Switching Synchronization States of a Ring of Coupled Chaotic Circuits with One-Direction Delay Effects

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***Abstract*—Synchronization states can be observed in coupled circuits. Further, interesting synchronization states were confirmed in coupled chaotic circuits containing time delay. In this study, we investigate the novel coupling methods and synchronization states observed in coupled chaotic circuits containing time delay. The novel coupling methods is a ring of coupled chaotic circuits with one-direction delay effects. Synchronization state observed the novel coupling methods is switching synchronization state. The switching synchronization state is changed by time delay of each subcircuit. We focus on relationships between switching synchronization state and the pattern of time delay. Moreover, we investigate the cycle of switching synchronization state.**

I. Introduction

Studies on synchronization state are extensively carried out in various fields [1]-[3]. Recently, in particular, synchronization states in chaotic oscillators are studied by many researchers. Behavior of chaotic oscillators is interesting. Then, chaos phenomena are quite dependent on initial values and not periodical and predictable. Moreover the synchronization states have caused very interesting phenomena. Synchronization and the related bifurcation of chaotic systems are good methods to describe various high-dimensional nonlinear phenomena in the field of natural science. However, many synchronization states of coupled chaotic oscillators have not been solved yet. The synchronization phenomena in electric circuit make clear the mechanism of the synchronization phenomena in our daily life. There are many nonlinear systems containing time delay, such as neural networks, control systems, meteorological systems, biological systems and so on in the natural world. Namely, it is considered that investigation of stability in such time-delay systems is significant [4]. Generation of chaos in time delayed system is reported self excited oscillation system containing time delay [5]. The oscillators have feedback systems which control gains in this study. This chaotic circuit can be easily realized by using simple electric circuit element and analyzed exactly. The coupling switch connects alternately with one subcircuit and other with a fixed time interval. On the other hand, there are examples of nonlinear phenomena, chaotic synchronization and so on [6]. In particular, many studies on synchronization of coupled chaotic circuits have been reported [7].

In this study, we devise the novel coupling method that takes advantage of features of the chaotic circuit containing time delay. The novel coupled method is utilizing the characteristics of the circuit having time delayed feedback. Then, we observe switching synchronization state. We carry out computer calculations for three coupled auto gain controlled oscillators containing time delay and investigate time delay of subcircuits effects a change of synchronization state and the time waveform.

II. Time Delayed Chaotic Circuit

Figure 1 shows the time delayed chaotic circuit. This circuit consists of one inductor L, one capacitor C, one linear negative resistor $-g$ and one linear positive resistor G of which amplitude is controlled by the switch containing time delay. The current flowing through the inductor L is i, and the voltage between the capacitor C is v. The circuit equations are normalized as Eqs. (1) and (2) by changing the variables as below.

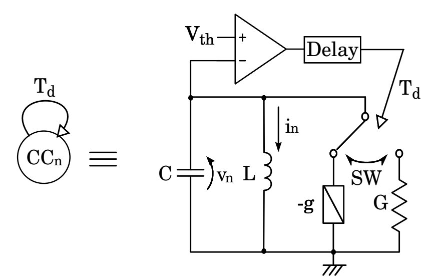


Fig. 1. Time delayed chaotic circuit.

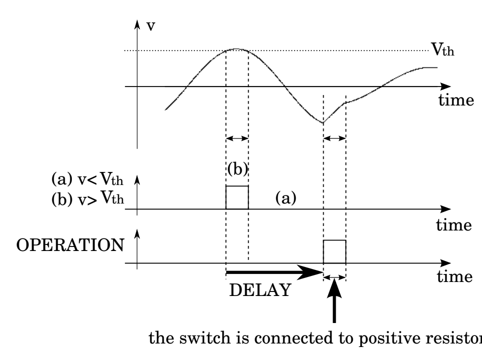


Fig. 2. Switching operation.

(A) In case of switch connected to $-g$,

$$\begin{cases} \dot{x} &= y \\ \dot{y} &= 2\alpha y - x, \end{cases} \quad (1)$$

(B) In case of switch connected to G,

$$\begin{cases} \dot{x} &= y \\ \dot{y} &= -2\beta y - x. \end{cases} \quad (2)$$

By changing the parameters and variable as follow:

$$i = \sqrt{\frac{C}{L}} V_{th} x,\ v = V_{th} y,\ t = \sqrt{LC}\tau,$$
$$g\sqrt{\frac{C}{L}} = 2\alpha \text{ and } G\sqrt{\frac{C}{L}} = 2\beta.$$

The switching operation is shown in Fig. 2, it controls the amplitude of the oscillator. This switching operation is included time delay. T_d denotes the time delay. First, the switch is connected to a negative resistor. In state of that, the voltage v is amplified up to while v is oscillating, the amplitude exceeds the threshold voltage V_{th} which is the threshold control loop. Second, the system memorize the time as T_{th} while v is exceeding the threshold voltage V_{th} and that state is remained for T_{th}. In subsequent the instant of exceeding threshold V_{th}, the switch stays the state for T_d. After that switch is connected to positive resistor during T_{th}. The switch does not immediately connect in the positive resistor however the switch is connected after T_d. A set of switching operations control the amplitude of v. By using mapping method to this circuit, we could derive the 1-dimensional Poincare map explicitly from each circuit, and the Poincare map was proved to have a positive Liapunov number with computer assistances [3].

III. System Including Time Delay in One Direction

The circuit in this study has characteristic time delays methods. We have devised coupled systems as shown in Fig. 3. This system is coupled by resistors R_0. It is called coupled systems and "a ring of coupled chaotic circuits with one-direction delay effects" By changing the parameters and variables as follows:

$$i_n = \sqrt{\frac{C}{L}} V_{th} x_n,\ v_n = V_{th} y_n,\ t = \sqrt{LC}\tau,$$
$$g\sqrt{\frac{C}{L}} = 2\alpha,\ G\sqrt{\frac{C}{L}} = 2\beta \text{ and } \gamma = R_0\sqrt{\frac{C}{L}}.$$

Here is case of coupled system by resistors R_0. The normalized circuit equations of the system are given as follows:

(A) In case of that switch is connected to $-g$,

$$\begin{cases} \dot{x}_n &= y_n \\ \dot{y}_n &= -x_n + 2\alpha y_n + \gamma(y_{n-1} - 2y_n + y_{n+1}), \end{cases} \quad (3)$$

(B) In case of that switch is connected to G,

$$\begin{cases} \dot{x}_n &= y_n \\ \dot{y}_n &= -x_n - 2\beta y_n + \gamma(y_{n-1} - 2y_n + y_{n+1}), \end{cases} \quad (4)$$

where $(n = 1, 2, 3)$ and $x_0 = x_3$, $x_4 = x_1$. In calculation result, in-phase synchronization state can be observed. When the coupling strength γ is larger than 0.1, full in-phase synchronization can be observed. However full in-phase synchronization can not be observed or synchronization is lost in case of small coupling strength γ.

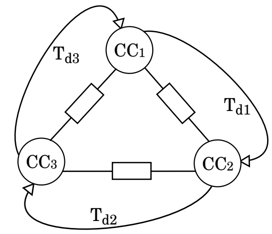


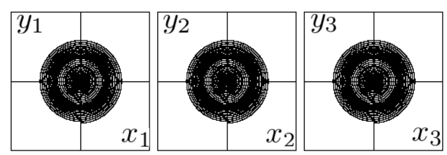
Fig. 3. System including time delay in one direction.

IV. Simulation Results

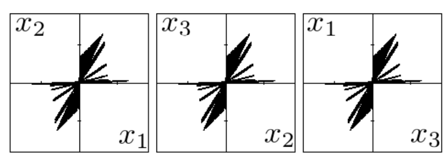
The proposed model has three time delay T_d. In this study, we consider that the coupled circuits have symmetric delay and asymmetric delay.

A. With symmetric delay effects

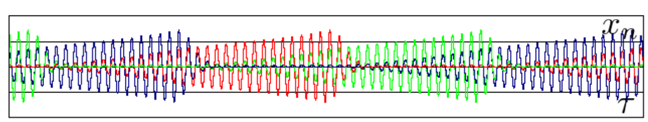
The simulation results are shown in the Figs. 4, 5 and 6. These figures can be obtained by difference of time delay T_{d1}, T_{d2} and T_{d3}. Figure 7 shows the magnified poincare section of switching synchronization with symmetric delay when T_{d1}, T_{d2} and T_{d3} change. Figure 8 shows the bifurcation and the cycle of switching synchronization with symmetric delay. The amplitude of x_n is switching sequentially in Figs. 4, 5 and 6 (c). These synchronization states are switching synchronization state. The order of switching amplitude of the loop is subcircuit1 - subcircuit3- subcircuit2. The amplitude is going divergence and convergence. Additionally the time of



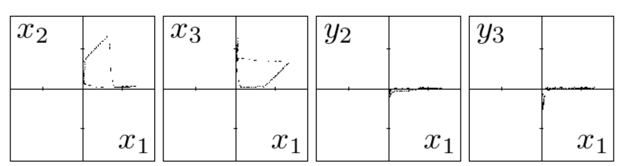
(a) Attractor



(b) Lissajous figure



(c) Time waveform



(d) Poincare section

Fig. 4. Simulation results with symmetric delay of $\alpha = 0.015$, $\beta = 0.5$, $\gamma' = 0.01$ and $T_{dn} = \pi$.

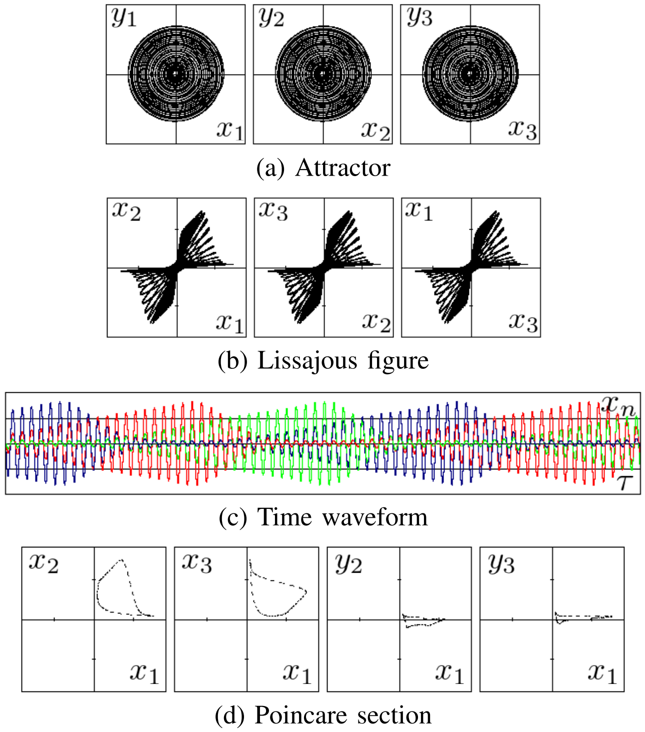


Fig. 5. Simulation results with symmetric delay of$\alpha = 0.015$, $\beta = 0.5$, $\gamma' = 0.01$ and $T_{dn} = 0.5\pi$.

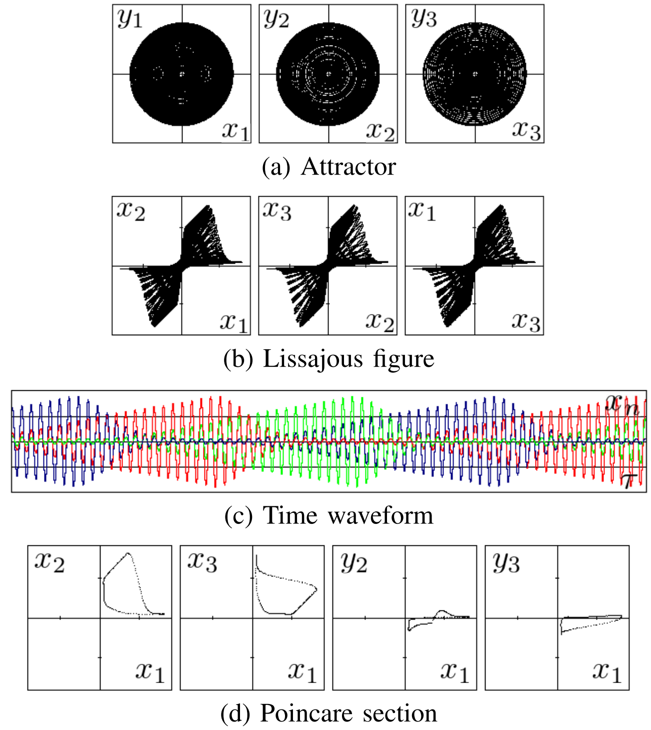


Fig. 6. Simulation results with symmetric delay of $\alpha = 0.015$, $\beta = 0.5$, $\gamma' = 0.01$ and $T_{dn} = 1.5\pi$.

divergence and convergence is different. Especially, the cycle of time waveforms is different in Figs. 4, 5 and 6 (c). Namely, the number of cycle between the maximum of the amplitude from next one is different. Figure 8 shows the bifurcation of switching synchronization with symmetric delay. When the attractor is high periodicity, the number of cycle is large.

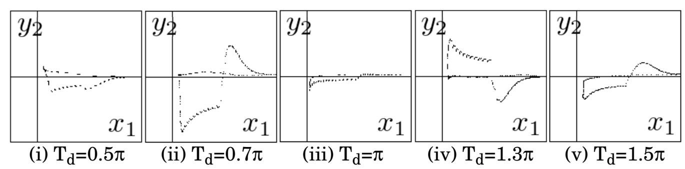


Fig. 7. Magnified poincare section with asymmetric delay of $\alpha = 0.015$, $\beta = 0.5$ and $\gamma' = 0.01$.

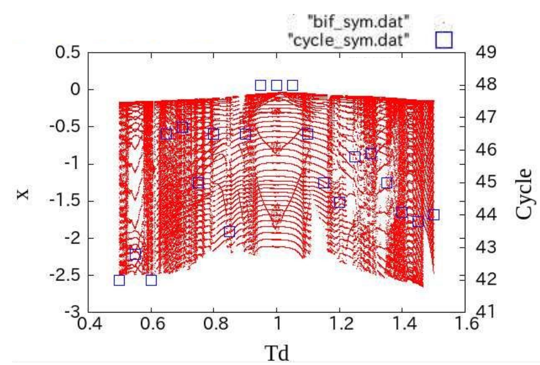


Fig. 8. Bifurcation and cycle with symmetric delay.

B. With asymmetric delay effects

Next we change T_{d1} and fix $T_{d2} = T_{d3} = 1.0\pi$. Figures. 9 and 12 show simulation results. We investigate cycle of switching synchronization with asymmetric delay. Figure 10 shows the magnified poincare section of switching synchronization with asymmetric delay. Figure 11 shows the bifurcation and the cycle of switching synchronization with asymmetric delay. This bifurcation is periodic. Figure 13 shows cycle of switching synchronization with symmetric or asymmetric delay. The variation of cycle with asymmetric delay is less than symmetric delay. The number of cycle is large when periodic bifurcation is observed. Namely, the cycle of switching synchronization state is stabilization when periodic solution can be observed.

V. Conclusion

In this study, we devised coupled systems that takes advantage of features of the time delayed chaotic circuit. We investigated synchronization state of a ring of coupled chaotic circuits with one-direction delay effects. As a result, some special synchronization states can be observed. We observed in-phase switching synchronization state. When time delay of subcircuits changes, the cycle of switching synchronization state changes. The switching of the amplitude can be observed by difference of time delay. The variation of cycle with symmtric delay is less than asymmtric delay. We consider the cycle of switching synchronization state is stabilization when periodic solution can be observed.

References

[1] G. Abramson, V. M. Kenkre and A. R. Bishop, "Ana-

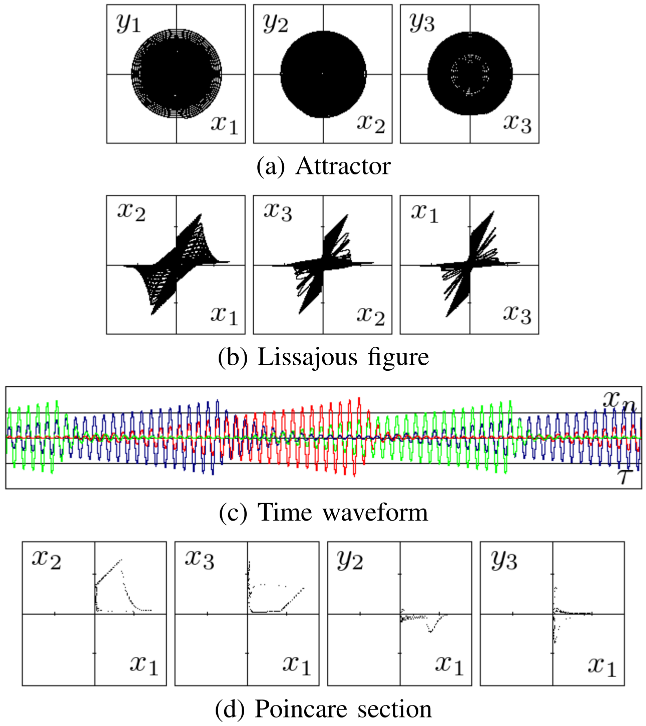


Fig. 9. Simulation results with asymmetric delay of $\alpha = 0.015$, $\beta = 0.5$, $\gamma' = 0.01$, $T_{d1} = 1.2\pi$ and $T_{d2} = T_{d3} = \pi$.

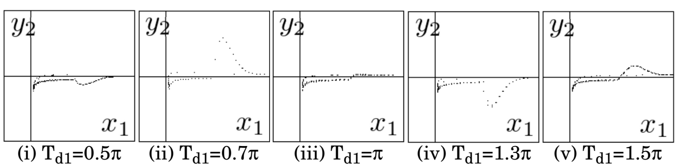


Fig. 10. Magnified poincare section with asymmetric delay of $\alpha = 0.015$, $\beta = 0.5$, $\gamma' = 0.01$ and $T_{d2} = T_{d3} = \pi$.

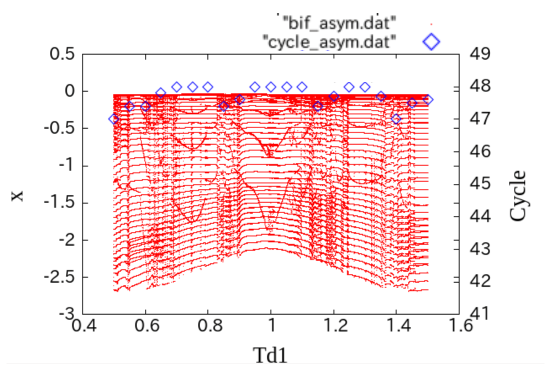


Fig. 11. Bifurcation and cycle with asymmetric delay.

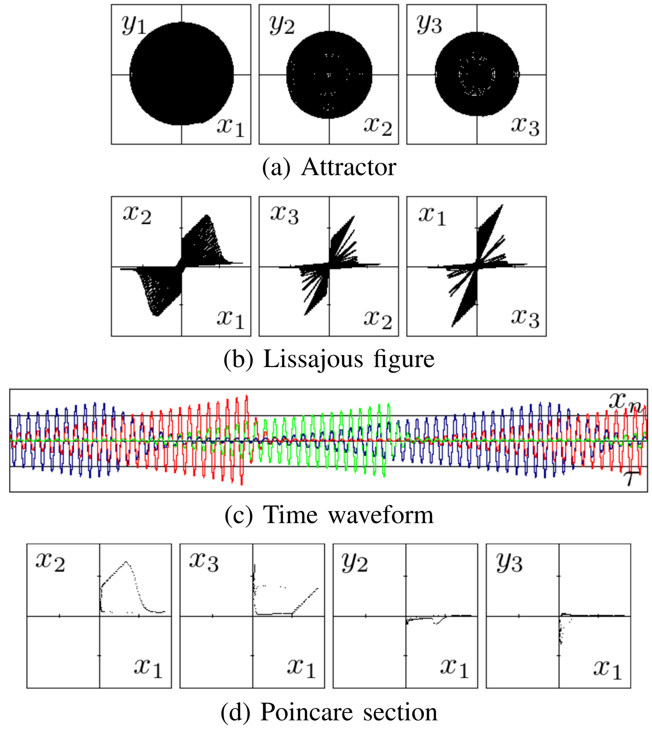


Fig. 12. Simulation results with asymmetric delay of $\alpha = 0.015$, $\beta = 0.5$, $\gamma' = 0.01$, $T_{d1} = 1.4\pi$ and $T_{d2} = T_{d3} = \pi$.

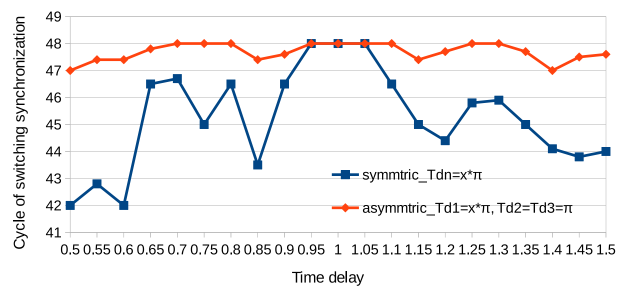


Fig. 13. Cycle of switching synchronization with symmetric or asymmetric delay.

lytic Solutions for Nonlinear Waves in Coupled Reactig Systems," Physical A: Statistial Mechanics and its Applications, vol. 305, no.3-4, pp. 427-436, 2002.

[2] C. M. Gray, "Synchronization Oscillators in Neural Systems: Mechanisms and Functions," J. Computational Neuroscience, vol. 1, pp. 11-38, Feb. 1994.

[3] Y. Zhang, G. H. Du and J. J. Jiang, "Synchronization Chaos by Impulsive Feedback Method," International Jounal of bifurcation and Chaos, vol. 11, no.8, pp. 2233-2243, 2001.

[4] X. Liu, "Stability of impulsive control systems with time delay. Mathematical and Computer Modelling," Vol.39, pp.511-519, 2004.

[5] T. Maruyama, N. Inaba, Y. Nishio and S. Mori, "Chaos in an Auto Gain Controlled Oscillator Containing Time Delay," Trans. IEICE, vol. J 72-A, pp. 1814-1820, Nov. 1989.

[6] T. Maruyama, N. Inaba, Y. Nishio and S. Mori, "Chaos in Self Oscillator Circuit Containing Time Delay," Proceedings of IEEE Midwest Symposium on Circuits and Systems (MWSCAS'90), vol. 2, pp. 1055-1058, Aug. 1990.

[7] L. M. Pecora and T. L. Carroll, "Synchronization in Chaotic Systems," Physical Review Letters, vol. 64, no.8, pp. 821-824, Feb. 1990.