

# Firefly Algorithm Existing Leader Fireflies

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**Abstract**—The community of human-beings, such as company and laboratory, is created as core on the leader of the community. In this study, we propose Firefly Algorithm existing leader fireflies. In our proposed FA, leader fireflies attract other fireflies within certain distance. We compare our proposed FA to the conventional FA with benchmark functions of CEC 2013. Numerical experiments indicate our proposed FA is more efficient algorithm than the conventional FA.

## I. INTRODUCTION

Evolutionary Computing (EC) is a subfield of artificial intelligence (AI) in computer science, and is based on biological mechanisms of evolution. EC technique mainly involves metaheuristic optimization algorithms such as Evolutionary Algorithm (EA) and Swarm Intelligence (SI). SI algorithm is one of stochastic algorithms. Stochastic algorithms have a deterministic component and a random component. Algorithms having only the deterministic component are almost all local search algorithms. There is a risk to be trapped at local optima such algorithms. However, stochastic algorithms are possible to jump out such locality. SI algorithms are based on the behavior of animals and insects. Representative examples are Particle Swarm Optimization (PSO) [1], Ant Colony Optimization (ACO), and Firefly Algorithm (FA) [2]–[4].

Human-beings belong to sub-society, which is called community such as company, laboratory and so on. The social animal creates a society and lives in the society. In addition, it is also believed that the animals are centered around a leader and have the society in the community. In other words, the community is created as core on the leader of the community. Moreover, in creating the community, the human-beings have some tendencies that human-beings easily gather around the leader, such as a CEO controlling the company, a supervisor controlling the laboratory and so on. In previous study, Haraguchi et al. have proposed a new type of Self-Organizing Map (SOM) algorithm, which is called Community SOM (CSOM) algorithm [7]. CSOM is composed of some leader neurons and its neighborhood neurons. They have applied CSOM for clustering and data extraction to various input data. They have confirmed that the number of communities created by CSOM is the same as the number of clusters, and the each community size depends on the number

of the input data in the cluster and the shape of the cluster. By numerical experiments, they have confirmed the effectiveness of CSOM in the application to the cluster extraction.

In this study, we propose new FA existing leader fireflies. These leader fireflies attract other fireflies within certain distance. We compare our proposed FA to the conventional FA with 28 benchmark functions of Congress on Evolutionary Computation (CEC) 2013 [6]. Numerical experiments indicate our proposed FA is more efficient algorithm than the conventional FA.

This study is organized as follows: first, we explain the conventional Firefly Algorithm in Section II, and then, we describe in detail of our proposed method in Section III. Followed by, we show numerical experiments. Finally, we conclude in this study.

## II. THE CONVENTIONAL FIREFLY ALGORITHM (FA)

Firefly Algorithm (FA) has been developed by Yang in 2007, and it was based on the idealized behavior of the flashing characteristics of fireflies. The conventional FA is idealized these flashing characteristics as the following three rules

- All fireflies are unisex so that one firefly is attracted to other fireflies regardless of their sex;
- Attractiveness is proportional to their brightness, thus for any two flashing fireflies, the less brighter one will move towards the brighter one. The attractiveness is proportional to the brightness and they both decrease as their distance increases. If no one is brighter than a particular firefly, it moves randomly;
- The brightness or light intensity of a firefly is affected or determined by the landscape of the objective function to be optimized.

Attractiveness of firefly  $\beta$  is defined by

$$\beta = (\beta_0 - \beta_{min})e^{-\gamma r_{ij}^2} + \beta_{min} \quad (1)$$

where  $\gamma$  is the light absorption coefficient,  $\beta_0$  is the attractiveness at  $r_{ij} = 0$ ,  $\beta_{min}$  is minimum value of  $\beta$ , and  $r_{ij}$  is the distance between any two fireflies  $i$  and  $j$  at  $\mathbf{x}_i$  and  $\mathbf{x}_j$ . The

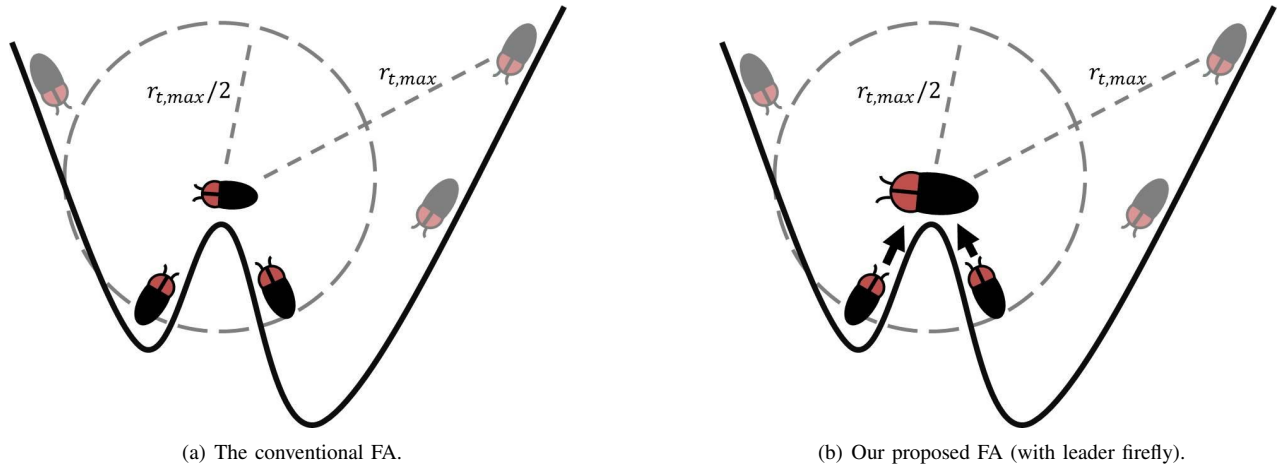


Fig. 1. Firefly operation with/without leader firefly.

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### Algorithm 1 Firefly Algorithm

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Objective function  $f(x)$ ,  $x = (x_1, \dots, x_d)^T$   
Initialize a population of fireflies  $x_i (i = 1, 2, \dots, n)$   
Define light absorption coefficient  $\gamma$   
**while**  $t < MaxGeneration$  **do**  
  **for**  $i = 1$  to  $n$ , all  $n$  fireflies **do**  
    **for**  $j = 1$  to  $n$ , all  $n$  fireflies **do**  
      Light intensity  $I_j$  at  $x_j$  is determined by  $f(x_j)$   
      **if**  $I_j > I_i$  **then**  
        Move firefly  $i$  towards  $j$  in all  $d$  dimensions  
      **end if**  
      Attractiveness varies with distance  $r$  via  $exp[-\gamma r]$   
      Evaluate new solutions and update light intensity  
    **end for**  
  **end for**  
  Rank the fireflies and find the current best  
**end while**  
Postprocess results and visualization

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movement of the firefly  $i$  is attracted to another more attractive firefly  $j$ , and is determined by

$$\mathbf{x}_i = \mathbf{x}_i + \Delta \mathbf{x}, \quad (2)$$

$$\Delta \mathbf{x} = \beta(\mathbf{x}_j - \mathbf{x}_i) + \alpha \epsilon_i, \quad (3)$$

where  $\mathbf{x}_i$  is the position vector of firefly  $i$ ,  $\epsilon_i$  is the vector of random variable, and  $\alpha(t)$  is the randomization parameter. The parameter  $\alpha(t)$  is defined by

$$\alpha(t) = \alpha(0) \left( \frac{10^{-4}}{0.9} \right)^{t/t_{max}}, \quad (4)$$

where  $t$  is the number of iteration.

Algorithm 1 shows pseudo code of the conventional FA for minimum optimization problems.

### III. FIREFLY ALGORITHM EXISTING LEADER FIREFLIES

Human-beings belong to sub-society, which is called community such as company, laboratory and so on. The community is created as core on the leader of the community.

Moreover, in creating the community, the human-beings have some tendencies that human-beings easily gather around the leader, such as a CEO controlling the company, a supervisor controlling the laboratory and so on. Therefore, we propose new FA existing leader fireflies in this study. These leader fireflies are defined from the beginning at random. In our proposed FA, normal fireflies and leader fireflies are attracted by the following three steps

- Normal fireflies are attracted to another more attractive normal fireflies;
- Leader fireflies are attracted to more attractive normal fireflies;
- Normal fireflies are attracted to leader fireflies regardless of their attractiveness, if the distance between normal firefly and leader firefly is  $r_{t,max}/2$  or less.

We define that  $r_{t,max}$  is the maximum distance from leader firefly at iteration  $t$ . In case of solving optimization problems by the conventional FA, the less attractive fireflies are attracted to the more attractive fireflies. There is a risk to be trapped at local optima. However, normal fireflies are attracted to less attractive leader fireflies in our proposed FA. We assume that normal fireflies falling into local optima escape by attracted to leader fireflies.

Figure 1 shows the concept of our proposed FA when the number of leader fireflies is 1. Big firefly indicates leader firefly and small fireflies indicate normal firefly. In Fig. 1(b), two normal fireflies within  $2/r_{t,max}$  from leader firefly are attracted to leader firefly.

### IV. NUMERICAL EXPERIMENTS

We compare our proposed FA to the conventional FA with benchmark functions of Congress on Evolutionary Computation (CEC) 2013 (see Table I).

The optimal solutions  $\mathbf{x}^*$  of these benchmark functions is shifted from 0, and the global optima  $f(\mathbf{x}^*)$  are not equal to 0. The search range of these functions is  $[-100, 100]^D$ , and

TABLE I  
2013 CEC BENCHMARK FUNCTIONS

No.	Name	$f(\mathbf{x}^*)$
Unimodal Functions		
1	Sphere function	-1400
2	Rotated High Conditioned Elliptic Function	-1300
3	Rotated Bent Cigar Function	-1200
4	Rotated Discus Function	-1100
5	Different Powers Function	-1000
Basic Multimodal Functions		
6	Rotated Rosenbrock's Function	-900
7	Rotated Schaffers F7 Function	-800
8	Rotated Ackley's Function	-700
9	Rotated Weierstrass Function	-600
10	Rotated Griewank's Function	-500
11	Rastrigin's Function	-400
12	Rotated Rastrigin's Function	-300
13	Non-Continuous Rotated Rastrigin's Function	-200
14	Schwefel's Function	-100
15	Rotated Schwefel's Function	100
16	Rotated Katsuura Function	200
17	Lunacek Bi Rastrigin Function	300
18	Rotated Lunacek Bi Rastrigin Function	400
19	Expanded Griewank's plus Rosenbrock's Function	500
20	Rotated Expanded Scaffer's F6 Function	600
Composition Functions		
21	Composition Function 1 (n=5, Rotated)	700
22	Composition Function 2 (n=3, Unrotated)	800
23	Composition Function 3 (n=3, Rotated)	900
24	Composition Function 4 (n=3, Rotated)	1000
25	Composition Function 5 (n=3, Rotated)	1100
26	Composition Function 6 (n=5, Rotated)	1200
27	Composition Function 7 (n=5, Rotated)	1300
28	Composition Function 8 (n=5, Rotated)	1400

TABLE II  
THE NUMBER OF LEADER FIREFLIES

number	1	2	3	4	5
unimodal	3	2	3	2	2
multimodal	9	6	4	6	5
composition	5	5	2	5	4
total	17	13	9	13	12

the dimension  $N$  is 30. The total number of fireflies is also 30. Each numerical experiment is run 50 times. In each test functions, the maximum number of iterations  $t_{max}$  is 1500.

We compare the number that our proposed FA wins the conventional FA in the comparison of average error value, when the number of leader fireflies is changed from 1 to 5 (see Table II).

Table II shows that the best number of leader fireflies is 1. When the number of leader fireflies is changed from 1 to 5, the results of basic multimodal functions change significantly. Only when the number of leader fireflies is 1, our proposed FA is more efficient algorithm than the conventional FA. Basic multimodal functions have a lot of local optima. We assume that fireflies of our proposed FA jump out locality easily. It is

because leader fireflies help normal fireflies falling into locality to be got out.

Table III shows the average error value, minimum value, maximum value and standard deviation of the conventional FA and our proposed FA when the number of leader fireflies is 1 (see Table III). Our proposed FA improves average error value of 5 functions ( $f_5, f_9, f_{19}, f_{21}, f_{27}$ ) nearly 5 percent. Moreover, improvement rates of  $f_3$  and  $f_{20}$  are more than 25 percent.

## V. CONCLUSION

In this study, we have proposed new Firefly Algorithm. In our proposed FA, leader fireflies exist. These leader fireflies attract normal fireflies within certain distance. We have compared our proposed FA to the conventional FA with 28 benchmark functions of 2013 Congress on Evolutionary Computation (CEC). Numerical experiments have indicated our proposed FA is more efficient algorithm than the conventional FA.

In the future work, we compare our proposed FA to another improved algorithms, and try to improve the rules of leader fireflies.

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TABLE III  
NUMERICAL EXPERIMENTS

$f$		FA	Proposed FA
$f_1$	avg	$6.48 \times 10^{-4}$	$6.58 \times 10^{-4}$
	min	$3.37 \times 10^{-4}$	$4.78 \times 10^{-4}$
	max	$1.06 \times 10^{-3}$	$1.03 \times 10^{-3}$
	std	$1.44 \times 10^{-4}$	$1.23 \times 10^{-4}$
$f_2$	avg	$1.08 \times 10^7$	<b><math>1.05 \times 10^7</math></b>
	min	$4.34 \times 10^6$	$2.29 \times 10^6$
	max	$2.48 \times 10^7$	$2.62 \times 10^7$
	std	$4.45 \times 10^6$	$4.38 \times 10^6$
$f_3$	avg	$1.25 \times 10^7$	<b><math>9.01 \times 10^6</math></b>
	min	$1.46 \times 10^4$	$1.44 \times 10^3$
	max	$7.17 \times 10^7$	$6.73 \times 10^7$
	std	$1.37 \times 10^7$	$1.31 \times 10^7$
$f_4$	avg	<b><math>1.15 \times 10^5</math></b>	$1.28 \times 10^5$
	min	$7.19 \times 10^4$	$7.04 \times 10^4$
	max	$1.80 \times 10^5$	$1.96 \times 10^5$
	std	$2.39 \times 10^4$	$2.64 \times 10^4$
$f_5$	avg	$3.89 \times 10^1$	<b><math>3.72 \times 10^1</math></b>
	min	$1.96 \times 10^{-2}$	$2.00 \times 10^{-2}$
	max	$1.32 \times 10^2$	$9.01 \times 10^1$
	std	$2.75 \times 10^1$	$2.06 \times 10^1$
$f_6$	avg	$2.73 \times 10^1$	<b><math>2.73 \times 10^1</math></b>
	min	$2.55 \times 10^1$	$2.57 \times 10^1$
	max	$2.89 \times 10^1$	$2.93 \times 10^1$
	std	$7.01 \times 10^{-1}$	$7.70 \times 10^{-1}$
$f_7$	avg	$1.16 \times 10^1$	<b><math>1.16 \times 10^1</math></b>
	min	$9.06 \times 10^{-1}$	$1.44 \times 10^0$
	max	$2.88 \times 10^1$	$2.68 \times 10^1$
	std	$6.00 \times 10^0$	$6.21 \times 10^0$
$f_8$	avg	$2.14 \times 10^1$	<b><math>2.14 \times 10^1</math></b>
	min	$2.10 \times 10^1$	$2.12 \times 10^1$
	max	$2.16 \times 10^1$	$2.16 \times 10^1$
	std	$9.32 \times 10^{-2}$	$8.55 \times 10^{-2}$
$f_9$	avg	$1.02 \times 10^1$	<b><math>9.70 \times 10^0</math></b>
	min	$4.81 \times 10^0$	$4.72 \times 10^0$
	max	$1.72 \times 10^1$	$1.47 \times 10^1$
	std	$2.54 \times 10^0$	$2.21 \times 10^0$
$f_{10}$	avg	<b><math>4.18 \times 10^{-1}</math></b>	$5.26 \times 10^{-1}$
	min	$5.47 \times 10^{-2}$	$7.66 \times 10^{-2}$
	max	$1.47 \times 10^0$	$1.88 \times 10^0$
	std	$3.46 \times 10^{-1}$	$4.76 \times 10^{-1}$
$f_{11}$	avg	<b><math>2.68 \times 10^1</math></b>	$2.83 \times 10^1$
	min	$1.29 \times 10^1$	$8.96 \times 10^0$
	max	$4.68 \times 10^1$	$5.57 \times 10^1$
	std	$7.92 \times 10^0$	$7.95 \times 10^0$
$f_{12}$	avg	<b><math>2.60 \times 10^1</math></b>	$2.71 \times 10^1$
	min	$1.29 \times 10^1$	$1.39 \times 10^1$
	max	$4.48 \times 10^1$	$5.37 \times 10^1$
	std	$6.95 \times 10^0$	$8.02 \times 10^0$
$f_{13}$	avg	<b><math>6.68 \times 10^1</math></b>	$7.54 \times 10^1$
	min	$3.74 \times 10^1$	$3.01 \times 10^1$
	max	$1.19 \times 10^2$	$1.47 \times 10^2$
	std	$1.84 \times 10^1$	$2.40 \times 10^1$
$f_{14}$	avg	$2.21 \times 10^3$	<b><math>2.16 \times 10^3</math></b>
	min	$7.39 \times 10^2$	$1.03 \times 10^3$
	max	$3.70 \times 10^3$	$3.59 \times 10^3$
	std	$4.53 \times 10^2$	$5.55 \times 10^2$
$f_{15}$	avg	<b><math>2.19 \times 10^3</math></b>	$2.33 \times 10^3$
	min	$6.82 \times 10^2$	$1.17 \times 10^3$
	max	$3.98 \times 10^3$	$3.77 \times 10^3$
	std	$6.04 \times 10^2$	$5.46 \times 10^2$
$f_{16}$	avg	$9.78 \times 10^{-2}$	<b><math>9.72 \times 10^{-2}</math></b>
	min	$4.14 \times 10^{-2}$	$4.23 \times 10^{-2}$
	max	$1.99 \times 10^{-1}$	$2.11 \times 10^{-1}$
	std	$3.29 \times 10^{-2}$	$3.67 \times 10^{-2}$
$f_{17}$	avg	<b><math>5.82 \times 10^1</math></b>	$6.05 \times 10^1$
	min	$4.50 \times 10^1$	$4.44 \times 10^1$
	max	$7.78 \times 10^1$	$8.50 \times 10^1$
	std	$7.09 \times 10^0$	$7.47 \times 10^0$
$f_{18}$	avg	$6.36 \times 10^1$	<b><math>6.33 \times 10^1</math></b>
	min	$4.61 \times 10^1$	$4.79 \times 10^1$
	max	$7.95 \times 10^1$	$8.89 \times 10^1$
	std	$7.33 \times 10^0$	$7.85 \times 10^0$
$f_{19}$	avg	$3.65 \times 10^0$	<b><math>3.50 \times 10^0</math></b>
	min	$2.03 \times 10^0$	$2.04 \times 10^0$
	max	$5.15 \times 10^0$	$4.79 \times 10^0$
	std	$6.58 \times 10^{-1}$	$7.10 \times 10^{-1}$
$f_{20}$	avg	$1.50 \times 10^1$	<b><math>5.40 \times 10^0</math></b>
	min	$1.50 \times 10^1$	$2.04 \times 10^0$
	max	$1.50 \times 10^1$	$1.50 \times 10^1$
	std	$8.75 \times 10^{-13}$	$4.24 \times 10^0$
$f_{21}$	avg	$3.36 \times 10^2$	<b><math>3.19 \times 10^2</math></b>
	min	$2.00 \times 10^2$	$2.00 \times 10^2$
	max	$4.44 \times 10^2$	$4.44 \times 10^2$
	std	$7.92 \times 10^1$	$7.86 \times 10^1$
$f_{22}$	avg	<b><math>3.01 \times 10^3</math></b>	$3.23 \times 10^3$
	min	$1.57 \times 10^3$	$1.31 \times 10^3$
	max	$6.00 \times 10^3$	$6.05 \times 10^3$
	std	$1.01 \times 10^3$	$1.09 \times 10^3$
$f_{23}$	avg	$4.05 \times 10^3$	<b><math>3.98 \times 10^3</math></b>
	min	$1.82 \times 10^3$	$1.37 \times 10^3$
	max	$5.63 \times 10^3$	$5.69 \times 10^3$
	std	$9.18 \times 10^2$	$1.02 \times 10^3$
$f_{24}$	avg	$2.22 \times 10^2$	<b><math>2.18 \times 10^2</math></b>
	min	$2.01 \times 10^2$	$2.01 \times 10^2$
	max	$2.39 \times 10^2$	$2.40 \times 10^2$
	std	$1.15 \times 10^1$	$1.24 \times 10^1$
$f_{25}$	avg	$2.35 \times 10^2$	<b><math>2.35 \times 10^2</math></b>
	min	$2.20 \times 10^2$	$2.19 \times 10^2$
	max	$2.61 \times 10^2$	$2.55 \times 10^2$
	std	$8.69 \times 10^0$	$8.22 \times 10^0$
$f_{26}$	avg	<b><math>2.84 \times 10^2</math></b>	$3.10 \times 10^2$
	min	$1.27 \times 10^2$	$2.00 \times 10^2$
	max	$3.30 \times 10^2$	$3.37 \times 10^2$
	std	$5.19 \times 10^1$	$2.48 \times 10^1$
$f_{27}$	avg	<b><math>4.39 \times 10^2</math></b>	$4.70 \times 10^2$
	min	$3.09 \times 10^2$	$3.13 \times 10^2$
	max	$6.72 \times 10^2$	$7.10 \times 10^2$
	std	$1.12 \times 10^2$	$1.06 \times 10^2$
$f_{28}$	avg	$3.16 \times 10^2$	<b><math>3.02 \times 10^2</math></b>
	min	$1.01 \times 10^2$	$1.01 \times 10^2$
	max	$1.44 \times 10^3$	$1.37 \times 10^3$
	std	$1.65 \times 10^2$	$1.64 \times 10^2$
$f$		FA	Proposed FA
best solution		11	17