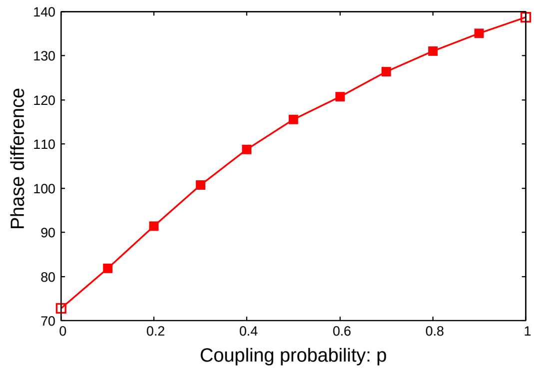
In the cases of Patterns-A, C and D, the new phase states can be observed between state-1 and state-2 in the range of $0.1 \leq p \leq 0.9$. While, in the case of Pattern-B, the phase states show almost similar value with state-2 when the coupling probability is smaller than 0.6. By increasing p, the new phase states can be obtained as shown in Fig. 7(b).

From these results, we can see that new synchronization states can be occurred by changing the network topology.

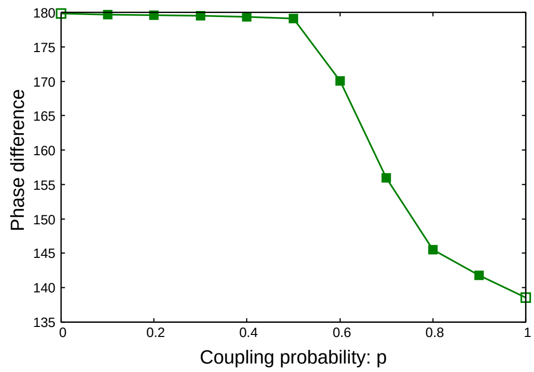
3. Conclusions

In this study, we have proposed new dynamical polygonal circuit system which is including the switching couplings, in order to understand the mechanism of high functional neural information processing in the human brain. In the proposed network, external signal is replaced with stochastic factor and we consider that the coupling strength is expressed by on/off as an extreme example. By using the computer simulations, we confirmed that new synchronization state can be occurred by switching the network topology.

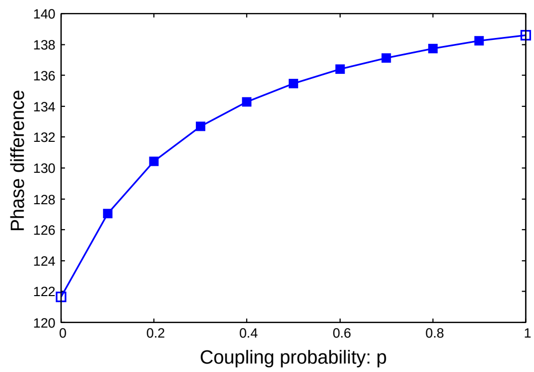
For the future works, we investigate the synchronization state in detail when the three switching couplings is changed with different iteration in the extended dynamical network. And, we would like to apply the proposed system to more large scale networks to model the existing biological complex networks.



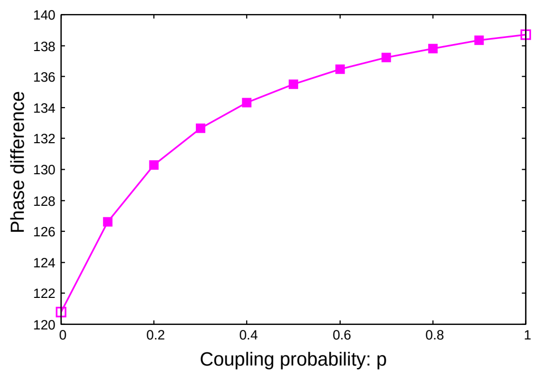
(a) Pattern-A.



(b) Pattern-B.



(c) Pattern-C.



(d) Pattern-D.

Figure 7: Observing new phase states by changing p.

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