Synchronization Phenomena in Star-Coupled van der Pol Oscillators by Adding Different Frequency Oscillators

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Abstract—**In many cases, mutual synchronization systems consisting of a large number of oscillators are used for practical model. In this work, we investigate synchronization phenomena observed by adding different frequency van der Pol oscillators coupled with star combination. By computer simulations, we confirm some of oscillators in the system are synchronized at anti-phase.**

I. Introduction

Synchronization has grown to a considerable research field in recent years. Synchronization phenomena in large populations of interacting elements are the subject of intense research efforts in physical, biological, chemical, and social systems. The synchronization in the research is one of the basic phenomena of nature and it is observed over a wide range of fields for example: human applause, digital telephony, video, digital audio, frogs, etc. In addition, in the organism, synchronization phenomena has been observed in the activity of the brain and the operation of the heart. It is considered to have become an important role. In the future, analysis of brain activity and analysis of heart activity are very important. Its application to engineering fields such as the realization of brain computer is expected.

Now, synchronization phenomena have been studied by many researchers in various field: two van der Pol oscillators coupled by chaotically varying resistor [1], synchronization in coupled van der Pol oscillators involving periodically forced capacitors [2], comparing two-layer CNN with van der Pol oscillators coupled by inductors [3], group synchronization of van der Pol oscillators with different frequencies [4], stochastic bifurcations in a bistable Duffing−van der Pol oscillator with colored noise [5], synchronization and anti-synchronization of chaos in an extended Bonhoffer−van der Pol oscillator using active control [6].

In coupled oscillators, synchronization phenomena depend on the type of coupling. When two identical oscillators are coupled, there are two basic possibilities of synchronization: in-phase synchronization and anti-phase synchronization.

In this study, we investigate the effect to three star-coupled oscillators by adding another oscillator with different frequency. In this work, we confirm the case of four oscillators and the case of five oscillators in the system. First, we change the parameter α of the fourth oscillator and investigate the effect to the star-coupled circuits with three oscillators. Next, we add the fifth oscillator and investigate the effect to the star-coupled oscillators. We consider that many unknown phenomena remain in such systems. Therefore, it is very important to investigate such systems.

II. Circuit Model with Four Oscillators

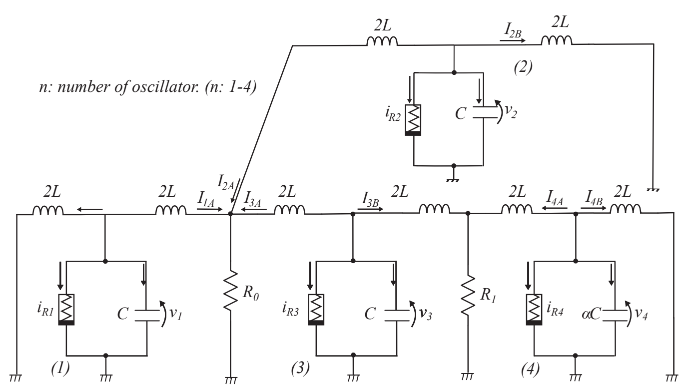

Fig. 1. Circuit model.

The circuit model used in the first pattern is shown in Fig. 1. Three van der Pol oscillators are connected as the star combination. In addition, we add a different frequency oscillator to the star-coupled van der Pol oscillators. We change the frequency of the $4th$ oscillator and investigate the influence of the $4th$ oscillator to the overall star circuit.

Firstly, the $v - i_{Rk}$ characteristics of the nonlinear resistors are defined as follows:

$$i_{Rk} = -g_1 v_k + g_3 v_k^3. \tag{1}$$

By changing the variables and the parameters:

$$v_k = \sqrt{\frac{g_1}{3g_3}}x_k,\ i_k = \sqrt{\frac{Cg_1}{3Lg_3}}y_k,\ \alpha = \frac{1}{\omega^2},\ t = \sqrt{L_1 C}\tau \quad (2)$$

and defining:

$$\delta_0 = R_0\sqrt{\frac{C}{L}},\ \delta_1 = R_1\sqrt{\frac{C}{L}},\ \varepsilon = g_1\sqrt{\frac{L}{C}}. \quad (3)$$

The normalized circuit equations are represented as follows:

$$\left\{\begin{array}{lcl} \dfrac{dx_k}{d\tau} & = & \varepsilon(x_k - \frac{1}{3}x_k^3) - y_k - z_k \quad (k=1,2,3) \\ \dfrac{dy_k}{d\tau} & = & \frac{1}{2}x_k - \frac{1}{2}\delta_0(y_1+y_2+y_3) \quad (k=1,2,3) \\ \dfrac{dz_k}{d\tau} & = & \frac{1}{2}x_k \quad (k=1,2,4) \\ \dfrac{dz_3}{d\tau} & = & \frac{1}{2}x_3 - \delta_1(z_3+y_4) \\ \dfrac{dx_4}{d\tau} & = & \omega^2(\varepsilon(x_4 - \frac{1}{3}x_4^3) - y_4 - z_4) \\ \dfrac{dy_4}{d\tau} & = & \frac{1}{2}x_4 - \frac{1}{2}\delta_1(z_3+y_4) \end{array}\right. \quad (4)$$

where ε is the nonlinear intensity.

III. Simulation Result

We use Runge-Kutta method to calculate the values of x, y and their phase difference. We investigate synchronization phenomena of the oscillators by using computer simulation with ε=0.03, δ_0=0.1 and δ_1=0.3. We investigate the change of varying ω. Figures 2 to 4 show the simulation results.

In Fig. 2, in the case of ω =1, the all four oscillators oscillated. Only between the $3rd$ oscillator and the $4th$ oscillator are synchronized at anti-phase. Consequently, we did not see the effects of ω to the star-coupled circuit. And then, as ω increases to 1.2, the oscillations of the $3rd$ and the $4th$ oscillators stop, namely oscillation death appears as Fig. 3. In that moment, the $1st$ oscillator and the $2nd$ oscillator synchronize at anti-phase. By these result, we can see the effect of α to the star-coupled oscillators.

In Fig. 4, the $4th$ oscillator oscillates again and the amplitude increase by increasing ω. When the value of ω above 1.2, frequency of the $4th$ oscillator becomes faster.

The amplitudes of the oscillator are shown in the Figs. 5 and 6. In Fig. 5, ω is changed inside a large range value [1.0,2.0]. As the result, the amplitudes of the $1st$ and the $2nd$ oscillator unchanged. Therefore, frequency is not effect to the star-coupled oscillators.

In Fig. 6, when the ω increase from 1 to 1.2, the amplitudes of the $3rd$ and the $4th$ oscillator become gradually smaller. Next, when the ω is increased to 1.4, the amplitudes of the $3rd$ and the $4th$ oscillator are perfect stop. When ω above 1.4, the amplitude of the $4th$ oscillator goes up. However, the amplitudes of the $3rd$ oscillator are almost unchanged.

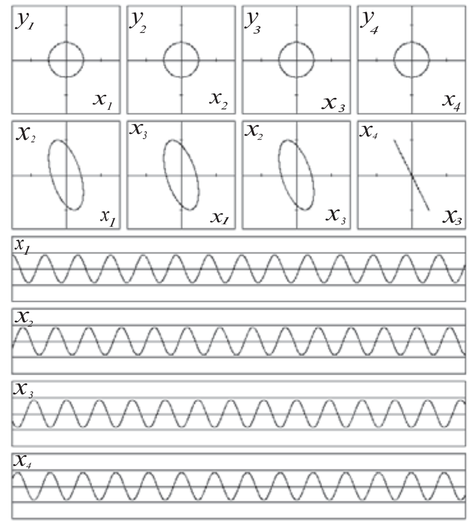

Fig. 2. Simulation result (ω=1).

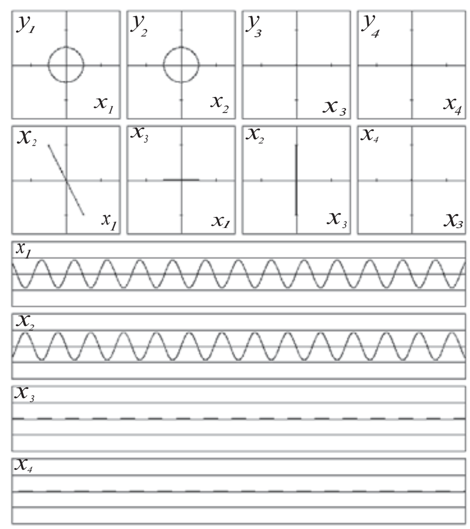

Fig. 3. Simulation result (ω=1.2).

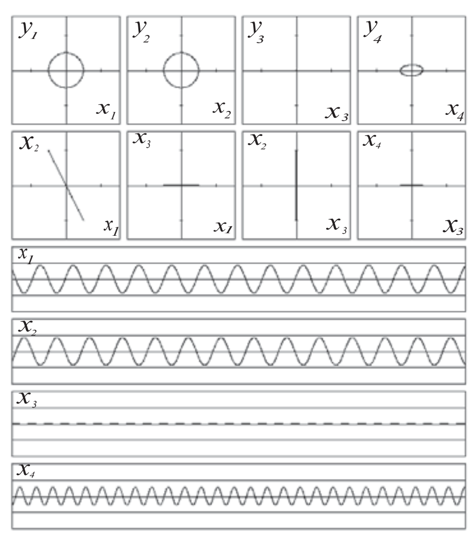

Fig. 4. Simulation result (ω=2).

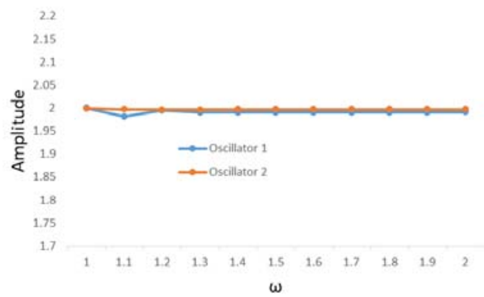

Fig. 5. Amplitude of oscillator 1 and oscillator 2.

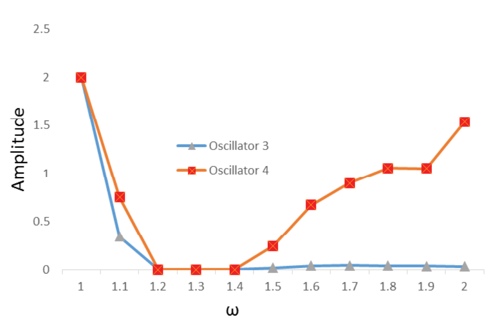

Fig. 6. Amplitude of oscillator 3 and oscillator 4.

IV. Five Oscillators Case

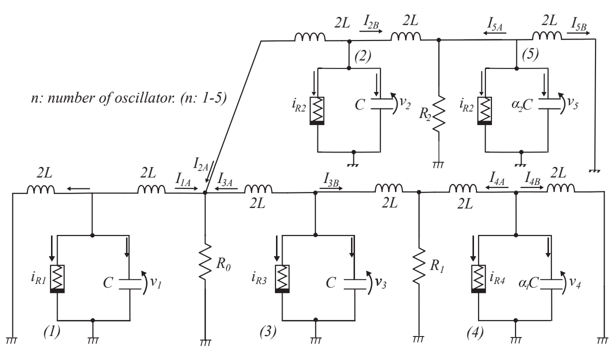

Fig. 7. Circuit model with five oscillators.

Next, we use a circuit model in Fig. 7. This system has five oscillators. We add the $5th$ oscillator in this system. We change the frequency of the $4th$ and the $5th$ oscillators and investigate their influence to the overall star-coupled oscillators.

In this system, the values of frequency α_1 and α_2 are normalized as Eq. (5)

$$\alpha_i = \frac{1}{\omega_i^2} \qquad (i = 1, 2) \tag{5}$$

We investigate synchronization phenomena with ε = 0.03, δ_0=0.1, δ_1=0.3 and δ_2=0.3 as varying ω_1 and ω_2. The simulation results are shown in Fig. 8 to Fig. 12.

In the case of ω_1=1 and ω_2=1, from before result we can imagine no effect from ω_1 and ω_2 to the star-coupled oscillators and that all of the five oscillators oscillate. Between the $3rd$ oscillator and the $4th$ oscillator, the $2nd$ oscillator and the $5th$ oscillator, the phase difference is synchronized at anti-phase.

In Fig. 8, the phase difference between the $3rd$ oscillator and the $4th$ oscillator, the $2nd$ oscillator and the $5th$ oscillator are quasi synchronized at anti-phase. The phase difference become asynchronous between the $1st$ oscillator and the $3rd$ oscillator, the $2nd$ oscillator and the $5th$.

In the next case, we increase only ω_1 to 1.1 and ω_2 remains 1. Now, we observe the effect of ω to the star-coupled oscillators. Oscillations of the $3rd$ oscillator and $4th$ oscillator become small.

From the result in Fig. 10, the $1st$, the $2nd$ and the $5th$ oscillators oscillate and the $3rd$ and the $4th$ oscillators stop, namely oscillation death appear. The phase difference between the $1st$ oscillator and the $2nd$ oscillator, the $2nd$ oscillator and the $5th$ oscillator become synchronized at anti-phase completely.

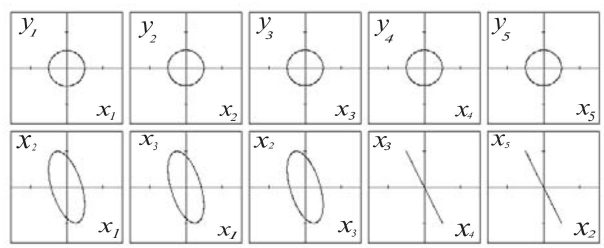

Fig. 8. Simulation result (ω_1=1, ω_2=1).

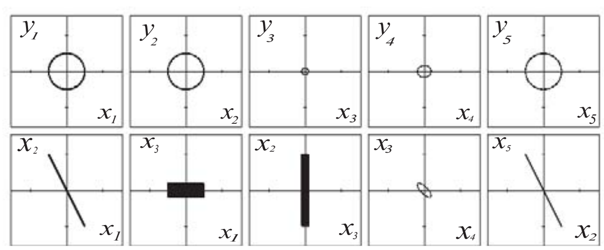

Fig. 9. Simulation result (ω_1=1.1, ω_2=1).

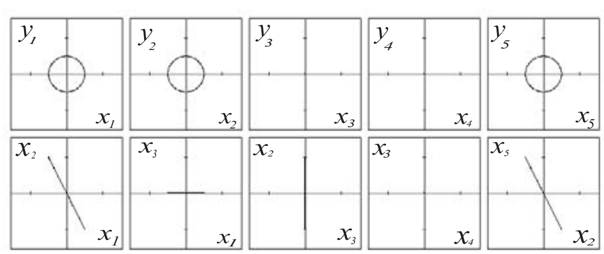

Fig. 10. Simulation result (ω_1=1.2, ω_2=1).

In Fig. 11, we increase not only ω_1, but also ω_2 to 1.2. Frequency is given a strong influence on circuit. Only the $1st$ oscillator still oscillate and the remaining oscillators stop.

In Fig. 12, if we still increase ω_2 to 2.5, oscillation of the $5th$ oscillator also starts again. Frequency of the $5th$ oscillator becomes faster when ω_2 is 1.2 and higher.

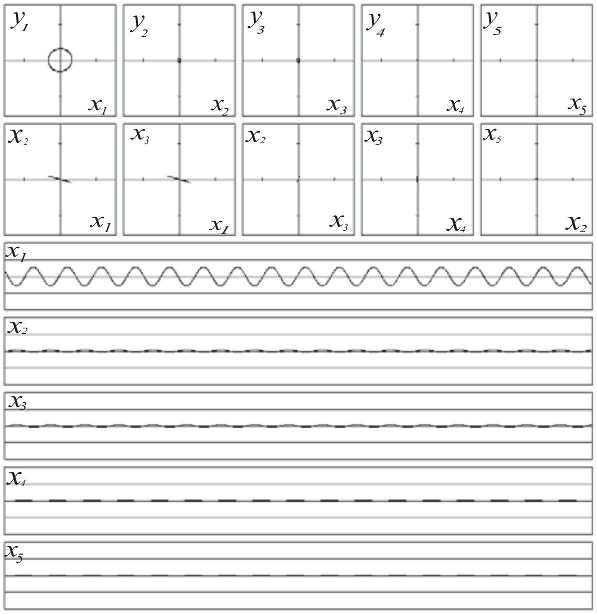

Fig. 11. Simulation result(ω_1=1.2, ω_2=1.2).

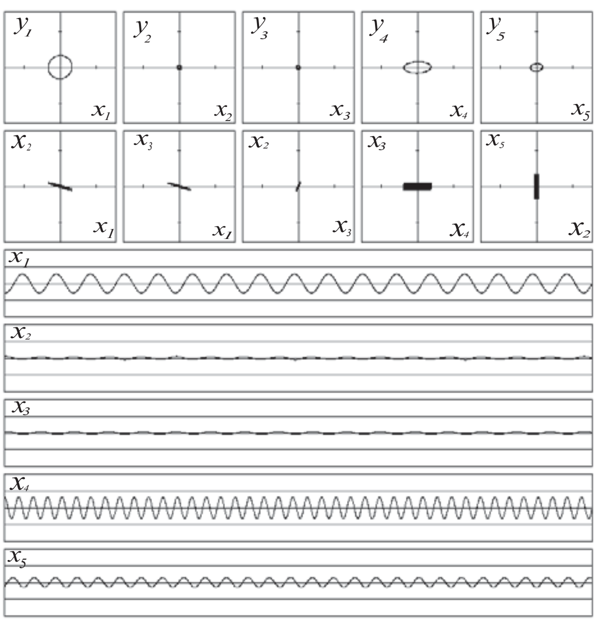

Fig. 12. Simulation result (ω_1=2.36, ω_2=1.6).

V. Conclusion

In this study, we have investigated synchronization phenomena and oscillators of four and five oscillators with different frequencies. By carrying out computer simulations, we confirmed oscillation of the oscillator stop in some range of the frequency. When the frequency of the $4th$ and $5th$ oscillator is varied, oscillation of some oscillator stop, namely oscillation death appear. By increasing the frequencies, the attractor becomes asynchronous states.

For the future work, we would like to increase the number of the oscillators to six and to change the star combination to the ring combination. We also would like to do real circuit experiments to confirm the results.

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